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The Gift of

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THE

DOCTRINE

OF

Combinations, Permutations,

AND

COMPOSITIONS

OF

QUANTITIES,

Clearly and ſuccinctly demonſtrated.

In tenui labor ——— VIRG.

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THIS ſmall Tract was intended to make a part of the Treatiſe of Algebra; but other matters had ſwelled the book to too large a ſize, to admit of this; which therefore was poſtponed to a further opportunity.

The Subject of it, which is the Doctrine of Combinations and Alternations, is a very entertaining and curious ſpeculation; by which, a great many very delightful problems may be reſolved: which will appear ſtrange and ſurprizing to them that are not verſed with theſe ſorts of calculations. The Doctrine thereof is here compleatly delivered, and in a little compaſs; and the rules demonſtrated in the plaineſt and eaſieſt manner, and therefore I hope it will prove beneficial and acceptable to the reader.

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Professor William H. Butts
10-14-1930

THE CONTENTS.

The Doctrine of Combinations, &c.

Page

Definitions — 1
Combinations, &c. — 2
Compoſition — 25

Chronology; or the Art of Reckoning Time.

Definitions — 7
Chronology — 8

Calculation, Libration, &c.

Definitions — — 1
Part I. Computing and weighing — 22
Part II. Geometrical Problems, and the meaſuring of Lines — — 45
Part III. The meaſuring of Areas and Surfaces 70
Part IV. The meaſuring of Solids — 95
Part V. Gauging — 131

The Art of Surveying.

Surveying — — 1
Appendix — — 1

DEFI-

Combinations, Permutations, &c.

DEFINITIONS.

DEFIN. I.

THE *Combination* of quantities, is ſhewing how oft a leſs number of quantities or things can be taken out of a greater, and combined together, without conſidering their places, or the order they ſtand in. This is alſo called *Election* or *Choice*.

Here every parcel muſt be different from all the reſt, and no two is to have preciſely the ſame quantities or things.

DEF. II.

Permutation, ſhews how many ſeveral ways, the places of any given number of things may be changed. This is alſo called *Variation*, *Alternation*, and *Changes*.

Here the only thing to be regarded is the order they ſtand in; for no two parcels are to have all the quantities placed in the ſame ſituation.

DEF. III.

Compoſition, is the taking a given number of quantities, out of as many equal rows of different quantities, one out of every row; and combining them together.

Here no regard is had to their places; and it differs from combination, in which there is but one row of things.

DEF. IV.

Combinations of the *ſame form*, are thoſe wherein are the ſame number of quantities, and the ſame repetitions. As *abcc*, *bbad*, *cffg*, &c. are of the ſame form. Alſo AB^3C^2, DA^2E^3, C^3ED^2, are of the ſame form; putting the numbers 2, 3 for the repetitions. But ABBC, AB^3, AACC, are of different forms.

Here obſerve, when any quantity is written with an index at the top, it ſignifies ſo many repetitions of that quantity, as a^2 ſignifies *aa*; a^3 ſignifies *aaa*; and b^4 ſignifies *bbbb*, &c.

PROP. I. *Prob.*

To find the number of Permutations or Changes of m things, all different from one another.

RULE.

Multiply continually, 1 × 2 × 3 × 4 × 5, &c. to *m* terms; and the product is the number of changes.

For 1 thing *a*, is capable but of one poſition *a*. And if there be 2 things *a* and *b*; they are only capable of theſe 2 variations *ab*, *ba*; whoſe number is 1 × 2.

If there be 3 things *a*, *b*, *c*; then any 2 of them, leaving out the 3d, will have 1 × 2 variations; therefore when theſe 3 are taken in, there will be 1 × 2 × 3 variations.

And if there be 4 things, every 3, leaving out the 4th, will have 1 × 2 × 3 variations. Then taking in ſucceſſively theſe four left out, and there will be 1 × 2 × 3 × 4 variations. And ſo on as far as you pleaſe.

Cor. 1. *$m \times$ changes of $m-1$ things $=$ changes of m things.*

Cor. 2. *The changes of $m-1$ things out of $m =$ changes of m things.*

For the changes of *abc* $= 1 \times 2 \times 3$, and taking in *d*, the changes of any 3 of *abcd* $= 1 \times 2 \times 3 + 3 \times 1 \times 2 \times 3 = 1 \times 2 \times 3 \times 4 =$ changes of *abcd*. And the ſame of others.

Example 1.

How many changes may be made of the words in this verſe? Tot tibi ſunt dotes, virgo, quot ſydera cœlo.

Anſ. $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$.

Example 2.

How many changes may be rung on 6 bells?

Anſ. $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ changes.

Examp. 3.

For how many days can 7 perſons be placed in a different poſition at dinner?

Anſ. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$; near 14 years.

PROP. II.

The number of permutations of m things taken n and n, or n at a time; is equal to m changes of $m-1$ things taken $n-1$ at a time.

For ſuppoſe theſe 5 things *abcde*, and firſt leave out *a*; then we ſhall have the four *bcde*, out of which let there be taken all the 2's, *bc*, *bd*, &c. and put $v =$ number of all the variations of every 2, out of the four quantities *bcde*.

Now let *a* be put in the firſt place of each of them, which will make *abc*, *abd*, &c; then each will conſiſt of three letters; and the number of changes will remain the ſame, whilſt *a* poſſeſſes the firſt place; that is, $v =$ number of variations of every 3 out of the 5 *abcde*, when *a* is firſt.

In like manner, if *b*, *c*, *d*, *e* be ſucceſſively left out, the number of the variations of all the 2's will alſo be v; and, putting *b*, *c*, *d*, *e* in the firſt place to make 3 quantities out of 5, there will ſtill be v variations, as before. But theſe are all the variations that can happen of 3 things out of 5, when *a*, *b*, *c*, *d*, *e* are ſucceſſively put firſt; and therefore the ſum of all theſe is the ſum of all the changes of 3 things out of 5. But the ſum of theſe is ſo many times v as is the number of things; that is, $5v$ or $mv =$ all the changes of 3 things out of 5. And it is evident, the ſame way of reaſoning may be applied to any numbers, m, n.

PROP. III. *Prob.*

Any number m of different things being given; to find how many permutations or changes can be made out of them, taken n by n, or n quantities at a time.

RULE.

Multiply continually $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times$ &c. to n terms, for the changes.

Suppoſe any number of things *a b c d e f g*; here $m = 7$, and let $n = 3$. Subſtract 1 from 3 and there remains 2, ſubtract 2 from 7 and there remains 5. Then ſet the numbers as follows,

n m
1 —— 5
2 —— 6
3 —— 7

Here it is evident, that the number of changes, made 1 by 1 out of 5 things, will be $5 = v$.

Then (Prop. II.) when $m = 6$, $n = 2$, the number of changes $= mv = 6 \times 5 = v$, a ſecond time.

Again, when $m = 7$, $n = 3$, the number of changes $= mv = 7 \times 6 \times 5$, that is, $= m \times \overline{m - 1} \times \overline{m - 2}$, continued to 3 or n terms. And the like may be ſhewn for any other numbers.

Cor. 1. *If* $V =$ *all the variations or changes of* m *things, taken* n *at a time; then,* $\overline{m - n} \times V =$ *all the changes of* m *things, taken* $n + 1$ *at a time.*

For when $m = 7$, $n = 3$, $V = 7 \times 6 \times 5$, and $m - n = 4$. But by this Prop. making $n = 4$, then the changes will be $= 7 \times 6 \times 5 \times 4$; that is, $= V \times \overline{m - n}$. And ſo for all other numbers.

Cor. 2. $nV = m \times$ *changes of* n *things out of* m $-$ *the changes of* $n + 1$ *things out of* m.

For $mV = m \times$ changes of n things out of m; from which ſubtracting $\overline{m - n}$. V (Cor. 1.) gives nV as above.

Ex. 1.

How many changes can be rung with three bells out of eight?

Anſ. $8 \times 7 \times 6 = 336$.

Ex. 2.

How many words can be made with 5 *letters of the alphabet, admitting all conſonants?*

Anſ. $24 \times 23 \times 22 \times 21 \times 20 = 5100480$.

PROP. IV. *Prob.*

Any number of things m, being given; whereof there are p things of one ſort, and q things of another, &c. To find how many changes can be made out of them all?

RULE.

Multiply and divide according to this formula.

$$\frac{1 \times 2 \times 3 \times 4 \times 5 \ldots \text{to } m}{1 \times 2 \times 3 \times \ldots \text{to } p \times 1 \times 2 \times 3 \times \ldots \text{to } q \text{ \&c.}}$$

For ſuppoſe *ab* two quantities, then there will be two changes, *ab*, *ba*. But if for *b*, you put *a*, then *aa* admits but of one alternation $= \frac{2.1}{2.1} = 1$. For *ab* and *ba*, being two variations, become *aa* and *aa*, which therefore will be but one.

In like manner *abc* affords ſix variations, but if it be reduced to *aaa* or a^3, then theſe ſix variations become but one $= \frac{1 \times 2 \times 3}{1 \times 2 \times 3} = 1$. And if *abc* be reduced to *aac*, the ſix variations will be reduced to only theſe three, *aac*, *caa*, *aca*; but half ſo many as before $= \frac{1 \times 2 \times 3}{1 \times 2} = 3$.

In like manner, if all theſe be different *abcd*, there will be 24 variations; but if they be all the ſame, as *aaaa* or a^4, then all theſe 24 will be but one $= \frac{1.2.3.4}{1.2.3.4}$. And if they be reduced to *aaab*, three of them being the ſame, there will but be four variations *aaab*, *aaba*, *abaa*, and *baaa* $= \frac{1 \times 2 \times 3 \times 4}{1 \times 2 \times 3} = 4$. And if the four quantities be *aabc*, the 24 will be reduced to $12 = \frac{1 \times 2 \times 3 \times 4}{1 \times 2} = 12$.

$= 12$; for it was ſhewn before that *aab* is reduced to 3, and ſince *c* may have four poſitions among the quantities *aab*; therefore the variations of all four *aabc*, will be 4×3 or 12.

After the ſame manner if we have *abbccc*, if they are all different, they will admit of 720 changes, but becauſe *b* occurs twice, and *c* thrice, it muſt be divided by 2, and then by 6, and the quotient will be $60 = \frac{1\times2\times3\times4\times5\times6}{1\times2\times1\times2\times3}$, as is eaſily tried. And the like may be ſhewn of any other ſuch quantities.

Cor. *Hence if any index p occurs twice or oftener, the ſeries muſt be divided by* 1.2.3 ... *to p twice, or as oft as that index occurs, in different quantities.*

For in that caſe we ſhall have a^3b^3, or *aaa bbb*, where *p* is 3. Or a^4b^4, where *p* is 4.

Examp. 1.

How many variations may be made of the letters in the word Bacchanalia?

Anſ. It will be in this form *abcdeffgggg*, and the changes will be

$$\frac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11}{1\times2\times1\times2\times3\times4.} =$$

831600.?

Ex. 2.

How many different numbers will the figures following make, 1220005555?

Anſ. $\frac{1\times2.3\times4\times5\times6\times7\times8\times9\times10}{\overline{1\times2}\times\overline{1\times2\times3}\times\overline{1\times2\times3\times4}}$

$= 12600.$

PROP. V. *Prob.*

To find the number of permutations of m *things, taken* n *at a time; in which there are* p *things of one ſort,* q *things of another,* &c.

RULE.

Firſt find all the different forms of combination of theſe m things, taken n at a time: and then how many combinations there are of each form.

Then by Prop. IV. find the number of changes in any form, which multiply by the number of combinations in that form.

Repeat the ſame for every diſtinct form; and the ſum of all the products gives the whole ſum of the changes required; as is evident from the nature of the Problem.

All the difficulty here is to find the combinations required; and this is eaſily done, when the quantities are but few, by joining together every two, every three, &c. till you come at the number n; and then reckoning how many different forms there are, having n at a time, and how many in each form.

But when the quantities are many and often repeated, this method would be impracticable; and then we muſt have recourſe to Prop. VIII. following to find theſe combinations.

Examp. 1.

How many alternations or changes can be made of every four letters out of theſe eight, aaabbbcc?

Set them down as if you were going to multiply them, and all the products accordingly; omitting ſuch as happen to be the ſame as ſome of the foregoing, and ſuch as are out of the queſtion.

a b c

a b c
a b c

aa ab ac bb bc cc
a b c

aaa aab aac abb abc acc bbb bbc bcc
a b c

aaab aaac aabb aabc aacc abbb abbc
abcc bbbc bbcc.

Here we have four of this form *aaab*, whoſe number of changes is 4, by Prop. IV.

And three of this form *aabb*, number of changes 6.

And three of this form *aabc*, number of changes 12.

Therefore $4 \times 4 = 16$
$3 \times 6 = 18$
$3 \times 12 = 36$

Number of changes 70

Examp. 2.

How many changes can be made of eight letters out of theſe ten; $a^4 b^2 c^2 de$?

Here the ſum of the indices muſt always be 8. And all the forms being found by Prop. VIII. following, and the combinations in each; put them down as in the following table, writing the indices of the quantities for the repetitions. Then find the changes (by Prop. IV.) for every eight, in theſe different forms, and multiply them ſeverally by the number of combinations in each form, as in the laſt column.

Forms.

Forms.	Numb. Combi.	Chang.	Chang. multip.
422	1	420	420
4211	6	840	5040
41111	1	1680	1680
3221	2	1680	3360
32111	2	3360	6720
22211	1	5040	5040
Sum of all the Changes,			22260

Ex. 3.

How many different numbers can be made out of 1 unit, two two's, 3 three's, 4 four's, and 5 five's; taken five at a time.

Put *a*, *b*, *c*, *d*, *e* for 5, 4, 3, 2, 1; then the things given will be $a^5\ b^4\ c^3\ d^2\ e$. Then by Cor. Prop. VIII. following, find all the different forms, theſe five things are capable of, and by that Prop. the number of combinations in each form; which ſet down as below. Then the changes in each form, being found by Prop. IV. muſt be multiplied into theſe combinations, as follows.

Indices.	Forms.	Combinations.
54321	5	1
$m = 15$	41	8
$n = 5$	32	9
$L = 5.$	311	18
	221	18
	2111	16
	11111	1

Then the number of alternations or changes will be,

1 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times 1 = 1 \times 1 = 1$

2 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 1} \times 8 = 5 \times 8 = 40$

3 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \times 5}{1 \cdot 2 \cdot 3 \times 1 \cdot 2} \times 9 = 10 \times 9 = 90$

4 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3 \times 1 \times 1} \times 18 = 20 \times 18 = 360$

5 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \times 1 \cdot 2 \times 1} \times 18 = 30 \times 18 = 540$

6 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \times 1 \times 1 \times 1} \times 16 = 60 \times 16 = 960$

7 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1} \times 1 = 120 \times 1 = 120$

Number of all the changes, 2111

Note, if 10 be to be taken at a time, the number of the forms of combination would be the ſame; but the combinations themſelves would be different.

PROP. VI.

If C be the number of combinations of m different things taken n at a time; then $\frac{m-n}{n+1}C$ *will be the number of combinations, taken* $n+1$ *at a time.*

Let *abcde* be the different things propoſed. Then all the different combinations, taken by one's, by two's, by three's, &c. will be as follows.

a ab

a	*ab*	4	*abc*	3	*abcd*	*abcde*
b	*ac*		*abd*		*abce*	—
c	*ad*		*abe*		*abde*	1
d	*ae*		*acd*	2	*acde*	
e	*bc*	3	*ace*		*bcde*	
—	*bd*		*ade*	1.	—	
5	*be*		*bcd*	2	5	
	cd	2	*bce*			
	ce		*bde*	1.		
	de	1	*cde*	1.		
	—		—			
	10		10			

1. Here the combinations of all the five ſimple letters, is evidently 5 or *m*.

2. Then all the quantities *abcde*, being combined with all the ſame 5, the whole number of combinations of two letters, will be 5 times 5 or 25, but the 5 ſquares are to be rejected, *aa*, *bb*, *cc*, *dd*, *ee*. Then there remains 20, which is 4 × 5 or $m \times \overline{m - 1}$, but out of theſe 20, every one is repeated twice over, as *ab* and *ba*, *ac* and *ca*, &c. Therefore $m \times \frac{m - 1}{2}$, or $C \times \frac{m - 1}{n + 1}$ will be the juſt number of the combinatious of 2 things, n being $= 1$.

3. Then if the laſt 10 or C, be combined with the 5 *abcde*, there will be produced 5 × 10 or 50 combinations of 3 letters. And out of theſe there muſt be rejected 4 with *aa*, and as many with *bb*, *cc*, *dd*, *ee*; that is, 5×4 or $20 = 2C$. Then there remains 30 combinations, or $\overline{m - n} \times C$. But each combination is thrice repeated, for three letters; for we ſhall have *abc*, *bac*, *cab*; and *abd*, *bad*, *dab*, &c. that is, they are as often repeated as there are letters; therefore taking a third part, $C \times \frac{m - 2}{3}$

or C $\times \frac{m-n}{n+1}$ = the juſt number of combinations, in this caſe = 10.

4. Again, if theſe laſt be again combined with the five, we ſhall have 5 × 10, or 50 combinations of four letters, in this example. But out of theſe, there will be ſix with *aa*, and as many with *bb*, *cc*, *dd*, *ee*; that is, 6 × 5 or 30, to be rejected. Then there remains 20 = 2 × 10 = $\overline{m-n}.$C. But theſe contain as many repetitions of each, as there are letters, that is, 4 or $n+1$. For amongſt them we have *abcd*, *bacd*, *cabd*, and *dabc*; and ſo of the reſt, therefore we muſt take only $\frac{1}{4}$ of theſe; and then we ſhall have $\frac{m-n}{4}$C for four quantities combined, which in this caſe is 5 = $\frac{m-n}{n+1}$C.

5. Then if theſe five be combined with *abcde*, we ſhall have 5 × 5, or 25 of all; but among them there will be four with *aa*, and as many with *bb*, *cc*, *dd*, *ee*; that is, 4×5, or 20 to be thrown out; then there remains 5, but all theſe 5 are but the repetition of the ſame letters differently placed; and therefore we muſt take the 5th part, or $\frac{5}{5}$ = 1, for the number of combinations = $\frac{m-n}{5}$C = $\frac{m-n}{n+1}$C, in our example.

And it is evident, that the ſame reaſoning may be applied to any number of diſtinct things whatſoever. And therefore if C be any number of combinations of n quantities, then $\frac{m-n}{n+1}$C will be combinations for $n+1$ quantities.

PROP.

PROP. VII. *Prob.*

To find the number of Combinations of m things, all different from one another, taken n at a time.

RULE.

Multiply continually $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$ &c. to n terms, for the number of combinations.

For let m be the number of the quantities *abcdef*, &c. then it is plain, that m will be the number of all the one's.

Then let $n = 1$, and $C = m$, and by the laſt Prop. the combinations of all the two's will be $C \times \frac{m-1}{2} = \frac{m}{1} \times \frac{m-1}{2}$.

Again, let $n = 2$, $C = \frac{m}{1} \times \frac{m-1}{2}$; then by the laſt Prop. the combinations of all the three's will be $C \times \frac{m-2}{3} = \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$.

Alſo if $n = 3$, $C = \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$, then the combinations of all the four's will be $C \times \frac{m-3}{4} = \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$. And ſo forward as far as you will.

Cor. 1. *The number of combinations of n things, taken out of m, is equal to the uncia or coefficient of the* $\overline{n+1}^{\text{th}}$ *term of a binomial, raiſed to the* m^{th} *power.*

For $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$ &c. to n terms is the $\overline{n+1}^{\text{th}}$ term of a binomial; by Cor. 1. Prob. V. Sect. I. Algebra.

Cor. 2. *The number of all the poſſible combinations, of all the one's, two's, three's, &c. is one leſs than the ſum of all the unciæ of a binomial raiſed to the* m^{th} *power, and that is equal to* $2^m - 1$. *This is the number of all the elections.*

For the ſum of the coefficients of the m^{th} power $=$ ſum of the coefficients of $\overline{1+1}^m = \overline{1+1}^m = 2^m$.

Cor. 3. *There is the ſame number of combinations of* n *things out of* m, as of $m - n$ *things out of* m.

For there is the ſame variety of taking n things, as of leaving n things; that is, of taking $m - n$ things.

Cor. 4. *There is but one combination of* m *things out of* m; *whether they be all different, or ſeveral of one or more ſorts.*

Cor. 5. *The number of permutations of* m *things taken* n *at a time* $=$ *the like number of combinations* $\times$ *into* 1.2.3.4.5 &c.. *to* n *terms.*

This is plain from this Prop. and Prop. III.

Ex. 1.

How many combinations can be made of ſix letters out of ten?

Anſ. $\frac{10\times9\times8\times7\times6\times5}{1\times2\times3\times4\times5\times6} = 210$.

Ex. 2.

How many combinations can be made with three letters of the alphabet?

Anſ. $\frac{24\times23\times22}{1\times2\times3} = 2024$.

SCHOL.

The number of diviſors of m quantities, $abcd$, &c. is $2^m - 1$. And the number of aliquot parts, $2^m - 2$. For the number of all the combinations of the one's, two's, three's, &c. is the ſame as the number of the diviſors. And the number of aliquot parts is one leſs; becauſe $abcd$ is a diviſor, but not a part of itſelf.

PROP. VIII. *Prob.*

To find the number of combinations of m things, taking n at a time; in which there are p things of one ſort, q things of another ſort, &c.

Here when any quantity is often repeated, as aaa, $bbbb$, &c. they are written thus a^3, b^4, &c. where the index ſhews the number of repetitions, of any quantity, or how often it is to be taken. In the former caſes of combination where the quantities are all diſtinct, the ſolution is had by a ſingle rule, or a ſingle operation. But this cannot be done where any of the quantities are repeated, for this requires ſeveral operations, for the different forms of combination, that n things are capable of, one for each form; whereas if they were all diſtinct, there could be only one form for all.

To proceed regularly, we muſt firſt find how many different forms n things will admit of, and what they are; and then how many combinations are in each form. And the ſum of all theſe combinations will be the number required.

RULE, *for the number of forms.*

1. Place the m things ſo, that the greateſt indexes may be firſt, and the reſt decreaſe in order. Then begin at the firſt letter, and join it with the

ſecond, third, fourth, &c. to the laſt; then take the ſecond letter and join it with the third, fourth, &c. and then take the third letter and proceed the ſame way; and then the fourth, and the fifth, &c. till all be done; always taking care to reject ſuch combinations as you have had before. Thus you have the combinations of all the two's.

Then for all the three's, join the firſt letter to every one of the two's, ſucceſſively; and then the ſecond, and third, &c. to the laſt; taking care to leave out ſuch combinations as are had before. Then you have all the combinations of the three's.

Proceed the ſame way to get the combinations of all the four's, all the five's, &c. till you come at n, whoſe combinations are required. So you will at laſt get all the ſeveral forms of combination of n things; and how many there is in each form.

But in particular caſes, ſhorter ways than this may often be found out.

2. To find the number of combinations in any form; let the quantities, both in the m things, and n things, be ranged according to the indexes, ſetting the greateſt firſt, as has been ſaid. Then if the quantities are not many, the number of combinations may be found out, and reckoned up, after a like manner, as their different forms were found out before; beginning at the firſt, and going gradually thro' them all. As if you have $a^3\ b^3\ c^2\ d$; and you would know how many are contained in it, of this form $aabc$. Here aa may be connected with bc, bd, cd; and there are three ſuch ſquares which may be connected in like manner, viz. aa, bb, and cc; therefore 3×3 or 9 will be the number of combinations in that form.

But when there are many terms, we muſt proceed to calculation in this manner. Let the things given whoſe number is m, be thoſe $a^p\ b^q\ c^r\ d^s$, &c. where p, q, r, s are the indices or repetitions, no matter whether equal or unequal. And let $a^t\ b^v\ c^w\ d^x$, &c. be the form to be combined, the indices being t, v, w, x, &c. either equal or unequal. And when a letter is but once expreſſed, its index will be 1. Then for the

RULE *for any ſingle form.*

Let A, B, C, D, &c. be the number of different letters in the m things expoſed, whoſe indices are equal to, or greater than (t, v, w, x, &c. reſpectively) the indexes in the n things.

Then the number of combinations in that form is $\frac{A\times\overline{B-1}\times\overline{C-2}\times\overline{D-3},\ \&c.}{RS,\ \&c.}$ continued to as many terms as there are different letters in the form propoſed; where $R=1\times 2$, or $1\times 2\times 3$, or $1\times 2\times 3\times 4$, &c. when any index in the form is twice, thrice, or four times, &c. repeated; and the like for S, &c.

3. The ſame operation is to be repeated for every form; and then the ſum of all, gives the number of combinations; being all that n things out of m, can admit of.

For ſuppoſe any number of things as $m = a^5b^3c^2d$, and the form propoſed. —— $a^3\ b^2\ c$

It is evident 3 (the index of a^3) is contained in 5 and 3, (the indexes of $a^5\ b^3$), which gives 2 caſe, where a cube can be taken out of the m things, here $A = 2$.

Then, 2 (the index of b^2) is contained in 5, 3, 2 (the three indexes of $a^5\ b^3\ c^2$); here $B = 3$. So there are three caſes where a ſquare can be taken, conſidered ſingly. But as ſome cube goes along with it, that takes away one caſe, and there remains $B-1$ or two caſes only. Now for every one of theſe, there will be A caſes, where the cube and ſquare are both contained in the m things: that is, the number of combinations with a cube and a ſquare will be $A \times \overline{B-1}$.

Again, 1 (the index of *c*) is contained in 5, 3, 2, and 1, (the indices of all the quantities $a^5 b^3 c^2 d$); whence C = 4; ſo that there will be four caſes, where (*c*) any ſimple quantity can be taken. But ſince a cube and a ſquare is to go along with it; theſe take up two places, and deſtroy two of the caſes; ſo that the number of caſes where a ſimple quantity can be connected with a ſquare and a cube (whoſe places are fixt, or ſpecies determined), will be $C-2$. And for every one of theſe, there will be $A \times \overline{B-1}$ caſes by varying the ſpecies of the cube and ſquare; ſo the number of theſe three combinations, will be $A \times \overline{B-1} \times \overline{C-2}$. And in like manner, if there had been a fourth quantity (*d*) to be combined, we ſhould have D = number of the ſimple caſes, and D — 3 the number of caſes, when going along with the other three. And then the whole number of combinations of quantities, with four different indices, would be $A \times \overline{B-1} \times \overline{C-2} \times \overline{D-3}$, for *d* muſt be ſuppoſed to have a different index from any of the reſt.

It is evident, this way of reaſoning is general, and may be applied to any collections of quantities whatever; where the indices of all the quantities *n*, in the propoſed form, are all different. But it matters not, whether the indices in the whole collection of quantities *m*, which are expoſed, be the ſame or different.

Now when any index in the propoſed form happens to be repeated ſeveral times, the former number muſt be divided by a correſpondent number, after a like manner as was ſhewn in Prop. IV. for the changes of things.

Let $m = a^3b^2c^2$, the form a^2b, then A = 3, B = 3, and $A \times \overline{B-1} = 6$. But if the form be

a^2b^2 or ab, in either caſe A and B remain the ſame, and $\frac{A \times \overline{B - 1}}{1.2} = 3$, which ſhews all the caſes that two ſquares, or two ſingle letters, can be taken out of m. Here $R = 1 \times 2$.

Again, let $m = a^5b^4c^3d^3$, and the form a^3b^2c; then $A = 4$, $B = 4$, $C = 4$, and $4.3.2 = 24$, the number of combinations of a cube and a ſquare and a ſingle letter. But if the form be a^2b^2c; A, B, C will be the ſame as before, and $\frac{4.3.2}{1.2 = R} = 12$ combinations of two ſquares and a ſingle letter, as is eaſily tried. And if the form be $a^2b^2c^2$; A, B, C, will ſtill be the ſame, and then $R = 1 \times 2 \times 3$, whence $\frac{4.3.2}{1.2.3} = 4$, the combinations of three ſquares; and it is evident by inſpection, there can be no more. And there would be the ſame number for the form abc. And the ſame will hold in all ſuch quantities, where the indexes are alike. For a repetition of the ſame index cauſes a certain number of repetitions among the quantities combined; which ought to be reduced in proportion. And what is ſaid of the repetition of one index, is equally true for the repetition of any other index, and then S comes into the calculation.

Cor. *Hence it appears, that before the required number of combinations can be found; all the different forms of combination muſt be had. And theſe may be obtained by Art.* 1. *before; or rather, when the quantities are many, by the following method.*

Here we muſt work by the indices. Put L = number of different letters.

Then ſet down all the indexes contained in the m quantities, taking the greateſt firſt. Then out of theſe muſt be made ſo many rows of figures as there are different forms; and the ſum of the figures in each row muſt make *n*, and no row muſt exceed L places. Now to get all theſe forms, ſet down the greateſt index firſt, and the next leſſer towards the right hand, and ſo on. Then proceed from one row to another, by breaking the leaſt numbers (thoſe towards the right) ſtill into leſſer numbers, till they be units. Thus proceed gradually figure by figure, from the right towards the left, till all the numbers be dimininiſhed as low as they will admit of.

Ex. 1.

Let the things propoſed be $a^3b^2c = m$, *to find the number of combinations made of every* 3 (*n*) *of theſe quantities.*

The indexes 3 2 1. L = 3.

Forms		combin.
	3	1
	2 1	4
	1 1 1	1
Sum of the combinations,		6

In this example there are 3 forms; 1. a cube; 2. a ſquare and a ſingle one; 3. three ſingle ones. The firſt admits of one combination only; the ſecond of 4, and the third of 1, whoſe ſum is 6.

Ex. 2.

Let $a^3b^3c^2$ *be propoſed; to find the number of combinations, taken* 4 *at a time?*

Indexes.	Forms.	Combin.
3 3 2	3 1	4
L = 3.	2 2	3
	2 1 1	3
	Sum of the combin.	10.

Ex. 3.

How many Combinations, in $a^4b^2c^2de$; *taking* 8 *at a time?*

Indexes.	Forms.	Combinations.
42211	422	1
$n = 8$	4211	6
$L = 5$	41111	1
	3221	2
	32111	2
	22211	1
	Combinations	13

In this example there is no three's to be combined with the four.

Ex. 4.

Suppose $a^5b^5c^4d^4e^4f^3g$ *be proposed; to find all the combinations, taken* 10 *at a time.*

Indices.	Forms.	Combinations.
5544431	55	1
$n = 10$	541	40
$L = 7$	532	40
	5311	100
	5221	80
	52111	100
	511111	12
		373

Forms.

Forms.	Combin.
442	40
4411	110
433	50
4321	400
43111	250
4222	50
42211	300
421111	125
4111111	5
	1320

Forms.	Combin.
3331	80
3322	90
33211	360
331111	75
32221	180
322111	240
3211111	30
22222	6
222211	45
2221111	20
	1126
	1320
	373
Combinations in all	2819

To give account of our proceedings in this Example, it may be obſerved, that all the figures decreaſe towards the right. In the firſt place, we take the greateſt 5 and 5, and then break the latter 5 into 4 and 1, or into 3 and 2, or into 3 and 1,1. Having done with 5,3, we take 5,2, and fill up (10) with 2,1, or with 1,1, 1.

Having done with 5, we take 4, and fill up with the greateſt numbers, not exceeding 4; as with 4,2, and 4,1,1. For if we had taken a bigger number than 4, we ſhall find it diſpatched before; and therefore we muſt always take an equal or a leſs number. In the next place we take 4,3, and fill up with 3, or 2,1, or 1,1,1; and thus we go on till all the right hand numbers are diſſolved into 1^s.

Having finiſhed thoſe with 4, we take 3, and put to it 3,3,1, the greateſt numbers that exceed not 3; and proceed as before; but we have now additional numbers, to do with, above 2. And 3 being done, we go to 2, and the whole collection ends with 2221111; for we cannot break 2 into 1 and 1, for the number of places would then exceed 7, which it cannot do.

All theſe forms being thus had, their ſeveral combinations are obtained, by the ſecond article, and ſet againſt them in the table. And this example may ſerve as a ſpecimen for other more difficult caſes.

SCHOLIUM.

When n, the number of things to be taken, is very great; you may firſt take the number of $m - n$ things; and find how many combinations there will be for them; for there will be juſt as many for one as for the other. And then having the combinations of $m - n$ things; by taking theſe ſeverally out of the whole, you will have the combinations of n things required. As if $m = a^3b^2c$, and $n = 4$, then $m - n = 2$, and all the two's will be *aa*, *ab*, *ac*, *bb*, *bc*. Theſe taken ſeverally out of a^3b^2c, will leave *abbc*, *aabc*, *aabb*, a^3c, a^3b; which are all the combinations of the four's. But the number of forms will not be the ſame in both, there being only two forms in the two's; and three forms in the four's Neither will the number of combinations, in the correſpondent forms, be the ſame.

PROP. IX.

The number of compoſitions of n things taken out of n rows, each row conſiſting of m things, is m^n, *or the* n^{th} *power of m.*

Let there be any number of rows as theſe annext. It is plain the number of ſingle things as *a*, *b*, *c*, *d*, is m or m^1.

a	*b*	*c*	*d*
e	*f*	*g*	*h*
i	*k*	*l*	*m*

Then the number of combinations of every 2, is had by joining each quantity in the ſecond row to all the quantities in the firſt, which will make as many times m as there are things in the ſecond row, or m times m, that is m^2; for all the two's.

Again, taking in, the third row; there will be as many times m^2, as there are things in the third row, that is m times mm, or m^3; for the compoſition of three things.

After the ſame manner if a fourth row was taken in; all the combinations of every four, would be m^4; and ſo on. And therefore univerſally when n rows are taken in, the number of combinations will be m^n, which is the number of compoſitions.

Cor. 1. *The number of compoſitions of all the one's, two's, three's, &c. to n, is* $\frac{m^n - 1}{m - 1} m$.

For $\frac{m^4 - 1}{m - 1} = m^3 + m^2 + m + 1$.

And $\frac{m^4 - 1}{m - 1} m = m^4 + m^3 + m + 1$, as will appear by diviſion, or in general, $\frac{m^n - 1}{m - 1} m = m^n + m^{n-1}$, &c. to m = compoſitions of all the one's, two's, three's, &c. to n.

Cor. 2. *Hence, may be found the compoſition of n things out of m, as follows. Involve m to the* n^{th} *power for the anſwer?*

Examp. 1.

How many compoſitions, of 3 letters out of 20?

Anſ. $20^3 = 8000$. But theſe ſhould be here, 3 different alphabets.

Ex. 2.

How many changes are there on throwing four dice?

Anſ. $6^4 = 1296$.

PROP.

PROP. X. *Prob.*

If there be m rows of quantities given, having the ſame quantities, and the ſame number of them, as a, b, c, d, &c. To find the number of compoſitions of m things taken out of the m rows; for any given form, of theſe quantities, as $a^t b^v c^w d^x$, &c.

RULE.

Put V = variations of the things $a^t b^v c^w d^x$,

And A = variations or alternations of the indexes, *tvwx*, both found by Prop. IV. Then will

AV = number of compoſitions required.

For let the different rows in the laſt Prop. be made all alike, or the firſt row *m* times repeated, as here And let the number of things propoſed be $a^3 b^2 c$; and firſt let the index 3 be fixt to *a*, and 2 to *b*, &c. Now ſince a^3 may be taken out of any 3 rows, and b^2 out of any two remaining rows; and *c* out of the laſt remaining row; therefore there will be ſo many ways of taking theſe letters, as each of them can be placed in different rows, that is, in different places or ſituations; for the varying the rows is the ſame thing as varying the places of the letters; and conſequently the number of different ways of taking them out of the ſeveral rows, is equal to the number of variations or permutations of theſe letters, to be found by Prop. IV. which number is = V. And this will hold as long as the indexes are fixt to theſe particular letters, and to none elſe.

a b c d
a b c d
a b c d
a b c d
a b c d
a b c d
a b c d

But ſince in any one form as $a^3 b^2 c$, that form will take in as many caſes, as there can be variations made in ſhifting the indices, from one letter to another; therefore there will be ſo many times V, as is the number of theſe variations. Therefore if

A =

A = number of variations or alternations, of the indices 3, 2, 1; or in general, of t, v, x, w. Then AV will be the whole number of compoſitions, that particular form will admit of.

Cor. 1. *Hence in any* $a^t b^v c^w d^x$ &c. *where the indices are fixt invariably to their particular letters; the number of compoſitions* V *will be,*

$$= \frac{1 \times 2 \times 3 \times 4 \ldots \text{to } m}{1.2.3 \ldots \text{to} \times 1.2.3 \ldots \text{to } v \times \&c}.$$

Cor. 2. *If* n = *number of different letters in any form* $a^t b^v c^w d^x$ &c. *Then the whole number of compoſitions for that form will be* =

$$\frac{1.2.3 \ldots \text{to } n}{RS\ \&c.} \times \frac{1.2.3 \ldots \text{to } m}{1.2 \ldots t \times 1.2 \ldots v \times \&c.}.$$

where R = 1.2, or 1.2.3 &c. according as any index is twice or thrice, &c. repeated; and the like for S, and ſo on.

Here follow ſome examples.

Examp. 1.

How many compoſitions are in the form $a^3 b^2 c$?

Here $n = 3$, $m = 6$, $t = 3$, $v = 2$, $w = 1$. And here is no repetition of indexes. Therefore

$$\text{Anſ. } 1.2.3 \times \frac{1.2.3.4.5.6}{1.2.3 \times 1.2} = 6 \times 60 = 360.$$

Ex. 2.

How many compoſitions in the form $a^2 b^2 c$?

Here $n = 3$, $m = 5$. $t = 2$, $v = 2$, $w = 1$. The index 2 is twice repeated. Therefore

$$\text{Anſ. } \frac{1.2.3}{1.2} \times \frac{1.2.3.4.5}{1.2 \times 1.2} = 3 \times 30 = 90.$$

Ex. 3.

To find the compoſitions in the form $a^3 b^3$.

Here $n = 2$, $m = 6$. $t,v = 3$, $w = 0$. The index 3 is twice repeated; and the letters a, b, thrice.

$$\text{Anſ.}\ \frac{1.2}{1.2} \times \frac{1.2.3.4.5.6}{1.2.3 \times 1.2.3} = 1 \times 20 = 20.$$

Ex. 4.

To find the number of compoſitions of the form $a^3b^3c^2d^2$.

Here $n = 4$, $m = 10$. $t,v = 3$, $w,x = 2$. The index 3 is twice repeated, as alſo the index 2. The letters, a, b, are thrice repeated; and c, d, twice.

$$\text{Anſ.}\ \frac{1.2.3.4}{1.2 \times 1.2} \times \frac{1.2.3.4.5.6.7.8.9.10}{1.2.3 \times 1.2.3 \times 1.2 \times 1.2} =$$
$$6 \times 25200 = 151200.$$

Ex. 5.

How many compoſitions in the form $a^5b^3c^3def$?

Here $n = 6$, $m = 14$; $t = 5$; $v,w = 3$, $x = 1$, &c. Here the index 3 is twice repeated, and the index 1 thrice. The letter a, 5 times repeated; and b and c thrice. Hence,

$$\text{Anſ.}\ \frac{1.2.3.4.5.6}{1.2.3 \times 1.2} \times \frac{1.2.3.4.5.6.7.8.9.10.11.12.13.14}{1.2.3.4.5 \times 1.2.3 \times 1.2.3}$$
$$= 60 \times 20180160 = 1210809600.$$ So prodigiouſly do the numbers increaſe in theſe operations.

SCHOL.

It may be obſerved, that if the multinomial A + B + C + D &c. be raiſed to the m^{th} power, and the given form, whoſe number of compoſitions is ſought, be found among the terms of that power; the numeral coefficient annext to it, will always be the number of variations V, that we want. See my Algebra, Schol. to Prob. LIX. B. I. Thus for the form a^3b^2c, look in the ſixth power, and you'll find $60A^3B^2C$; therefore $60 = V$. Alſo in the fifth power, you'll find $30A^2B^2C$, therefore $V=30$, for the variations of a^2b^2c. Alſo with a^3b^3 in the ſixth power, you'll find 20 for V; and ſo of others. And this being known, this doctrine may be ſerviceable for finding the unciæ of any term of a multinomial. As ſuppoſe you have the 4th term of the 7th power of $A+B+C$ &c. which is A^4B^3, A^5BC, and A^6D,

The variations of A^4B^3 is $\frac{1.2.3.4.5.6.7}{1.2.3.4\times1.2.3}=35.$

and the variations of $A^5BC=\frac{1.2.3.4.5.6.7}{1.2.3.4.5}=42;$

and the variations of $A^6D=\frac{1.2.3.4.5.6.7}{1.2.3.4.5.6}=7;$

therefore the complete term is $35A^4B^3+42A^5BC+7A^6D$. And the ſame may be done for any other term, if the letters that compoſe it be known. The doctrine of Combinations and Permutations, is likewiſe applicable to many other purpoſes, particularly to ſeveral problems of chance, which for brevity's ſake I omit.

FINIS.

CHRONOLOGY:

Or, the ART of

RECKONING TIME.

DESCRIBING

The ſeveral Diviſions of TIME in common Uſe; the Nature and Original of CYCLES and PERIODS, and the moſt remarkable AERA's; and the manner of Computing the Numbers of ſuch CYCLES, or the Common Notes, for any Year; the moveable Feaſts; the Places of the Sun and Moon; and the Nature of the Calendar.

Founded upon

ASTRONOMICAL PRINCIPLES.

To which is added,

A Short CHRONOLOGICAL TABLE.

—— res antiquæ laudis et artis
ingredior, ———— VIRG.

LONDON:
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MDCCLXX.

THE

PREFACE.

I Suppoſe little or nothing need be ſaid in praiſe of an art, which we ſtand in need of, and are forced to practice every day; and ſuch is the art of Chronology. *Without this art we could not tell how time goes on, nor underſtand the difference of times and ſeaſons. We daily want to know the riſing and ſetting of the ſun and moon, the increaſe or decreaſe and the length of days, the moon's changing and full, and ſuch like things.*

There is ſuch a near connection between Aſtronomy and Chronology, that ſome authors treat of them together; and in this Treatiſe I have laid down ſuch aſtronomical problems as are always wanted in Chronology; which are here ſolved or rather reckoned, after a ſhort way for common uſe. But to have them accurate, muſt be performed more laboriouſly by Aſtronomy.

As Chronology is of ſuch ſervice in reckoning time, and every body has ſome buſineſs or other with time; therefore every body ſhould at leaſt know the uſe of the Calendar, and the common Chronological notes; and be acquainted with ſo much Chronology, as to be able to point out the ſeveral times and ſeaſons, as they are diſtinguiſhed in the country where he lives. Therefore I have here put together the eaſieſt elements of

Chronology, or thoſe of moſt frequent uſe, being ſuch as relate to the civil computations of time.

Chronology is abſolutely neceſſary to be known by all ſuch as want to underſtand hiſtory; without which he will wander in the dark, and be always at a loſs. For as the Hiſtory tells us the perſon that acts, and Geography gives us the place of action, ſo Chronology muſt give us the time of acting; without which we ſhall have very imperfect notions of what we read. For this reaſon, I have added a Chronological table, containing the moſt memorable actions that have happened, and the moſt remarkable events; and the times when the moſt famous men were alive.

It is true, the Chronology of ancient times is very obſcure and uncertain; for though the facts are moſtly true, yet theſe being annexed to wrong periods of time, has occaſioned infinite perplexities and inconſiſtencies in Chronology, which ſtood in great need of amendment. For this reaſon Sir I. Newton *ſet about rectifying the times, which he did with wonderful ſagacity; and has given us a table of ancient tranſactions, adjuſted to their proper periods of time, as far as he had any* data *to work by, and perhaps as far as it is poſſible to be done in this age. By this means he has made a great many events in hiſtory, ſurpriſingly to agree. This he effected by reaſoning from the lengths of generations of men, and of the reigns of kings, and by aſtronomical obſervations, where they could be had; and theſe are infallible guides.*

For to confirm the time of any tranſaction, ſeveral characters, taken from Aſtronomy, are of eſpecial uſe; as the eclipſes of the ſun and moon, the conjunctions of the ſuperior planets, the acronical riſing or ſetting of the ſtars, the occultation of the fixt ſtars by the moon, the places of the equinoctial points, and ſuch like. Theſe, when taken notice of by any hiſtorian, go a great way in determining the time of any action or event, then mentioned.

But ſome perverſe people, bigotted to another way, and reſolved to oppoſe all improvements; not content with what Sir I. Newton *had done, have ventured to write againſt him; tho' they cannot tell how to give a better ſyſtem, or clear the common ſyſtem of Chronology from its known inconſiſtencies. In what they advance, they only give us the chaff and the corn together; ſome truths mixt with endleſs falſhoods, abſurdity and nonſenſe; knowing no way to ſeparate them, and wanting Sir* Iſaac's *philoſophical ſieve; ſo they blunder on in the old road, and ſtick faſt among their abſurdities, not knowing how to extricate themſelves. Some of them attempt to confute Sir* Iſaac's *aſtronomical computations, which perhaps may go down with ſuch as are no judges; but will be laughed at by ſuch as are; for when things are examined, they appear to be moſt wretched computers. The way to confute him is to invalidate his proofs, which they can never do. Therefore the table I have annext is moſtly taken from his table, as being evidently the moſt conſiſtent ſcheme that has yet appeared.*

W. Emerſon.

CHRONOLOGY.

DEFINITIONS.

DEF. I.

CHRONOLOGY is the art of meaſuring time, and adjuſting all actions and events, to the ſeveral points of time in which they happened. And theſe parts of time are years, months, weeks, days, hours, minutes, ſeconds, &c.

DEF. II.

A *Year* is the time wherein the ſun (apparently) moves round the earth in its orbit, or the earth (really) moves round the ſun. This is the moſt obvious and ſenſible meaſure of large portions of time.

DEF. III.

The *aſtronomical* or *periodical Year*, is the time wherein the earth moves round the ſun, from any ſtar or fixt point in its orbit, to the ſame point again, and conſiſts of 365 days, 6 hours, 9 minutes, 14 ſeconds.

DEF. IV.

The *tropical Year* is the time wherein the earth (or ſun) performs its revolution thro' all the ſeaſons; which being ended, they all return again as before. It conſiſts of 365 days, 5 hours, 48 min. 57 ſec.

DEF. V.

The *Egyptian Year* is the ancient year uſed by the *Egyptians*, and now uſed by the *Perſians*, and is juſt 365 days continually. This being ſhorter than the tropical year by near 6 hours, a day is loſt in 4 years; and in about 1500 years, the beginning of the year moves backward thro' all the ſeaſons. This year is the moſt convenient for aſtronomical calculations, as it is always the ſame, having no intercalary days to interrupt it.

DEF. VI.

The *Julian Year* is the year inſtituted by *Julius Cæſar*, 46 years before Chriſt. It contains 365 days for three years, which are called *Common Years*; and 366 days, every fourth year, which is called *Leap Year*; whence the Julian Year is 365¼ days at a mean. But this year being longer than the tropical year by full 11 minutes, a day is gained in 130 years; or the ſeaſons come ſooner by a day in 130 years. And in 47540 years the beginning of the year would move forward, thro' all the ſeaſons of the year.

DEF. VII.

The *Gregorian Year*, is the year inſtituted by Pope *Gregory*, in 1582; it conſiſts of 365 days for three years, and 366 for the fourth, which is leap year, like the Julian. But then every hundredth year, that is, not diviſible by 4, muſt be only a common year, inſtead of a leap year.

This ſort of year is neareſt the tropical year of any, and agrees nearly with the apparent motion of the ſun, and is deſigned to keep pace with the tropical year. But it is very unfit for aſtronomical calculations, being perplext with ſo many intercalations.

It was contrived to be brought to the ſun's motion as it was at the time of the council of Nice, in 325; which was done by dropping 10 days out of the Calendar in the month of October, In the laſt century it was 10 days before the Julian account; and in the preſent century, is 11 days before the Julian year; and gains one day more, every following century, except in ſuch as are exactly diviſible by 4. But it is ſtrange that the Pope ſhould bring back the beginning of the year, only to the Nicene council; when it ought to have been brought back to the time of Julius Cæſar, when the error was firſt begun; at which time the vernal equinox was on March 24 or 25. And then inſtead of 10 days, 13 or 14 ſhould have been cut off.

This year was brought into uſe in England in the year 1752; before which, the Julian account was always uſed. It was effected by ſtriking out 11 days in September, calling the third day the fourteenth, and ſo on. And the year to begin on Jan. 1.

DEF. VIII.

The *civil Year* is the year which is in common uſe in any nation. Thus the Julian year was uſed in England till 1752, and after that the Gregorian year was to be always uſed.

Different nations begin the year at different times. The ancient *Egyptians* fixt its beginning in Auguſt; the *Athenians* and *Grecians*, at the ſummer ſolſtice; ſome places in *Germany* and *Italy*, at the vernal equinox; and others at the autumnal equinox. But moſt of the *Europeans* begin it at the firſt of January; as England does by a late act of parliament.

The year is divided into 12 months; as alſo into four quarters, for the four ſeaſons of the year; ſpring, ſummer, autumn, and winter.

DEF. IX.

The *lunar Year* conſiſts of 12 lunations, or 354 days; it is uſed by the *Turks* and *Mahometans*. It runs thro' all the ſeaſons in 32 years.

DEF.

DEF. X.

The *luniſolar Year*, is the moſt ancient year of all; it conſiſted of 12 lunations or months of 30 days each; which fell ſhort of the ſolar year, and therefore they were obliged to add a month, whenever they found it too ſhort for the return of the ſeaſons; and to omit a day or two in an intercalary month, when they found it too long for the courſe of the moon.

DEF. XI.

A *Year current* is a year running on, and not yet compleated. The ſame of days, &c.

DEF. XII.

A *Month* is the 12th part of the year.

DEF. XIII.

A *ſolar Month* is the time of the ſun's paſſing thro' a ſign of the zodiac.

DEF. XIV.

A *lunar* or *ſynodical Month*, is the time between one conjunction of the ſun and moon to another. It conſiſts of 29 days, 12 hours, 44 min. at a mean lunation.

DEF. XV.

Periodical Month, is the time of the moon's revolution, till it came again to the ſame point, or the ſame ſtar. It is 27 days, 7 hours, 43 min.

DEF. XVI.

Civil or *calendar Month*, is the length of 30 or 31 days, according as is ſet down in the Calendar.

DEF. XVII.

A month of weeks; this conſiſts of four weeks; and 13 of theſe months make a year, wanting a day.

DEF. XVIII.

A week is a collection of ſeven days, the 1ſt day is ſunday, 2 monday, 3 tueſday, 4 wedneſday, 5 thurſday, 6 friday, 7 ſaturday; then they come about again in the ſame order. Theſe days have their names from the ſeven planets.

DEF. XIX.

A day is the time wherein the ſun ſeems to make one revolution. This is the moſt ſenſible part of time for meaſuring the ſmall parts of it.

DEF. XX.

A natural day, is the time wherein the ſun moves round the earth, from one meridian to the ſame again. This is longer by about four minutes, than the time of the earth's rotation.

Different people begin the day differently. The *Aſtronomers* and *Arabians* begin the day at noon the preceding day, and end it at noon again. The *Jews*, *Italians*, *Athenians*, *Auſtrians* and *Bohemians*, begin their day at ſun-ſet the preceding evening, and reckon till ſun-ſet again. The *Babylonians* begin and end their day at ſun-riſe. The ancient *Egyptians*, and moſt *Europeans*, begin the day at midnight preceding, and reckon to the midnight following.

DEF. XXI.

An *artificial or ſhining day*, is the time the ſun is above the horizon, or the time between ſun-riſe and ſun-ſet. And *night* is the time of the ſun's abſence.

DEF.

DEF. XXII.

An *hour*, is the 24th part of a natural day. The *Italians* reckon to 24 hours; moſt others reckon to twice 12.

DEF. XXIII.

Planetary hour, is the 12th part of the time of the ſun ſhining; as alſo the 12th part of the time of his abſence.

The Jews and ancient Romans divided the day into 12 hours, whether long or ſhort; and begun to reckon their hours at ſun-riſe, and ended at ſun-ſet, when they began to reckon the hours of the night, which they alſo divided into 12 parts or hours. Whence the hours of their day were longer than the hours of their night in ſummer; and ſhorter in winter. The Turks uſe this way at preſent.

DEF. XXIV.

A *minute* is the 60th part of an hour; a *ſecond* the 60th part of a minute; and ſo of *thirds*, *fourths*, &c.

DEF. XXV.

A *period* or *cycle*, is a certain ſpace of time, or a revolution of a certain number of years, which being ended, it begins anew. Thus,

The *Julian period* conſiſts of 7980 years; the product of the numbers, 19, 28, 15. This period was 4713 at the beginning of the Chriſtian Era.

The *Dionyſian* or *Victorian period* is 532 years, being the product of 19 and 28. This period was 457 at the beginning of the Chriſtian Era. And,

DEF. XXVI.

An *Era* is a particular account or reckoning of time, from ſome remarkable point; to which account all events and memorable actions are referred. This has a beginning but no ending, as a period has.

The following are the moſt remarkable Era's, according to the common account.

	Years before Chriſt.
Of the world, ——	4000
Noah's flood, —	2294
Of the Olympiads (Greek)	776
Building of Rome (Roman)	753
Nabonaſſar, ——	747
Of Chriſt's birth, —	2

	Years after Chriſt.
Diocleſian perſecution,	284
Hegira, among the Turks,	622
Geſdegird, among the Perſians,	632

D E F. XXVII.

Epocha is the firſt point of time, or the beginning of an Era.

D E F. XXVIII.

Stile, the method of reckoning time. The *Julian ſtile* is the *old ſtile*, in uſe from the time of Julius Cæſar. The *Gregorian ſtile*, is that begun by Pope *Gregory*, called the *new ſtile*; and uſed in *England* ſince 1752.

D E F. XXIX.

The *Calendar* is a day book for a year, divided into twelve parts for the twelve months, and each month into its proper number of days, all regularly numbered; which numbers are to ſhew the day of the month.

The names of the months from the beginning to the end of the year, and the days in each are, January 31, February 28, March 31, April 30, May 31, 31, June 30, July 31, Auguſt 31, September 30, October 31, November 30, December 31: in leap year February hath 29. The number of days in each month may be remembered by theſe lines,

30 days hath September,
April, June, and November,
February has 28 *alone,*
And all the reſt have 31.

Againſt the days of the month in the Calendar, are ſet all the remarkable days, holy days, feſtivals, term times, with ſuch Phænomena of the ſun and moon, as are uſeful for meaſuring and determining the ſeveral times and ſeaſons of the year, but eſpecially all the fixt days; for if the moveable days are inſerted, it is more properly an *Almanack*, and laſts only for a year; whilſt the Calendar is perpetual.

Every week is divided into ſeven days, which are denoted by the firſt ſeven letters of the alphabet, A, B, C, D, E, F, G; A is ſet at the firſt day of January, B at the ſecond, C the third, &c. all following in order, and keep their places invariable. The letter that falls on a ſunday is called the Dominical letter.

Since a common year has 365 days, or 52 weeks and one day; therefore A will ſtand againſt the laſt day of the year, as well as againſt the firſt. Therefore if any year begins on a ſunday it will alſo end on a ſunday; and therefore the next year will begin on a monday; and therefore the next year, to that, on a tueſday; and the next, on a wedneſday; and ſo on till a leap year happens, when February has 29 days, and then that year goes forward, or leaps two days. And hence it follows, that the ſunday letter for each ſucceeding year moves one letter back, in this order, A, G, F, E, &c. till leap year, and then it moves two letters back.

In the yearly Diarys or Almanacks, the Sunday letter is commonly marked a red letter, to denote it belongs to Sunday. For all nations ſet apart one day in the week for publick worſhip. The *Chriſtians* obſerve the firſt day in the week, or Sunday, which they call the Lord's day. The *Grecians*, Monday; the *Perſians*, Tueſday; the *Aſſyrians*, Wedneſday; the *Egyptians*, Thurſday; the *Turks*, Friday; the *Jews*, Saturday, which they call the Sabbath.

Beſides the periods or cycles before mentioned, there have been ſeveral others in uſe among the ancients; as a *Jubilee* 49 years, uſed by the Jews. An *Olympiad* 4 years, uſed by the Greeks. A *Luſtrum* 5 years, uſed by the Romans; and the *Roman Judicature* 15 years. An *Age* or *Century* 100 years.

SCHOLIUM.

I ſhall here make ſome remarks concerning the calendar and the diviſion of the year; not only in their preſent form, but as formerly uſed.

Before the time of *Julius Cæſar*, the months conſiſted alternately of 29 and 30 days, by which means the new moon kept to the firſt day of every month, when once rightly adjuſted; and their year always begun with the new moon neareſt the vernal equinox. But afterwards, by the order of *Julius Cæſar*, the months were made to conſiſt of 30 and 31 days alternately, and the laſt of 29; which made up the year, or 365 days. And the year begun at March. The months and days were as follows,

March	31	July	31	November	31
April	30	Auguſt	30	December	30
May	31	Septem.	31	January	31
June	30	October	30	February	29, and

30 in leap year.

But this regular and uniform method was altered by *Auguſtus Cæſar*, who made Auguſt 31, and February 28, and the reſt as we have them now. But tho' the ſaid uniform method of computing, was altered at the pleaſure and humour of a Roman tyrant; yet it is ſtrange that none reſtored it again after his death, upon account of its ſimplicity.

This Julian year with its months thus diſpoſed, has ſeveral advantages; as 1. the year beginning at March; the intercalary day, in leap year, would come in at the end of the year, where it ought to be, and would diſturb nothing. Thus in leap year, Feb. 30 would be the laſt day of the year. 2. From this diſtribution of the months into days, it will follow, that if the day of the moon's changing in March, is known for any year, it will be known in all the following months, according to her mean motion, by continually adding a day for every month paſt March; becauſe each month taken one with another, is a day longer than the moon's ſynodic revolution. 3. The year beginning at March, it would be near the time when the ſun is in the vernal equinox, which is the beginning of all aſtronomical computations.

It is certain, the more ſimple any account of time is, the more eaſy and regular will any computation be, that depends upon it. But the year now in uſe is quite otherwiſe. For the beginning of the year is fixt to the firſt of January; and the intercalary or additional day, which comes in every fourth or leap year, is added at the end of February; which in effect makes two beginnings of the year. For at the beginning of January, there is a change of the dominical letter, and of every other thing depending on it. And on the firſt of March there is another change of the dominical letter, and of all other things relating thereto. So that one fragment of the year is regulated by one rule; and another, by another rule; which is a manifeſt abſurdity. For it muſt needs appear abſurd to any body, to ſee an additional day thruſt into the middle of a year, where there is no room for it. But this is not all, for in the times following *Julius Cæſar*, this intercalary day was abſurdly thruſt into the middle of a month; and the 24th of February was to be reckoned twice over; as if it were poſſible for one day to be two, or two days to be only one day.

It may be ſaid for excuſe, that moſt of Europe follows that way of reckoning, and therefore we in England, for the conveniency of correſponding with them, ought to follow the ſame. However, all nations that obſerve this ſort of year, muſt labour under the like abſurdities.

When Pope Gregory altered the civil year, and impoſed it upon all Chriſtendom to obſerve; he ſhould have rectified it to the time of Chriſt's birth, and not to the time of the council of *Nice*. For our epocha is not dated from the Nicene council, but from the birth of Chriſt. And at that time the vernal equinox was not on the 21ſt of March, but on the 24th or 25th.

When the Proteſtant ſtates of *Germany* came to a reſolution to alter the Calendar, they would not receive the Pope's mandate, but did it in their own way, after this manner. They ordered that eleven days ſhould be left out of February after the 18th day, in the year 1700. So that inſtead of writing Feb. 19, they ſhould write March 1. And that the time of Eaſter for the future ſhould be determined by aſtronomical calculation; which was to be the firſt ſunday after the firſt full moon after the vernal equinox. Or the ſunday after, when the full moon fell on a ſunday.

As to our Calendar, as now ordered by the Stile act, it keeps the ſame form, yet it differs in this particular, that the golden numbers are ſet to the full moons in each month, and not to the change, as formerly; which variation ſeems to have no manner of advantage in it, but a manifeſt diſadvantage. For it is more material to know the moon's change, than the full, in every month except March.

To enquire particularly what ſort of a year, or what kind of computation, had been the moſt convenient; we ſhall mention ſeveral ways that it may, or rather might have been ordered. And,

1. As the year conſiſts of 365 days, 5 hours, 49 minutes very near. It might have been ordered to conſiſt of 365 days for three years, and 366 every fourth year, as the Julian year does. And placing the odd day at the end of December; beginning the year with January. Or,

2. The Julian account might be preſerved, and the odd day put at the end of February; and the year to begin at March. Then altering the dominical letters in January and February, ſo as to be continued from December in alphabetical order.

3. The year might always have begun on the ſame day of the week, as ſuppoſe on ſunday, which would have much facilitated any computation by the day of the month. But it would have happened that we ſhould always have, at the end of the year, two ſaturdays, and ſometimes three. But I know of no inconvenience this could have been; and it would have had that advantage, that the firſt day of the week would alſo have been the firſt day of the year.

4. But this might at leaſt have been done, viz. continuing the Julian year as it was, only beginning the year at March 1. For the Julian year is very near the ſolar year, and is very commodious for calculation, being a mean between the aſtrono-

mical and tropical years. And is attended with no other inconvenience, but a ſmall alteration of the ſeaſons, the terms, fairs, &c. which might have been mended by putting them forward about a week in a thouſand years, by an act of parliament. Whilſt ſhifting the year to agree with theſe, is like ſhifting the fire to a man, to ſave him the labour of going to it. But be the convenience or inconvenience what it will, we muſt be forced to take up with what we have. However we know this, that we had but one account to follow before, and now we have two to follow.

PROB. I.

To find how many years it is after leap year.

By the Gregorian account, which is the account we alſo follow; every hundredth year that is diviſible by 4, is a leap year, and the others not. And moreover, every common year that is diviſible by 4, is leap year, and the reſt not. Whence,

RULE.

Divide the year of our Lord by 4, and the remainder ſhews how long it is after leap year: if 0 remains it is leap year.

This holds for both Julian and Gregorian accounts. But in the Gregorian, no even hundreds are leap years, but ſuch as can be divided by 4.

Example 2.

Let the year be 1769.

4)1769(442
16
——
16
16
——
9
8
——
1 the firſt after leap year.
——

PROB. II.

To find on what day of the week, a given day of the month falls on.

RULE.

The initial letters of the words in the following two lines, are the dominical letters for the firſt day of each month in order; whence it is eaſy to reckon any day by the dominical letter.

A dram did give Ben eaſe;
Good cheer, for all did freeze.

Examp.

On May 16, 1769; *the Sunday letter is* A, *and* B *(Ben) is May* 1*ſt, whence* A *is the 7th, therefore the* 7, 14, 21, 28 *are all Sundays; and therefore the* 16*th is Tueſday.*

PROB.

PROB. III.

To find the cycle of the ſun, or the number of it, for any year.

The cycle of the ſun, or rather of the Sunday letter, is a period of 28 years; in which time it ſhews all the variations of the dominical letter. And this period being ended, all the numbers and correſpondent letters, return again in the ſame order, for ever, by the Julian ſtile; but only till the century is out, by the Gregorian. Its principal uſe is to find the Sunday letter; but it is alſo an ingredient in the Julian period.

RULE.

Add 9 to the year of Chriſt, and divide the ſum by 28; the remainder is the number of the cycle. If 0 remains, take 28.

Examp.

Let the year be 1769.

$$\begin{array}{r} 9 \\ \hline 28)\,1778\,(63 \\ 168 \\ \hline 98 \\ 84 \\ \hline 14 \end{array}$$

rem. 14 the ſun's cycle.

SCHOL.

The cycle of the ſun continues always the ſame, and alſo the connection of the numbers, with the letters, according to the Julian form. But the connection of the numbers with the letters, by the Gregorian ſtile, is variable; any cycle laſting but 100 years, and then a new cycle is to take place.

PROB. IV.

To find the dominical letter or letters, for any year.

1 RULE.

Add 17 to the year of the Lord, and divide the ſum by 28; the remainder being found in the old cycle below, ſhews the dominical letter, for the *new ſtile.* If 0 remain, take 28. This rule holds only till 1800.

For the *old ſtile*, ſeek the number of the ſun's cycle (found by the laſt Prop.) in the old cycle below, and againſt it is the dominical letter. This holds for ever.

Note, there are two letters in leap year; one holds till the end of February; the other to the year's end.

The old Cycle.

1 GF	5 BA	9 DC	13 FE	17 AG	21 CB	25 ED
2 E	6 G	10 B	14 D	18 F	22 A	26 C
3 D	7 F	11 A	15 C	19 E	23 G	27 B
4 C	8 E	12 G	16 B	20 D	24 F	28 A

To make this rule general for the new ſtile; inſtead of 17, between the beginning of 1800 and 1900, add 5, between 1900 and 2100, add 21; from 2100 to 2200, add 9, &c. And in general, add 16 more at every hundred years forward, except ſuch as are diviſible by 4. But after 7 additions of 16, the ſame numbers will return again; that is, after 8 hundred years.

Example.

What is the dominical letter for 1769 ?

```
   1769
     17
28)1786(63
   168
   ----
    106
     84
   ----
     22 = A in the old
cycle, for new ſtile.
```

But for old ſtile, 14 (the number of the cycle) ſtands againſt D.

To ſhew the truth of this rule for all future ages. Here we keep to the old cycle and poſition of the letters, without alteration; and find new additional numbers for every ſucceeding century; becauſe every century (except the 4th) is a common year, and therefore the Julian account is diſturbed at theſe times, and requires an alteration.

In the Julian account the additional number is perpetually 9, becauſe it was 9 at the beginning of the Chriſtian era. And as the whole contrivance was arbitrary, the firſt year of the cycle was made a leap year, and ſet to the letter GF, the 2d is E; the 3d D; the 4th C, the 5th BA, being leap year again; and ſo on till it come to 28, when it begins again. For if there was only a ſingle change of the dominical letter every year, the cycle would be completed in 7 years. But ſince in every 4th year there is a change of two letters, therefore this cycle will conſiſt of 4 times 7, or 28 years; which being completed, all the letters return again in the ſame order.

Now to make a tranſition from the Julian to the Gregorian account. The ſtile was altered in

1752, a leap year; and the Julian letters ED, that is, D for the end of the year. Now by dropping 11 days, any week day is, by that means, moved 11 days forward in the alphabet; or rather (throwing out 7) 4 days forward. Therefore the Sunday letter, from D (in the Julian) becomes A (in the Gregorian); and being leap year, the letters will be BA. Now againſt DE in the cycle ſtands 25, and againſt BA is 5, which is 8 places beyond 25; but $9 + 8 = 17$; and as we added 9 to the year to find the Julian letter, we muſt now add 17 (which is 8 more), to find the Gregorian letter, by the old cycle. And this holds till the beginning of the next century, or between 1700 and 1800.

In the year 1800, the laſt rule would give 25 (ED) in the cycle, but ſince it is not to be a leap year, it will be only E. Therefore we muſt ſeek in the old cycle for FE, to the end that 4 years after, in the cycle, may be a double letter; and then all will go regularly forward till the next century. Accordingly againſt 13 we find EF. Then counting forward from DE to EF, we ſhall find EF to be 16 places beyond, and $17 + 16 = 33$, or rather (rejecting 28) 5. Therefore ſeeing before, we added 17 to the year; we muſt now add 5 to get the number and letter, between 1800 and 1900.

In the year 1900, by this laſt rule, we ſhall find 1 (GF) in the cycle; but not being leap year it muſt only be G; therefore ſeek AG, which is at 17, which is 16 before GF; therefore $5 + 16$ or 21 is the number to be added, which holds from 1899 to 2100; becauſe 2000 being leap year, makes no alteration.

In the year 2100, we ſhall find 21 (CB), which muſt only be C; then find DC in the cycle, which is 16 further; and $21 \times 16 - 28 = 9$, the additional number for 2100 +, till 2200.

Thus adding 16 continually for every 100 years (except leap years), gives ſo many new rules. But after 800 years, we ſhall fall upon the ſame number of the cycle as at firſt; and the ſame numbers will return again; for 7 × 16 = 4 × 28. Therefore the numbers to be added, in theſe ſeveral periods of time will be 17, 5, 21, 9, 25, 13, 1, Then 17, 5, &c. over again.

2 RULE.

Add 9 to the year of the Lord, and divide the ſum by 28; the remainder being found in the I new cycle below, ſhews the dominical letter for the new ſtile, till 1800. Or find it in the II new cycle, ſhews it after till 1900. When 0 remains take 28.

For the old ſtile; thus,

Divide the year its 4th and 4, by 7;
What's left ſubſtract from 7, the letter's given.

I new Cycle for 1700, &c.			
1	DC	15	G
2	B	16	F
3	A	17	ED
4	G	18	C
5	FE	19	B
6	D	20	A
7	C	21	GF
8	B	22	E
9	AG	23	D
10	F	24	C
11	E	25	BA
12	D	26	G
13	CB	27	F
14	A	28	E

II new Cycle for 1800, &c.			
1	ED	15	A
2	C	16	G
3	B	17	FE
4	A	18	D
5	GF	19	C
6	E	20	B
7	D	21	AG
8	C	22	F
9	BA	23	E
10	G	24	D
11	F	25	CB
12	E	26	A
13	DC	27	G
14	B	28	F

Examp.

To find the Sunday letter for 1769?

```
   1769
      9
28)1778(63
   168
   ---
     98
     84
     --
     14 = A the ſunday letter in the
```

I new Cycle.

It has been ſhewn before, that at the beginning of every century, 16 is to be continually added, for a new additional number. Or that the number, finding the dominical letter in the old cycle, is 16 places further on. But this is equivalent to making a new cycle, ſetting the letters, found at the 17th number, in the firſt place; the 18th in the ſecond; and ſo on in order. And this muſt be done every 100 years; or a new cycle made for each century.

It was likewiſe ſhewn, that in changing the Julian for the Gregorian ſtile, that 8 was to be added to the Julian additional number, which is the ſame thing as reckoning 8 places forward from the firſt, and making that place, which is the 9th, the beginning of the I new cycle. Thus in the old cycle CD is the 9th place, and in the I new cycle it is the firſt; and all the reſt follow in order. Again the 17th place of the I new cycle is DE, which therefore muſt begin the 2d new cycle. And if you would make a third, it muſt begin with EF; and a fourth with FG; and ſo on. Therefore in changing from the 1ſt to the 3

3d, 5th, 7th, &c. hundred year; as alſo from the 2d to the 4th, 6th, 8th, &c. you move 4 places forward in the laſt cycle. For 16 + 16 = 32, which exceed 28 by 4. But here it muſt be remembered, that I except ſuch hundreds as are leap years, becauſe they make no alteration.

The firſt method ſhewed you to find the letter, by finding new additional numbers, retaining the ſame cycle; and the latter ſhows how to find it, by finding new cycles, retaining the ſame additional number. But the firſt method is eaſier, becauſe a new number is eaſier found than a new cycle; but either way will do for the preſent age.

3 RULE.

Add to the year its 4th part; then divide by 7, and ſubtract the remainder from 7, gives the number of the letter, from 1700 to 1800.

After the beginning of the year 1800, 1900, 2100, &c. add the 4th part + 6, 5, 4, &c. reſpectively.

And in general for every 100 years (not leap years) abate 1. But after 7 ſuch hundred years, the ſame numbers return again.

Note, there being two letters for leap year, this rule finds that, for the latter part of the year. And if the year has an exact 4th part, it is leap year.

Examp.

Find the letter for 1769.

```
 1769
  442
 ─────
7)2211(315
  21
 ───
   11
    7
  ───
   41    7
   35    6
   ──    ─
    6.   1 = A the letter.
```

In any century, by reaſon of the leap years, 5 letters go over in 4 years, reckoning from any fixt time. Let us reckon from 1700, when the letters were DC. Then in 1701, 2, 3, &c. the letters are B, A, G, &c. Therefore, if to any year (after 1700) its 4th part be added, the ſum ſhews the number of letters elapſed, ſince that time. And that ſum divided by 7, the remainder ſhews what letter it is, beginning at B, and reckoning backwards. The letters being numbered will ſtand thus,

B,	A,	G,	F,	E,	D;	C,	B,	A
1	2	3	4	5	6	7	8	9

To 1700 add its 4th part 425, then the ſum 2125 divided by 7, there will 4 remain. If 0 had remained, we might have taken the year of our Lord inſtead of the year after 1700, without any more to do. But ſince by uſing the year of our Lord inſtead of the year after 1700, the remainder will be more by 4; therefore to accommodate this rule to the year of our Lord, we muſt begin to number four places ſooner, and then we ſhall fall upon the ſame letter either way. The numbering will be thus,

F,	E,	D,	C,	B,	A,	G,	F,	E,	D,	C,	&c.
1	2	3	4	5	6	7	8	9	10	11	
						0	1	2	3	4	

That is, the year of our Lord divided by 7, the remainder will denote the letter as above. But ſubtracting theſe remainders from 7, we ſhall find for A, 1; for B, 2; for 3, C; &c. in the order of the alphabet, as laid down in the rule.

In 1800 (not being leap year) and the following, the letter falls 1 ſhort; therefore the remainder muſt be diminiſhed by 1, or the year itſelf diminiſhed by 1; for the ſame reaſon, 1900 and the following years, muſt be diminiſhed by 2; 2100, &c. by 3, and ſo on. Or it comes to the ſame thing if 1800, &c. be increaſed by 6, 1900, &c. increaſed by 5, 2100, &c. increaſed by 4, and ſo on. For 6, 5, 4 are the compliments of 1, 2, 3, to 7.

Cor. 1. *The ſun's cycle being known, the Gregorian letter is known, by adding* 8, *and finding the ſum in the old cycle, gives the letter*; *till* 1800.

Cor. 2. *The old dominical letter being given, that for the new ſtile is had by adding* 4 *letters. This holds till* 1800.

A Table ſhewing the Sunday letter till 1800.

years	lett.	years	lett.	years	lett.	years	lett.
						1790	C
1760	FE	1770	G	1780	BA	91	B
61	D	71	F	81	G	92	AG
62	C	72	ED	82	F	93	F
63	B	73	C	83	E	94	E
64	AG	74	B	84	DC	95	D
65	F	75	A	85	B	96	CB
66	E	76	GF	86	A	97	A
67	D	77	E	87	G	98	G
68	CB	78	D	88	FE	99	F
69	A	79	C	89	D	1880	E

PROB. V.

To find the golden number for any year.

The *golden number or prime*, is a cycle of 19 years, increaſing 1 every year, till it be completed; after which it begins again. This period of 19 years being ended, the new moons, and full moons, fall on the ſame day of the month, as they did 19 years before; and therefore it is called the *moon's cycle*. To find it,

RULE.

Divide the year of our Lord by 19, and to the remainder add 1, gives the golden number, for both old and new ſtile.

Examp.

What is the golden number for 1769.

19)1769(93
171.
59
57
2
add 1
3 the golden number.

A Table of the golden Numbers till 1800.

years	G.N.	years	G.N.	years	G.N.	years	G.N.
1760	13	1770	4	1780	14	1790	5
61	14	71	5	81	15	91	6
62	15	72	6	82	16	92	7
63	16	73	7	83	17	93	8
64	17	74	8	84	18	94	9
65	18	75	9	85	19	95	10
66	19	76	10	86	1	96	11
67	1	77	11	87	2	97	12
68	2	78	12	88	3	98	13
69	3	79	13	89	4	99	14
						1800	15

PROB. VI.

To find the Epact for any year.

The *epact* is the number to be added to the lunar year, to make it equal to the ſolar. This is a cycle depending on the golden number, and is likewiſe compleated in 19 years. It increaſes 11 every year, rejecting 30 when it is above that; but the epact never exceeds 29. Its uſe is for finding the new moons in March, and from thence in every other month.

1 RULE.

Take 1 from the golden number; and then multiply the remainder by 11, and divide the product by 30, and the remainder is the epact, for the *new ſtile*. When 0 remains take 29.

This Rule holds from 1700 to 1900, after which the epact ſo found, muſt be leſſened by 1, and for every 240 years after, it muſt be leſſened by 1 more, continually.

For the *old ſtile*, multiply the golden number by 11, and divide by 30, for all years.

Examp.

What is the Epact for 1769?

The golden number 3
ſubſ. 1

2
11

30)22(0
0

22 the epact.

A ſynodic revolution of the moon is 29.53 + days, and 12 ſuch revolutions are 354.37 days; and the Julian year is 365.25 days, which exceeds theſe 12 revolutions by 10.88 days. Hence if the moon changes at a certain time in one year, the next year ſhe will change 10.88 days ſooner; and the next; 21.76 days ſooner; and the next, 32,64 —29.

— 29.53, that is 3.11 days ſooner; and ſo forward. Therefore in any number of years after, if 10.88 be multiplied by that number, and the product divided by 29.53; the remainder will ſhew how many days the change happens before the fixt time in the firſt year. But becauſe we do not want to know the fractions or parts of days, but only in general, what day the change happens on; therefore, inſtead of 10.88 and 29.53, we may take the round numbers 11 and 30, which will give nearly the ſame remainders.

After 19 years are expired, the cycle begins again, and the changes happen on the ſame day, and almoſt on the ſame hour, as they did 19 years before. For in 19 years, the moon performs 235 revolutions, and $235 \times 29.53 + = 6939.69$ days, and $19 \times 365\frac{1}{4} = 6939.75$; the former falling ſhort only about .06 of a day, which is $1^h\ 28^m$. So that in 19 years, the new moons happen near an hour and a half ſooner; which in a little more than 300 years, will be a day ſooner. And ſince the Gregorian is ſhorter than the Julian year by three days in 400 years, or nine days in 1200 years; and the new moons happen ſooner by near four days in 1200 years; it is plain, the new moons will happen later in the Gregorian account by (9 — 4 or) near 5 days in 1200 years; or 1 day in 230 years. Therefore the numbers of this cycle being once adjuſted to the golden number, for the year 1700, it will continue the ſame for above 200 years. And hear the firſt number of the epact is 0 or 29, and then all the reſt follow of courſe; whence we get the rule above. And ſince the epacts are to ſhew how long before a fixt day the moon changes; it is plain the epact is to be diminiſhed 1 in every 230 years.

2 RULE.

Seek the golden number for the year in the following new table of epacts, and againſt it, is the epact ſought. This table continues in force till the year 1900; after which the epact muſt be leſſened by 1.

Examp.

Suppoſe 1769. *The golden number for this year is* 3, *againſt which is* 22, *the epact for the ſaid year.*

3 RULE.

Diminiſh the golden number by 1, ſeek the remainder in the firſt column of the old table below, againſt which is the epact.

Ex.

In 1769, *the golden number is* 3, *therefore againſt* 2 *is* 22 *the epact.*

Old Table.		New Table.	
G.Num.	Epact.	G.Num.	Epact.
1	11	1	29
2	22	2	11
3	3	3	22
4	14	4	3
5	25	5	14
6	6	6	25
7	17	7	6
8	28	8	17
9	9	9	28
10	20	10	9
11	1	11	20
12	12	12	1
13	23	13	12
14	4	14	23
15	15	15	4
16	26	16	15
17	7	17	26
18	18	18	7
19	29	19	18

Cor. *Having the epact for old ſtile, that for the new ſtile may be found by ſubtracting* 11 *from the old one.*

A Table of Epacts till 1800.

years	epact	years	epact	years	epact	years	epact
1760	12	1770	3	1780	23	1790	14
61	23	71	14	81	4	91	25
62	4	72	25	82	15	92	6
63	15	73	6	83	26	93	17
64	26	74	17	84	7	94	28
65	7	75	28	85	18	95	9
66	18	76	9	86	0.29	96	20
67	0.29	77	20	87	11	97	1
68	11	78	1	88	22	98	12
69	22	79	12	89	3	99	23
						1800	4

PROB. VII.

To find the day of the moon's changing in March.

RULE.

Add 1 to the epact, and ſubtract the ſum from 30, the remainder is the day of the change.

Exam.

The change is required in March 1768.

Epact 11
add 1

12
30

18 day.

In the year 1700, the moon changed on March 20, near four in the afternoon, by the Gregorian ſtile. The epact then muſt be 9, that 9 + 1 or 10 being ſubſtracted from 30, there may remain 20, for the day of the change. But the golden number is 10; therefore ſetting the epact 9 againſt the golden number 10, all the other numbers in the table of epacts will follow of courſe.

PROB. VIII.

To find the day of Eaſter full moon.

Eaſter full moon is the firſt full moon after the 21ſt day of March, and is called the *paſchal full moon*, or *Eaſter limit*. And it is denoted by the number of days from the beginning of March.

1 RULE.

By the laſt Prob. find the change day in March for the given year, to which add 15; and if the ſum be more than 21, this will be Eaſter full moon: if leſs, add 30 to it, and it gives Eaſter limit.

Examp.

Find Eaſter full moon for the year 1769.

The day of change March	7
add	15
Eaſter full moon	22d of March.

2 RULE.

Find the epact for the year, and ſubtract it from 44; if the remainder is equal or greater than 21, it is the day of full moon: if leſs, add 30 to it, and it gives Eaſter limit.

For the *old ſtile*, ſubtract the old epact from 47.

Examp.

Let the year be 1769.

44
epact 22

Eaſter limit 22d day of March.

For the new moon in March is 30 — 1 + epact = 29 — epact; and adding 15 days for full moon, 29 + 15 — epact, or 44 — epact = time of full moon from the beginning of March.

3 RULE.

Find the epact in the firſt column of the following Table, againſt which in the ſecond column is the day of the paſchal full moon; and the letter for that day. This Rule holds till 1900. N. Stile.

Epacts.	Paſ. full Moon.	
1	12 April	D
3	10 April	B
4	9 April	A
6	7 April	F
7	6 April	E
9	4 April	C
11	2 April	A
12	1 April	G
14	30 March	E
15	29 March	D
17	27 March	B
18	26 March	A
20	24 March	F
22	22 March	D
23	21 March	C
25	18 April	C
26	17 April	B
28	15 April	G
0.29	13 April	E

Examp.

In 1769, *the epact is* 22, *againſt which we have* March 22 *for the full moon, and its letter* D.

This table is conſtructed by the 2d Rule.

SCHOL.

It is ordered by Act of Parliament, that the paſchal full moon is to be the firſt full moon after the 21ſt of March. But it had been more agreeable to the cœleſtial motions, to have made it the firſt full moon after the vernal equinox. For the equinox is not on March 21, but on March 20. For this reaſon, the rule laid down in the Act, ſometimes fails, as it did in 1761, when the Eaſter full moon fell on March 20, a little after the equinox; being agreeable to the Table, but different from the Rule.

PROB. IX.

To find Eaſter day in any given year.

Eaſter day (by Act of Parliament) is the firſt Sunday after the firſt full moon, next after March 21; and if that full moon be on a Sunday, Eaſter day is the Sunday following.

But inſtead of March 21, it ſhould have been the *vernal equinox*, to make the Rule univerſally true.

Or *Eaſter day* is the firſt Sunday after the paſchal full moon, or Eaſter limit.

1 RULE.

Add 3 to the day of Eaſter full moon, or Eaſter limit (found by the laſt Prob.), and divide the ſum by 7, and ſubſtract the remainder from the dominical letter, (borrowing 7 if need be); add this laſt remainder to Eaſter full moon, gives Eaſter day, reckoned from the beginning of March, N. S.

Note, If the laſt remainder be 0, take 7 for it; and in leap year, take the letter which ſerves for the latter part of the year.

The ſame rule ſerves for the *old ſtile*, uſing the Julian Calendar, &c.

Exam.

When does Eaſter fall, in 1769?

Eaſter limit 22 March
3
7)25(3
21
4
Letter A = 1 or 8
Laſt Rem. 4
22
Eaſter day 26 March.

For ſince March begins with D, adding 3 to Eaſter limit, makes the reckoning commence at A. And dividing the whole by 7, the remainder ſhews how far Eaſter limit was advanced in the alphabet A, B, C. Let R be that remainder, S the Sunday letter; then S — R the laſt remainder, ſhews how much the reckoning falls ſhort of Sunday; which therefore muſt be added to the limit, to find Sunday; and the ſum will be the diſtance of Eaſter day from the beginning of March.

Golden Numb.	Months, Days.	Sun. Let.
14	Mar. 21	C
3	22	D
—	—23	E
11	24	F
—	—25	G
19	26	A
8	27	B
—	—28	C
16	29	D
5	30	E
—	—31	F
13	Apr. 1	G
2	2	A
—	— 3	B
10	4	C
—	— 5	D
18	6	E
7	7	F
—	— 8	G
15	9	A
4	10	B
—	—11	C
12	12	D
1	13	E
—	—14	F
9	15	G
—	—16	A
17	17	B
6	18	C
	19	D
	20	E
	21	F
	22	G
	23	A
	24	B
	25	C

2 RULE.

Find the golden number in the firſt column of this Table; againſt which in the 2d column ſtands the day of Eaſter full moon. Then look in the 3d column for the Sunday letter next following, and the day of the month, ſtanding againſt it in the 2d column, is Eaſter day. This holds from 1700 till 1900, N. S.

Note, If the full moon happens on a ſunday, the next ſunday is Eaſter day. In leap year, take the letter for the end of the year.

Example.

To find Eaſter day in 1769.

The golden number is 3, and ſunday letter A. Againſt 3, ſtands Mar. 22 and D; and looking down to A, it ſtands againſt Mar. 26, which is Eaſter day.

The ſame Table will ſerve to find Eaſter from 1900, till the beginning of the year 2200, by only placing all the numbers in the firſt column (except the two laſt), a line lower. And for each following 100 years, a line lower ſtill.

This Table is taken from the act 1750, for altering the ſtile; and muſt always ſtand good; even tho' it diſagree with aſtronomical calculations.

3 RULE.

3 RULE.

In the following Table, find the golden number on the ſide, and go croſs over, 'till you come under the ſunday letter at top; and there you will have the day, and the month towards the left hand, when Eaſter day is. N. S.

A Table to find Eaſter till 1900.

gold. num.	A	B	C	D	E	F	G
1	Apr. 16	17	18	19	20	14	15
2	Apr. 9	3	4	5	6	7	8
3	Mar. 26	27	28	29	23	24	25
4	Apr. 16	17	11	12	13	14	15
5	Apr. 2	3	4	5	6	Mar. 31	Apr. 1
6	Apr. 23	24	25	19	20	21	22
7	Apr. 9	10	11	12	13	14	8
8	Apr. 2	3	Mar. 28	29	30	31	Apr. 1
9	Apr. 16	17	18	19	20	21	22
10	Apr. 9	10	11	5	6	7	8
11	Mar. 26	27	28	29	30	31	25
12	Apr. 16	17	18	19	13	14	15
13	Apr. 2	3	4	5	6	7	8
14	Mar. 26	27	28	22	23	24	25
15	Apr. 16	10	11	12	13	14	15
16	Apr. 2	3	4	5	Mar. 30	31	Apr. 1
17	Apr. 23	24	18	19	20	21	22
18	Apr. 9	10	11	12	13	7	8
19	Apr. 2	Mar. 27	28	29	30	31	Apr. 1

Examp.

In 1769, the golden number is 3, and dominical letter A.

Under A and againſt 3, is March 26 for Eaſter day.

This Table is eaſily compoſed from the laſt, thus. Let the golden number be 1; againſt it we find Apr. 13, and the letter E; which being ſunday letter, Eaſter ſunday is Apr. 20. But the ſunday letters being F, G, A, B, C, D; we find Apr. 14, 15, 16, 17, 18, 19, for Eaſter ſunday, reſpectively. And ſo of the reſt.

A Table for finding Eaſter day till 1800.

year	Eaſt. d.	year	Eaſt. d.	year	Eaſt. d.	year	Eaſt. d.
1760	Apr. 6	1770	Apr. 15	1780	Mar. 26	1790	Apr. 4
61	Mar. 22	71	Mar. 31	81	Apr. 15	91	Apr. 24
62	Apr. 11	72	Apr. 19	82	Mar. 31	92	Apr. 8
63	Apr. 3	73	Apr. 11	83	Apr. 20	93	Mar. 31
64	Apr. 22	74	Apr. 3	84	Apr. 11	94	Apr. 20
65	Apr. 7	75	Apr. 16	85	Mar. 27	95	Apr. 5
66	Mar. 30	76	Apr. 7	86	Apr. 16	96	Mar. 27
67	Apr. 19	77	Mar. 30	87	Apr. 8	67	Apr. 16
68	Apr. 3	78	Apr. 19	88	Mar. 23	98	Apr. 8
69	Mar. 26	79	Apr. 4	89	Apr. 12	99	Mar. 24
						1800	Apr. 13

Cor. 1. *Hence, the times of all the moveable feaſts, that depend upon Eaſter, may be known.*

Septuageſima Sunday is 9 weeks	before Eaſter.
Sexageſima Sunday is 8 weeks	
Shrove Sunday, or *quinquageſima*, is 7 weeks —	
Shrove Tueſday, *Aſh-wedneſday*, next following	
Quinquageſima is 6 weeks	
Palm Sunday a week	
Good Friday 2 days	
Low Sunday is 1 week	after Eaſter.
Rogation Sunday is 5 weeks	
Aſcenſion day, or *Holy Thurſday*, the Thurſday after Rogation,	
Whit-ſunday is 7 weeks	
Trinity Sunday is 8 weeks	

Then follow all the Sundays after Trinity in order. The Sundays between Aſh-wedneſday and Eaſter, are called Sundays in Lent; and the Sundays between Eaſter and Whit-ſunday, are called Sundays after Eaſter.

Note, The diſtance of Eaſter Sunday from March 21, is by ſome called, the *Number of Direction.*

Cor. 2. *From the Table, Rule* 2, *it appears, that Eaſter can never fall ſooner than Mar.* 22, *nor later than Apr.* 25.

SCHOLIUM.

I have not ſhewn how to find any of the common chronological notes or numbers, for years before Chriſt; as it is more material to know what is to come, than what is paſt. But that is eaſily done, by reckoning backward, ſo far as we need to go. Remembering that the firſt year current before Chriſt, is the next (backward) to the firſt year current after Chriſt. Before 1582 there was no Gregorian year, and 46 years before Chriſt, there was no Julian year. And to what purpoſe can it be, to ſeek for any note, according to either Julian or Gregorian year, at a time when there was no Julian or Gregorian year exiſting.

If the arithmetical rule, for finding Eaſter, happens to differ from the table in the 2d rule, it muſt be ſet aſide. For that table is ſettled by the ſtile act, to be the general rule; altho' the act at the ſame time lays down another, which is ſometimes inconſiſtent with the table. For the act ſays, that Eaſter day, is to be the firſt Sunday after the firſt full moon, next after the 21ſt of March. Which rule would have made Eaſter a month later than by the table, in the year 1761. Whereas if it had been mentioned, *after the firſt full moon, next after*

after the vernal equinox, which was the 20th of March; then the rule would have agreed with the table. And more inſtances of this kind may happen.

But after all the fuſs that has been made for finding a rule for the obſervation of Eaſter; the preſent rule is certainly a very ill contrived one; and had been better fixt to ſome Sunday, at a certain time of the year. Jeſus Chriſt did but ſuffer once, and riſe once; which therefore muſt have been at one certain time of the year. And if that time could be known, then the neareſt Sunday to that, would moſt properly be Eáſter Sunday; which would have been eaſily known. But our rule makes it vary 5 weeks from the time, as obſerved in different years.

It is extremely probable that his paſſion was in the year 34, on Friday the 14th day of the Month *Niſan*, which by the Julian account, was on Friday April 23; and at that time the equinox was on March 24. But with us now, the equinox is upon the 20th of March, which is 4 days ſooner. Therefore bringing the paſſion back to our way of reckoning, it will fall on April 19, and conſequently the reſurrection would be on April 21. Therefore if the neareſt Sunday to April 21, had been ſet apart for Eaſter day, it would have been an exceeding eaſy rule, and very near the true time; and I think this time might have been as well commemorated by an eaſy and plain rule, that comes near the true time; as a perplext one, that runs further from it.

PROB. X.

To find in what year Eaſter day ſhall fall on the 25th of April.

Suppoſe the preſent year to be 1769; the cycle of the ſun is 14, dominical letter A, golden number 3.

But by the laſt table in Prob. IX. when Eaſter day falls on April 25, the dominical letter is C, and golden number 6. Therefore look into the I new cycle (Prob. IV), and reckon from 14 to all the places where you find C, which will be after 4, 10, 15, 21 places; and that is 4, 10, 15, 21 years after 1769. Then if *a* be the revolutions of the ſun's cycle, *e* the revolutions of the golden number; and ſince the golden number 6 exceeds the preſent golden number (3) by 3; the time will be expreſſed by any of the following equations. Therefore if *a* or *e* be found, the time will be known.

$$\left.\begin{array}{l} 1.\ 28a+4=19e+3 \\ 2.\ 28a+10=19e+3 \\ 3.\ 28a+15=19e+3 \\ 4.\ 28a+21=19e+3 \end{array}\right\} = \text{number of years hence.}$$

Therefore by reduction, we get the following equations.

$$\begin{array}{l} 1.\ 28a+1=19e. \\ 2.\ 28a+7=19e. \\ 3.\ 28a+12=19e. \\ 4.\ 28a+18=19e. \end{array}$$

Theſe equations being reſolved, and the leaſt values of *a* found, we have 1. $a=2$, 2. $a=14$, 3. $a=5$, 4. $a=17$. Here 2 being the leaſt value of a, $28a+4=60$, and $1769+60=1829$

for the year; which ſhews, it will not happen in this century.

Again in the year 1800, the ſun's cycle is 17, and (by the II new cycle, Prob. IV.) the dominical letter E, and the golden number 15. Therefore in II new cycle (Prob. IV.) reckon from 17 to all the places of C, and we ſhall find 2, 13, 19, 24. And ſince $6 + 19 - 15 = 10$. Therefore we ſhall have the following equations.

$$\left.\begin{array}{l} 28a + 2 = 19e + 10 \\ 28a + 13 = 19e + 10 \\ 28a + 19 = 19e + 10 \\ 28a + 24 = 19e + 10 \end{array}\right\} \text{or} \left\{\begin{array}{l} 28a - 8 = 19e. \\ 28a + 3 = 19e. \\ 28a + 9 = 19e. \\ 28a + 14 = 19e. \end{array}\right.$$

By the firſt equation $a = 3, 22$, &c; by the ſecond $a = 6$; by the 3d, $a = 18$, and by the 4th, $a = 9$. Taking $a = 3$, $28a + 2 = 86$ years, and therefore 1886 is the firſt year that this thing will happen, and the only year in the next century; for all the other values of a are too big, as running into the next century after.

And in the ſame manner we muſt proceed, if any other day be aſſigned for Eaſter.

SCHOLIUM.

If this Prob. was to be reſolved by the Julian account, it need not be repeated, becauſe we are ſtopt by no centuries, as in the Gregorian account. But in the Julian, time runs on in the ſame way for ever. And therefore after 532 years, which is the period for Eaſter, or the Dionyſian period, all the days and years return again in the ſame order, in the Julian account.

PROP. XI.

To find the day of the month, when the ſun enters the 12 Signs.

RULE.

Beginning at March for the Sign Aries. Then
The days, for the firſt four couples, will be,
20, 21, 22, 23.
There's four more months, for them will be ſeen,
22, 21, 19 *and* 18.

That is,

In March and April, on the 20th, for ♈ and ♉.
In May and June, on the 21ſt, ♊, ♋.
In July and Auguſt, on the 22d, ♌, ♍.
In September and October, on the 23d, ♎, ♏.
In November, on the 22d, ♐.
In December, on the 21ſt, ♑.
In January, on the 19th, ♒.
In February, on the 18th, ♓.

Cor. *Hence will be known the place of the ſun, on any day; by reckoning a degree for every day from the entrance.*

Exam.

What is the ſun's place Oct. 10? The ſun enters Libra on the 23d of September, and October 10 is 17 days after; therefore the ſun's place is ♎ 17°.

PROB. XII.

To find the time of ſun-riſing or ſetting at any time, for the middle of England.

RULE.

Remember that on March 20, and September 23, the ſun riſes and ſets at 6. Then at 1, 2, or 3 months diſtance, reckon 1, 2 or $2\frac{1}{2}$ hours ſooner or later; and ſo in proportion.

Examp.

On May 1, the time of ſun-ſet is required.

On Apr. 20th (being a month from March 20th) the ſun riſes one hour ſooner, that is, at 5; and May 1ſt, (being $\frac{1}{3}$ of a month after) gives 20 minutes more. So the time of riſing, May 1ſt, is 4^h 40^m; and ſetting, 7^h 20^m.

PROB. XIII.

To find the day of the moon's changing in any month; or her age, on any day thereof.

1 RULE.

To the Epact add the number of the month beginning at March, and take the ſum from 30 (or 60), gives the day of the change: or that ſum added to the day of the month, gives her age, rejecting 30 if it exceeds 30.

Note, In January and February the epact of the preceding year muſt be uſed.

This Rule ſometimes varies a day, owing partly to the inequality of the months, and partly to the irregular motion of the moon.

Examp.

To find the change in July 1769, and her age the 20th day.

Epact	22		27
Month	5		20
Subt.	27		47
From	30	Sub.	30
Change	3d day		17 her age.

The cycle of epacts is ſo contrived, that at the beginning of it, the moon changes at the beginning of March; and likewiſe at the end of it, about the 29th or 30th day. Whence in any following year, the epact will ſhew the number of days the change happens, before the end of March. And ſince the months are each, about a day longer than the time of one lunation; therefore we muſt add ſo many days as there are months paſt, from March; whence that ſum taken from 30 gives the day of the change. And that day ſubtracted from a given day of the month, gives her age; that is, the given day — 30 + ſum = age.

2 RULE.

Look for the month in the new Calendar at the end, and find the golden number for the year in the ſecond column; againſt it ſtands the day of the change.

If you uſe the Calendar in the ſtile act, the golden number finds the full moon.

Exam.

To find the change of the moon in July 1769.

The golden number for the year is 3, which in July ſtands againſt the 3d day of the month, for the change day.

3 RULE.

Look for the golden number in the proper month in the old Calendar, againſt which is the old change day; then reckon ſix days further, and you have the day of the change by the new ſtile.

Exam.

To find the change in July 1769.

The golden number is 3, and ſtands at the 27th day. Reckoning ſix further, falls on the 2d of

Auguſt. Or rather ſeek 3 in June, which ſtands at the 27th day; from which reckoning ſix more, it falls on the 3d of July for the change.

For in the old Calendar, the moon is 4 or 5 days back, and the new ſtile eleven days forward; therefore the change is, by the new ſtile (11—5=) 6 days forward of the old ſtile, or ſometimes (11—4 or) 7 days.

PROB. XIV.

To find the time of the Moon's ſouthing.

RULE.

Take 8 tenths of the moon's age in days, gives the hour of her ſouthing.

Examp.

On July 10, 1769, the moon is ſeven days old.

$$\begin{array}{r} 7 \\ .8 \\ \hline 5.6 \end{array} = 5^h\ 36^m, \text{ the ſouthing.}$$

For the ſynodic revolution of the moon being $29\frac{1}{2}$ days, whilſt ſhe gains 24 hours of the ſun; therefore in one day ſhe will gain $\frac{24}{29\frac{1}{2}} = .81$ of an hour. And in any number of days, ſo many times .81, or 8 tenths.

Cor. *To find the time of high water at London Bridge, add three hours to the time of her ſouthing.*

PROB. XV.

To find the Moon's place in the Zodiac, on any day.

Find the ſun's place for that day, to which add 4 tenths of the moon's age in days; that is, ſo many ſigns; and you have the moon's place.

Example.

To find the moon's place July 20, 1769.

Moon's age	17	
	.4	
	6.8	
or	6°	24′
Sun's place	3	28
Moon's place	10	22
Or Aquarius		22°

For in $29\frac{1}{2}$ days the moon gains 12 ſigns of the ſun; and in one day ſhe leaves the ſun $\frac{12}{29\frac{1}{2}}$ or $\frac{4}{10}$ of a ſign. And in any number of days, ſhe will be ſo many times $\frac{4}{10}$ of a ſign before him, or more exactly .407, before him.

Cor. *The moon leaves the ſun* 12 *degrees, or rather* 12.2 *degrees, in a day. And* $\frac{2}{5}$ *the moon's age gives the moon's diſtance from the ſun, in ſigns.*

PROB. XVI.

To find the time of moon riſing or ſetting.

1 RULE.

Find the moon's ſouthing, or how many hours ſhe ſouths after noon. Take half this number of hours, and reckon ſo many months forward from the time given. Find the hour of ſun-ſet, for the time you reckon to; this will be the moon's ſemidiurnal arch, or half the time of her ſhining.

Then add the ſemidiurnal arch to the ſouthing, gives the ſetting; or ſubtract it, and it gives her riſing.

Note, between the change and the full, you need only find her ſetting, for ſhe riſes in the day-time. And from the full to the change, her riſing; for then ſhe ſets in the day.

Example.

July 20th, 1760.

Moon's ſouthing 13 . 6 = 13^h 36^m.
half — 6 . 8 months.
6 months from July 20, is Jan. 20th.
And 6.8 months from July 20th, is Feb. 14.
The ſun ſets on Feb. 14th, at 5^h 0^m ſubſt.
from moon's ſouthing, 13 36
moon riſe 8^h 36^m, at night.

For when the moon ſouths 12 hours after the ſun, ſhe is got ſix ſigns from him, and then is where the ſun would be ſix months after; and whatever her ſouthing be, ſhe is advanced ſo far, as the ſun would be, after half that number of months. And when ſhe is there, her ſemidiurnal arch is equal to the ſun's, or equal to the hours of ſun-ſet. And this is the arch the moon has to deſcribe after her ſouthing.

2 RULE.

Find the moon's place in the zodiack, and the ſun's ſemidiurnal arch when he is there; which will alſo be the moon's.

Then to $\frac{8}{10}$ the moon's age, add the ſemidiurnal arch, for the ſetting; or ſubſtract it, for the riſing.

Examp.

On July 10th, 1769. The moon's age is 7, and her place 6^{s} 12°, and her ſemidiurnal arch 5^{h} 40^{m}.

$\frac{8}{10}$ her age 5.6 = 5^{h} 36^{m}
add — 5 40

Moon ſets at 11^{h} 16^{m} at night.

For $\frac{8}{10}$ of her age is the time of her ſouthing, and her ſemidiurnal arch is the ſame as in the firſt Rule.

Cor. 1. *Hence the time of the moon's ſhining is twice the ſemidiurnal arch found above.*

Cor. 2. *From hence it is eaſy to know how long the moon ſhines after ſun-ſet, or before ſun-riſe. Thus,*

To the ſouthing add the ſemidiurnal arch, and ſubtract the time of ſun-ſet; for the hours of ſhining after ſun-ſet.

And to the time of ſun-riſe add the ſemidiurnal arch, and ſubtract the ſouthing (after 12 is abated), and you have the hours of ſhining before ſun-riſe.

For the moon's ſetting — the ſun's ſetting =, hours of ſhining after ſun-ſet. And ſun-riſe — moon-riſe = hours of ſhining before ſun-riſe.

SCHO-

Scholium.

Theſe rules will ſometimes vary an hour or more, which is occaſioned by the moon's latitude, and irregular motion. The ſix laſt Problems are properly aſtronomical; but having ſo near a relation to Chronology, are put here; and ſerve well enough for computing theſe things for ordinary uſe, and are eaſily wrought.

PROB. XVII.

To find the Roman indiction.

The *Roman indiction*, was a term or period of fifteen years, of uſe among the Romans. For once in fifteen years, they collected a tribute or tax from ſuch countries as they had conquered. But this period is of no manner of uſe now, but only as it is an ingredient in the Julian period. To find it,

RULE.

Add 3 to the year of Chriſt, and divide by 15, the remainder is the indiction. If 0 remains, take 15.

Exam.

For the year 1769
add 3

1772

15)1772(118
15

27
15

122

122
120

2 the indiction.

SCHOL.

There is an old rule which will give the Golden number, Cycle of the Sun, and Roman Indiction, all at once; 'tis this,

When 1, 9, 3, *to the year has added been,*
Divide by 19, 28, 15.

PROB. XVIII.

To convert one Era or period into another.

RULE.

We muſt firſt know what year of any Era or period, anſwers to the firſt year of Chriſt (or any other Era); or what year of Chriſt anſwers to the firſt year of any period or Era. Then 1 ſubtracted from that year, gives the reducing number.

Or thus, find how many years one Era begins before the other; and that will be the reducing number; which being added or ſubſtracted as the caſe requires, gives the correſpondent year complete in all caſes; and likewiſe the year current in all caſes, except when they run contrary ways; and then a unit muſt be added to one of the numbers to make them correſpond.

Ex. 1.

What year of the Julian period anſwers to 1769 *of the Chriſtian Era?*

The Julian period begun 4713 years before Chriſt,
therefore to 1769
add 4713

Anſwer, 6482d year. I. P.

Examp. 2.

What year of Chriſt anſwers to the 4635th year of the Julian period?

Here 4713 ſhould be ſubtracted from 4635, but it being bigger, we muſt ſubſtract the other from it, and the remainder will be years before Chriſt.

4635
—4713

— 78 years compleat.

But becauſe the year of Chriſt is running backward, it will be the 79th year current before Chriſt.

Ex. 3.

What year of the Era of Nabonaſſar anſwers to 1769 of the Chriſtian Era?

The Chriſtian Era began 747 years after that of Nabonaſſar.

To 747
add 1769

2516 year

Ex. 4.

What year of Chriſt anſwers to the 20th of Nabonaſſar?

20
—747

—727 years compleat.
before Chriſt 728 years current.

Ex. 5.

What year of the Hegira of the Turks, anſwers to the 30th year before the Yezdegird of the Perſians?

The Hegira begun 10 years before the Yezdegird.
from 30
ſub. 10

20 year current or compleat.

SCHOL.

In uſing any period, if the number be more than the period; divide by that period, and the remainder is the number of the year.

PROB. XIX.

Having the cycles of the ſun and moon; to find the year of the Dionyſian period.

RULE.

Multiply the cycle of the ſun by 57, and the cycle of the moon by 476; add the products together, which divide by 532, the remainder is the year required.

Examp.

Let the cycle of the ſun be 17, *and of the moon* 11.

$17 \times 57 = 969$
$476 \times 11 = 5236$

532)6205(11 periods.
532

885
532

the year 353 of the period.

Inveſtigation. Find two numbers $28x$ and $19y$, that being reſpectively divided by 28 and 19, will leave no remainder; but divided alternately by 19 and 28 will leave 1; that is, $\frac{28x}{19}$ leaves 1, and

$19y$

$\frac{19y}{28}$ leaves 1. Whence $\frac{28x-1}{19}$ = a whole number, $= x + \frac{9x-1}{19}$, therefore $\frac{9x-1}{19}$ = a whole number $= p$, and by reduction, $x = \frac{19p+1}{9} = 2p + \frac{p+1}{9}$; therefore $\frac{p+1}{9}$ = a whole number $= q$; whence $p = 9q - 1$; the leaſt value of q is 1, therefore $p = 8$, $x = \left(\frac{19p+1}{9} = \right) 17$, and $28x = 476 = A$.

Again, $\frac{19y-1}{28}$ = a whole number $= p$, then $y = \frac{28p+1}{19} = p + \frac{9p+1}{19}$, and $\frac{9p+1}{19}$ = a whole number $= q$, and $p = \frac{19q-1}{9} = 2q + \frac{q-1}{9}$; therefore $\frac{q-1}{9}$ = a whole number $= r$, and $q = 9r + 1$; and the leaſt value of r is 0, then $q = 1$, $p = \left(\frac{19q-1}{9} = \right) 2$, $y = \left(\frac{28p+1}{19} = \right) 3$; and $19y = 57 = B$.

Now if A be multiplied by any number n, and dividing by 19, it will leave n remaining, but divided by 28 will leave 0.

Alſo if B be multiplied by any number m, and divided by 28, it will leave m remaining; but divided by 19 it leaves 0.

Therefore $nA + mB$ divided by 28, will leave m (17); and divided by 19, will leave n (11); therefore $nA + mB$ is the year of the period. But ſince this may exceed 28×19 or 532; therefore divide by 532, and the remainder is the year of the period.

Cor. *The year of the Dionyſian period being given; the cycles of the ſun and moon are found, by dividing reſpectively by 28 and 19; for the remainder is the number of the cycle.*

PROB. XX.

Having the Cycles of the Sun, Moon, and Indiction; to find the year of the Julian period.

RULE.

Multiply the cycle of the ſun by 4845,
the cycle of the moon by 4200,
the cycle of indiction by 6916,
divide the ſum of the products by 7980, the remainder is the year of the Julian period.

Examp.

Let the Sun's Cycle be 17, *the Moon's* 11, *the Indiction* 6.

$$\begin{array}{r} 4845 \times 17 = 82365 \\ 4200 \times 11 = 46200 \\ 6916 \times 6 = 41496 \\ \hline \end{array}$$

$$\begin{array}{r} 7980)170061(21 \text{ periods.} \\ 15960. \\ \hline 10461 \\ 7980 \\ \hline \text{the year } 2481, \text{ of I. P.} \\ \hline \end{array}$$

Inveſtigation. Since $28 \times 19 \times 15 = 7980$, and $28 \times 19 = 532$, $28 \times 15 = 420$, $19 \times 15 = 285$. Firſt find three numbers $532\,x$ (A), $420\,y$ (B), $285\,z$ (C); that being reſpectively divided 532, 420, and 285, will leave 0 remaining. But the firſt divided by 15 leaves 1, the ſecond by 19 leaves 1, the third by 28 leaves 1.

Proceeding as in the laſt Problem, you will find $x = 13$, $y = 10$, $z = 17$; and $532x = 6916 = A$; $420y = 4200 = B$; and $285z = 4845 = C$; as may be eaſily tried by dividing.

Therefore, as in the laſt, $nA + mB + pC$ divided by 15, 19, and 28 reſpectively, will leave the remainders n, m, and p; and therefore $nA + mB + pC$ is the year of the period, or the remainder, after it is divided by 7980. For the quotient only ſhews how many periods are elapſed.

Cor. *The year of the Julian period being given, the cycles of the ſun, moon, and indiction are found, by dividing by* 28, 19 *and* 15 *reſpectively; the remainder is the number of the cycle.*

PROB. XXI.

To explain the Calendar, its nature and conſtruction.

A Calendar is a table containing 12 months, with all the days of theſe months, ſet down in order. And the months are alſo divided into weeks by the 7 firſt letters of the alphabet. On the top of the following Calendar, you have the name of each month; in the firſt column are the days of the month, numbered 1, 2, 3, &c. In the ſecond column you have the letters of the week days, and the golden numbers. In the third column are the holidays, and other remarkable fixt days, ſet againſt the proper days of the month.

Among the letters, that which ſtands for Sunday at the beginning of the year, will be the Sunday letter for all the year, and will point out all the Sundays. And the like for the reſt of the letters, the ſame letter always denoting the ſame day of the week quite thro' the year.

The

The CALENDAR.

MARCH.				APRIL.			
mon. days.	letters	gol. n.	Remarkable days.	mon. days.	letters	gol. n.	Remarkable days.
1	D	9	St. David.	1	G		
2	E		Chad B.	2	A	17	
3	F	17		3	B	6	Richard.
4	G			4	C		St. Ambroſe.
5	A	6		5	D	14	
6	B	14		6	E	3	
7	C			7	F		
8	D	3	Philemon.	8	G	11	
9	E	11		9	A	19	
10	F	19		10	B		
11	G			11	C	8	
12	A	8	Gregory.	12	D		
13	B			13	E	16	
14	C	16		14	F	5	
15	D	5		15	G		Perpetua.
16	E			16	A	13	
17	F	13	St. Patrick.	17	B	2	
18	G	2	Edward.	18	C		
19	A			19	D	10	
20	B	10	St. Cuthbert.	20	E	18	
21	C	18	Benedict.	21	F		
22	D			22	G	7	
23	E	7		23	A	15	St. George M.
24	F			24	B		
25	G	15	*Lady day.*	25	C	4	*St. Mark.*
26	A	4		26	D		
27	B			27	E	12	
28	C	12		28	F	1	
29	D	1		29	G		
30	E			30	A	9	Jacob.
31	F	9					

The CALENDAR.

MAY.

mo. da.	lett.		Remarkable days.
	g.	n.	
1	B	17	*St. Ph. & Ja. May day.*
2	C		
3	D	6	
4	E		
5	F	14	
6	G	3	Silvanus.
7	A		
8	B	11	
9	C	19	
10	D		
11	E	8	
12	F	16	
13	G	5	
14	A		
15	B	13	
16	C	2	
17	D		
18	E	10	
19	F		Prudent. V.
20	G	18	
21	A	7	
22	B		
23	C	15	
24	D	4	
25	E		
26	F	12	
27	G	1	Bede, V.
28	A		
29	B	9	
30	C		
31	D	17	

JUNE.

mo. da.	lett.		Remarkable days.
	g.	n.	
1	E	6	
2	F		
3	G	14	
4	A	3	
5	B		Boniface, M.
6	C	11	
7	D	19	
8	E		
9	F	8	
10	G		
11	A	16	*St. Barnabas.*
12	B	5	
13	C		
14	D	13	
15	E	2	
16	F		
17	G	10	St. Alban, M.
18	A	18	
19	B		
20	C	7	
21	D	15	
22	E		
23	F	4	
24	G	12	*St. John Bapt.*
25	A		
26	B	1	
27	C	9	
28	D		
29	E	17	*St. Peter.*
30	F	6	Lucina.

The CALENDAR.

JULY.				AUGUST.			
mo. da.	lett. g.	n.	Remarkable days.	mo. da.	lett. g.	n.	Remarkable days.
1	G			1	C		*Lammas day.*
2	A	14		2	D	3	
3	B	3	Cornelius.	3	E	11	
4	C			4	F		
5	D	11		5	G	19	
6	E			6	A		
7	F	19	Tho. a Becket.	7	B	8	
8	G	8		8	C	16	
9	A			9	D		
10	B	16		10	E	5	St. Laurence M.
11	C	5		11	F	13	
12	D			12	G		Clara V.
13	E	13		13	A	2	
14	F	2		14	B	10	
15	G		Swithin B.	15	C		
16	A	10		16	D	18	
17	B			17	E		
18	C	18		18	F	7	Helena.
19	D	7		19	G	15	
20	E		Margaret V&M.	20	A		Bernard.
21	F	15		21	B	4	
22	G	4	St. M. Magdalen	22	C	12	
23	A			23	D		
24	B	12	Chriſtiana.	24	E	1	St. Bartholomew
25	C	1	*St. James.*	25	F	9	
26	D		St. Ann.	26	G		
27	E	9		27	A	17	
28	F	17		28	B	6	Auguſtine.
29	G		Martha.	29	C		
30	A	6		30	D	14	
31	B	14		31	E	3	

The

The CALENDAR.

SEPTEMBER.

mo. da.	lett. g.	n.	Remarkable days.
1	F		Giles.
2	G	11	
3	A		
4	B	19	
6	C	8	
5	D		
7	E	16	
8	F	5	Nat. V. Mary.
9	G		
10	A	13	
11	B	2	
12	C		
13	D	10	
14	E		h. Rood d. h. cr.
15	F	18	
16	G	7	
17	A		Lambert B& M.
18	B	15	
19	C	4	
20	D		
21	E	12	*St. Matthew.*
22	F		
23	G	1	
24	A	9	
25	B	17	
26	C		S. Cyprian.
27	D	6	
28	E		
29	F	14	*St. Michael.*
30	G	3	St. Jerome.

OCTOBER.

mo. da.	lett. g.	n.	Remarkable days.
1	A		Remigius, B.
2	B	11	
3	C	19	
4	D		Francis.
5	E	8	
6	F		Faith V. & M.
7	G	16	Julia.
8	A	5	
9	B		S. Dennis.
10	C	13	
11	D	2	
12	E		
13	F	10	
14	G	18	
15	A		
16	B	7	
17	C	15	Etheldred V.
18	D		*St. Luke.*
19	E	4	
20	F		
21	G	12	Urſula.
22	A	1	
23	B		
24	C	9	
25	D	17	Criſpin.
26	E		
27	F	6	
28	G	14	*St. Simon & Jude.*
29	A		
30	B	3	
31	C	11	

The CALENDAR.

NOVEMBER.

mo. da.	lett.	g. n.	Remarkable days.
1	D		*All Saints.*
2	E	19	All Souls.
3	F	8	
4	G		
5	A	16	
6	B	5	Leonard.
7	C		
8	D	13	
9	E	2	
10	F		
11	G	10	St. Martin B&C.
12	A		
13	B	18	Brice.
14	C	7	
15	D		Machute.
16	E	15	
17	F	4	Hugh, B.
18	G		
19	A	12	Elizabeth.
20	B		Edmund.
21	C	1	
22	D	9	Cecilia.
23	E		St. Clem. B & M.
24	F	17	
25	G	6	Catharine.
26	A		
27	B	14	
28	C	3	
29	D		
30	E	11	*St. Andrew.*

DECEMBER.

mo. da.	lett.	g. n.	Remarkable days.
1	F	19	
2	G		
3	A	8	
4	B		Barbara.
5	C	16	
6	D	5	Nicholas, B.
7	E		
8	F	13	
9	G	2	
10	A		
11	B	10	
12	C	18	
13	D		Lucy, V. & M.
14	E	7	
15	F	15	
16	G		
17	A	4	
18	B	12	
19	C		
20	D	1	
21	E		*St. Thomas.*
22	F	9	
23	G	17	
24	A		
25	B	6	*Chriſtmas day.*
26	C	14	*St. Stephen.*
27	D		*St. John Evan.*
28	E	3	*Innocents.*
29	F	11	
30	G		S. Roger.
31	A	19	Silveſter, B.

Cor. *Hence it will be no hard matter to compoſe an Almanack, from what has been laid down before.*

For in the firſt place you muſt have a Calendar, which is the baſis of all the work; in this, the fixt days are already placed. Then the chronological notes for the year; then all the moveable days are next to be inſerted, and likewiſe the law days; all the ſundays, and holidays, whether fixt or moveable, are put in red letters for diſtinction. The law terms are theſe,

Eaſter term begins 17 days after Eaſter, and ends the Monday following Aſcenſion day.

Trinity term begins 12 days after Whitſuntide, and continues 19 days.

Michaelmas term begins the 9th or 10th of October, and ends the 28th or 29th of November.

Hillary term begins Jan. 23 or 24, and ends Feb. 12 or 13.

Likewiſe the Univerſity terms may be put in if any one pleaſes.

The next thing to be done, is to put into ſeparate columns the riſing and ſetting of the ſun, the riſing, ſouthing, and ſetting of the moon, and likewiſe the change and full, and quarters of the moon; and ſuch other aſtronomical matters as may be thought uſeful.

But to have an Almanack compleat, all the Celeſtial Phenomena ought to be put in, ſuch as the Sun and Moon's places and declinations; the heliocentric and geocentric places of all the planets, with their latitudes; the moon's place and latitude; the riſing, ſouthing and ſetting of the planets; the eclipſes of the luminaries; the aſpects of all the planets; the places and eclipſes of Jupiter's ſatellites; the ſouthing of the ſtars, the beginning and end of twilight; the equation of time, the ſun's right aſcenſion, and other ſuch like things. Moſt of theſe things vary every year, and therefore an Almanack laſts but for one year.

There are four things, which ought always to be kept in memory: the year, the month, the day of the month, and day of the week.

I ſhall now inſert a Chronological Table, ſhewing the times when the moſt noted events happened, which are mentioned in Hiſtory; ſuch as the changes of Kingdoms, the riſe and fall of Monarchies, the lives of famous men, and ſuch like things. And tho' the Chronology of antient times is very uncertain, yet it may not be amiſs to know within a little, when ſuch or ſuch a tranſaction happened; altho' by reaſon of the great diſtance of time, it cannot be known ſo accurately as we could wiſh.

A CHRONOLOGICAL TABLE.

The Time is ſhewn in Years before and ſince CHRIST.

	ye. bef. Chriſt.
THE Creation of the World according to *Moſes*; *Adam* made. By ſome Accounts 4000, by others 5508, by others 3956 years,	3950
Seth, the ſon of Adam, born	3820
Enos, the ſon of Seth, born	3715
Cainan, the ſon of Enos, born	3625
Mahaleel, the ſon of Cainan, born	3555
Jared born, the ſon of Mahaleel,	3490
Enoch born, the ſon of Jared,	3328
Mathuſalem born, the ſon of Enoch,	3263
Lamech born, the ſon of Mathuſalem,	3076
Adam dies, aged 930 years,	3020
Noah born, the ſon of Lamech,	2894
Noah begins to build the ark,	2414
Mathuſalem dies, aged 969 years, the oldeſt man,	2294
The ark finiſhed. *Noah*'s flood, or the univerſal deluge, that drowned the world,	2294
The tower of *Babel* building, and the confuſion of languages,	2131
The Chaldeans now began to obſerve the ſtars.	
Noah dies, 950 years old,	1944
The vocation of *Abraham*,	1927
A famine cauſes *Abraham* to go into Egypt,	1926
Sodom and *Gomorrah* burnt,	1903
Iſaac born, the ſon of Abraham,	1902
Jacob born, the ſon of Iſaac (afterwards called *Iſrael*)	1842
Joſeph born, the ſon of Jacob,	1751

	ye. bef. Chriſt.
Job living, the ſon of Iſſachar.	
A great famine cauſes *Jacob* and his family to go into *Egypt*, — —	1712
Jacob dies in Egypt, 147 years old,	1695
Joſeph alſo dies in Egypt, aged 110 years,	1641
Moſes born, — —	1577
The king of *Egypt* orders all the male children of the Hebrews to be killed,	1573
The children of *Iſrael* go out of Egypt, and Pharoah and his army drowned in puring them, — —	1496
Joſhua defeats the Gibeonites; and the ſun ſtands ſtill, — —	1458
Moſes dies, 120 years old, —	1457
Joſhua dies, 110 years old, —	1433
Gideon, judge of Iſrael, defeats the Midianites, — —	1271
Ruth.	
Abimelech kills his 70 brethren, and governs the Jews, — —	1239
Mephres, king of upper Egypt,	1125
Miſphragmuthoſis reigned over all Egypt, and drove out the ſhepherds,	1120
Ogyges, his flood. Eli governs Iſrael.	
Sampſon kills himſelf, with 3000 Philiſtines,	1116
The Philiſtines conquer *Iſrael*, and take the ark, — —	1100
Samuel judges Iſrael, —	1094
Athens governed by Archons, —	1080
Ducalion king of Theſſaly. —	
Amoſis or *Tethmoſis* the 2d king of all *Egypt*,	1070
Saul king of *Iſrael*,	1070
Saul defeated by the Philiſtines, kills himſelf, — — —	1060
Sparta built. — — —	
Samuel dies, — —	1060

David

	ye. bef. Chriſt.
David made king,	1059
David conquers the Edomites, who fly into *Egypt*,	1048
Hieram king of *Tyre* furniſhes David with timber,	1048
Tyre, *Aradus* built.	
Cadmus and *Europa*.	
Acriſius marries Eyridice,	1047
The Phœnicians that fled from David, carry their arts and ſciences into Greece, &c. and begin to ſail on the Mediterranean,	1045
Ducalion's flood drowned Theſſaly.	
Ammon reigns in Egypt, has fleets of ſhips in the Red Sea and Mediterranean,	1034
The Egyptians begun to obſerve the ſtars.	
The luniſolar year altered, and made to conſiſt of 365 days.	
Ceres teaches the Greeks to ſow corn,	1030
Perſeus.	
Solomon reigns, and marries the daughter of Ammon, king of Egypt,	1019
Solomon's temple founded,	1015
Minos reigns in Crete. His father Aſterius (Saturn) flies into Italy.	
Seſac, the ſon of Ammon, invades Arabia Felix, and ſets up pillars by the Red Sea,	1010
Solomon's temple finiſhed,	1009
Seſac reigns in Egypt,	1002
Dædalus invents the ſaw, wimble, &c. and gives a beginning to theſe arts in Europe,	989
Syſiphus reigns in Corinth,	983
Solomon dies,	979
Rehoboam reigns in Iſrael. Iſrael divided, and ten tribes revolt.	

	ye. bef. Chriſt.
Seſac rifles the temple at Jeruſalem, and the king's palace. Invades Syria and Perſia, Jeroboam ſubject to him, —	974
Bacchus and *Ariadne*.	
Seſac invades India, ſets up pillars on the Ganges, — —	971
Theſeus reigns at Athens, overcomes the Minotaur, — —	968
Licurgus.	
Seſac conquers Thrace, kills king Licurgus, he had Ethiopians in his army commanded by Pan; and Lybian women commanded by Minerva, —	967
Seſac baffled by the Greeks and Scithians,	965
Minos ſlain by the king of Sicily,	964
Hercules born.	
Helle drowned in the Helleſpont,	962
Abijam reigns in Judea.	
Seſac ſlain by his brother Japetus or Neptune, — —	956
Aſa king of Judah reigns.	
Orus reigns in Egypt and routs the Lybians.	
The *Ethiopians* invade Egypt, and drown Orus in the Nile, — —	947
Menes, or *Amenophis*, reigns in Ethiopia,	946
The ſhip *Argo* built by the Greeks,	939
Chiron makes the conſtellations.	
Theſeus, king of Athens, ſtole Helena,	938
Hippocrates.	
The *Argonautic expedition* to Colchis,	937
Ahab, king of Iſrael, reigns,	925
Paris, king Priam's ſon of Troy, ſtole Helena, — — —	918
Jehoſhaphat king of Judah reigns.	
Obadiah, *Elias*.	
Troy beſieged by the Grecians, —	914

	ye. bef. Chriſt.
Troy taken,	904
Dido builds Carthage. *Pigmalion* reigns at Tyre,	883
Æneas alive.	
Heſiod, *Homer*,	870
Mœris reigns in Egypt,	860
Jonah.	
The *Heraclides* return into Peloponneſus,	825
Cephren reigns in Egypt, and built another great pyramid,	824
Hoſea, *Amos*, *Iſaiah*, *Joel.*	
Pul founds the Aſſyrian empire,	790
Aſychis reigns in Egypt; which breaks into ſeveral kingdoms,	788
Ipitus reſtores the Olympiads. And from this æra, the Olympiads are reckoned,	776
Semiramis flouriſhes,	760
Sanchoniathon, *Micah.*	
Pul, king of Aſſyria, dies,	747
Tiglapileſar ſucceeds Pul, and reigns at Nineve.	
Nabonaſſar reigns at Babylon,	747
The *Egyptians* carry their Aſtronomy to Babylon, and found the æra of Nabonaſſar.	
Ahaz reigns in Judea.	
Damaſcus taken by Tiglapileſar, king of Aſſyria,	740
Hezekiah reigns in Judea.	
Salmanaſſar ſucceeds Tiglapileſar,	729
Habakkuk.	
Salmanaſſar takes Samaria, and carries the ten tribes captive to Nineve,	721
Sennacherib reigns over Aſſyria,	719
Syracuſe built by Archias.	
Nahum, *Tobiah*, or *Tobit.*	

	ye. bef. Chriſt.
Sennacherib ſlain. The Medes revolt from the Aſſyrians,	711
Confucius the Chineſe philoſopher.	
Aſſerhaddon reigns in Aſſyria, he built Tarſus and Anchiale in one day,	711
Lycurgus,	710
The *Corinthians* begin to build ſhips with three orders of oars,	697
Tirhakah reigns in Egypt,	687
Aſſerhaddon invades Babylon,	681
The *Jews* conquered by Aſſerhaddon, their king Manaſſeh carried priſoner to Babylon,	673
Aſſerhaddon invades Egypt, and ſubdues it,	671
Aſſerhaddon dies,	668
Amaſis defeats Apres, and reigns in Egypt.	
Saoſduchinus king of Aſſyria reigns,	668
Nebuchadnezzar defeats Arphaxad, the Mede,	661
Manaſſeh returns from captivity.	
Judith cut off the head of Holofernes, Nebuchadnezzar's general,	660
The oldeſt ſea fight between the Corinthians and Corcyreans; in which the Corcyreans are worſted,	657
Pſammiticus, by conquering the other eleven kings, becomes king of all Egypt,	655
The firſt Meſſenian war,	652
Charops the firſt decennial archon at Athens,	647
Saoſduchinus dies.	
Chyniladon reigns in Aſſyria,	643
Joſiah reigns in Judea,	640
Phraortes, king of the Medes, killed in the Aſſyrian wars,	636
Aſtyages king of the Medes,	636
Jeremiah, *Zephaniah*.	
Cyrene built by Battus,	633

	ye. bef. Chriſt.
Rome built by Romulus, (by the common account 752), — — —	627
Nabopolaſſar revolts from the king of Aſſyria, and reigns over Babylon, —	625
Pſammiticus conquered by Cambiſes king of Perſia, — — —	621
Tarentum built by Phalantus, — —	620
Pſammiticus dies, — — —	617
Nechaoh, or *Pharaoh Necco*, reigns in Egypt,	617
Cyaxeres reigns over the Medes, —	611
Joſiah, king of Iſrael, ſlain by Nechaoh,	609
Solon.	
Nebuchadnezzar reigns at Babylon,	606
Nebuchadnezzar defeats Neco.	
Nineve deſtroyed, and the Aſſyrian empire,	609
Daniel.	
The ſecond Meſſenian war,	607
Nebuchadnezzar invades Syria and Judea, under Nebopolaſſar his father, — —	606
Nebuchadnezzar king of Babylon, —	604
The kingdom of Macedon founded by Carannas and Perdiccas, — —	596
Zedekiah reigns in Judea.	
Thales Mileſius.	
Cyaxeres makes war againſt the Lydians,	590
Judith.	
Arphaxad reigns over the Medes, —	589
Nebuchadnezzar burnt the temple of Solomon, and carried the Jews captive to Babylon, — — — —	588
Meſſana built, — — —	588
The *Aſſyrians* beaten by the Medes, under Aſtyages, — — —	586
A total eclipſe of the ſun, predicted by *Thales* put an end to the war between the	

	ye. bef. Chriſt.
the Medes and Libians, May 28th,	585
Baruch.	
Nebuchadnezzar invades Egypt, and conquers it,	566
Darius the Mede reigns.	
Hagai, *Zechariah.*	
Solon, archon of the Athenians, makes laws for them,	562
Cyrus firſt king of Perſia,	560
Nabonadius king of Babylon,	555
Negropolaſſar defeated by Cyrus,	551
Cræſus king of Lydia conquered by Cyrus king of Perſia. So ended the kingdom of the Lydians,	550
Cyrus takes Sardes,	544
Cyrus takes Babylon,	538
Belſhazzar, the laſt king of the Babylonians.	
Cyrus overcomes Darius the Mede, and tranſlates the empire to the Perſians,	536
The *Jews* return from captivity.	
Cyrus dies,	529
Cambiſes, king of Perſia, took Memphis,	524
Subdued the Ethiopians,	523
Darius, the ſon of Hyſtaſpes, reigns in Perſia,	521
The *Magi* are ſlain.	
Zoroaſter, *Eſther*, *Pindar*, *Anacreon.*	
The ſecond temple built at *Jeruſalem* at the command of *Darius*,	520
Tarquin the laſt king of the Romans expelled, and conſuls elected,	508
Anaxagoras.	
The *Athenians* beat the Perſians at Marathon,	485
Xerxes reigns (called alſo *Ahaſuerus*)	485
Xerxes defeated by the Greeks,	484

Xerxes

	ye. bef. Chriſt.
Xerxes paſſes over the Helleſpont, fought Leonidas king of Sparta, who beat him,	480
Athens burnt by Mardonius, Xerxes's general, — —	478
Artaxerxes Longimanus reigns, —	464
Eſdras.	
Artaxerxes kills his brother, —	463
Ezra returns into Judea, — —	457
Nehemiah.	
Herodotus, *Meton.*	
The *Peloponneſian* war begins, —	431
Hippocrates, *Democritus.*	
Darius Nothus reigns, — —	424
Thucidides.	
The *Jewiſh* hiſtory ends, —	412
Ninias, the Athenian admiral, frightened by ſeeing an eclipſe of the moon,	412
Artaxerxes Mnemon reigns, —	405
Ezra, *Socrates*, *Plato*, *Zenophon*, *Pappus.*	
The end of the *Peloponneſian* war, —	404
Philip, king of Macedon, reigns,	362
Malachi, *Nehemiah*, *Eudoxus*, *Ariſtotle.*	
Artaxerxes Orchus reigns, — —	359
Arogus reigns, — —	338
Darius Codomannus reigns in Perſia,	336
Alexandria in Egypt built, —	335
Alexander the Great conquers the Perſians, and tranſlates the empire to the Greeks,	332
Darius Codomannus, the laſt king of Perſia, ſlain, — —	331
Alexander the Great died at Babylon,	324
Ptolomy Soter,	320
Eratoſtenes, *Epicurus.*	
Antigonus routed by the Syrians, —	316
Agathocles king of Sicily, —	316

An

	ye. bef. Chriſt.
An eclipſe of the ſun happened Aug. 15, when *Agathocles* was croſſing the ſea againſt the Carthaginians, — —	316
Euclid, *Manetho*, *Beroſus*.	
Ptolomeus Philodelphus reigns in Egypt, *Aratus*.	277
The *Arundelian* marbles compoſed, —	264
Firſt *Punic* or *Carthaginian* war, —	264
Quintus Fabius Pictor, the oldeſt Latin hiſtorian, — —	224
The *Chineſe* wall built, — —	222
The ſecond *Punic* war, — —	217
Hannibal, the Carthaginian, defeats the Romans at Pavia, — —	216
Hannibal defeats the Romans at Canno, *Archimedes*.	215
Syracuſe taken by Marcellus, —	213
Antiochus Epiphanes began to reign in Syria,	175
Antiochus plunders the temple at Jeruſalem, and murders the Maccabees, — —	173
Macedon becomes a Roman province. A lunar eclipſe, — — —	168
Judas Maccabeus routs Antiochus, —	166
Terence, *Hipparchus*, *Hippocrates*.	
The third *Punic* war, — —	149
Carthage and Corinth burnt by the Romans,	146
The hiſtory of the *Maccabees* ends, —	135
Lucilius, *Galen*, *Polibius*.	
Civil wars at *Rome* between Pompey and Cæſar, — — —	82
Jeruſalem taken by Pompey, — —	62
Cataline's conſpiracy, *Cicero*, *Varro*, —	61
The Triumvirate of *Pompey*. *Rome* enſlaved,	58
Cæſar conquers the Gauls, — —	53
—— invades Britain, — —	52
Julius Cæſar defeats Pompey, — —	49

	ye. bef. Chriſt.
Pompey ſlain by Cæſar, in Pharſalia,	46
Saluſt.	
Alteration of the year by *Julius Cæſar*, where he corrects the calendar,	46
Julius Cæſar burns the famous library at Alexandria, containing 700,000 volumes,	44
— killed in the Senate houſe by *Brutus* and *Caſſius*,	43
Herod made king of Judea,	40
Brutus and *Caſſius* being defeated kill themſelves.	
Diodorus Siculus.	
The triumvirate of *Mark Anthony*, *Lepidus* and *Auguſtus*,	40
Jeruſalem taken by Herod,	37
Anthony loſes the battle of Actium,	31
Cleopatra dies, and Egyyt becomes a Roman province,	29
Virgil, *Horace*, *Ovid*, *Lucretius*, *Livy*, *Dionyſius*, *Cornelios Nepos.*	
Auguſtus Emperor, and ſole maſter of the world,	28
Jeſus Chriſt born,	2
Herod murders the innocent children.	

The vulgar æra of Chriſt begins,	ye. aft. Chriſt.
Trogus Pompeius.	
Auguſtus dies; Tiberius ſucceeds,	14
Jeſus Chriſt crucified,	34
Caius Caligula Emperor,	37
Philo, *Seneca.*	
Claudius Emperor,	41
Nero Emperor,	54
Sets *Rome* on fire for ſport,	64
Galba Emperor,	69

	ye. aft. Chriſt.
Vitellius Emperor,	70
Flavus Veſpaſian Emperor,	71
——He ſent his ſon *Titus* into Judea, who deſtroyed Jeruſalem. Since that, the Jews are diſperſed all over the world.	
Joſephus, *Pliny*, *Quintus Curtius*.	
Titus Veſpaſian Emperor,	79
Domitian Emperor,	81
Cornelius Tacitus.	
Cocceius Nerva Emperor,	96
Suetonius Tranquillus, *Ptolomy.*	
The temple of *Pantheon* in Rome beaten down with thunder,	110
Adrian Emperor,	117
Plutarch.	
Arrian.	
The *Picts* wall built,	123
Appian, *Dion Caſſius.*	
Antoninus Pius Emperor,	138
Diogenes Laertius, *Juſtin.*	
M. A. Antoninus Emperor,	161
Commodus Emperor,	180
Sep. Severus Emperor,	193
Philoſtratus.	
A. Baſtianus Caracalla Emperor,	211
Op. Macrinus Emperor,	217
Lucius Florus.	
Herodian,	250
Diocleſian Emperor,	284
Conſtantine the Great Emperor,	306
Euſebius.	
The council of *Nice* held,	325
Conſtantinople rebuilt,	330
Julian Emperor,	337
St. Jerome, *Rufinus.*	
Valentinian and *Valens* Emperors,	364

	ye. aft. Chriſt.
Epiphanes.	
Theodoſius Emperor,	379
Zozimus, *St. Auſtin.*	
Honorius Emperor of Rome,	396
Rome taken by the Goths,	410
The monarchy of *France* founded,	420
Venice built,	421
Valentinian III. Emperor,	425
Theodoret.	
Auguſtulus, the laſt Emperor of Rome,	476
Arthur king of Britain,	516
Juſtinian Emperor,	527
The Chriſtian æra begun to reckon by,	532
The ſeat of the empire removed to *Conſtantinople*,	550
Procopius.	
Tiberius II. Emperor,	575
Mahomet's flight to Medina,	622
Damaſcus and *Jeruſalem* taken by the Saracens,	636
Conſtantius Emperor,	699
Ven. *Bede.*	
Juſtinian II. Emperor, but expelled after,	686
Theodoſius III. Emperor,	714
The *Spaniards* defeat the Moors at Tours,	716
Pepin king of France.	
Charles the great, king of France,	767
Alphonſus king of Spain, who freed it from the tribute of 100 virgins,	793
Charlemaign Emperor of Rome,	800
Egbert firſt ſole monarch of England,	819
Ethelwolf king of England,	837
Alfred, king of England, founds the univerſity of Oxford,	874
The *Normans* ſettle in Normandy,	884
Eudo king of France drives out the Normans,	889
Lewis IV. Emperor, the pope ſets up another,	900

	ye. aft. Chriſt.
Alexander, Emperor of the Eaſt,	911
Athelſton king of England,	924
Otho the great, Emperor of the Weſt,	938
J. Zimiſca, Emperor of the Eaſt,	969
Otho II. Emperor of the Weſt,	973
Edward (the martyr) king of England,	975
The *Danes* invade England and are beaten,	1002
Henry (duke of Saxony) choſen Emperor,	1008
Edmund (Ironſide) king of England,	1016
Guido Aretinus, invents the notes of muſic,	1022
Harald the Dane, king of England,	1036
Edward (the Confeſſor) king of England,	1042
Harrold uſurps the crown of England,	1066
William the conqueror king of England,	1066
The *Turks* conquer Perſia, take Jeruſalem, &c.	1070
The holy war againſt the Turks,	1096
Jeruſalem regained from the Turks,	1099
Henry V. Emperor of Germany,	1106
J. Comnenus, Emperor of Greece,	1118
E. Comnenus, Emperor at Conſtantinople,	1143
Henry II. king of England,	1154
Saladine, ſultan of the Turks,	1170
London bridge built of ſtone,	1187
Jeruſalem retaken by the Turks,	1187
Richard I. king of England,	1189
Henry VI. Emperor of Germany,	1190
Baronius.	
John, king of England, and granted Magna Charta,	1199
Henry III. king of England,	1216
The inquiſition firſt erected againſt the *Albagenſes*,	1222
Frederic, Emperor of Germany, recovers Jeruſalem,	1229
Henry III. confirms Magna Charta,	1253

Richard

	ye. aft. Chriſt.
Richard (brother to king Henry) Emperor,	1256
The *Barons* take king Henry priſoner, for breaking Magna Charta,	1258
M. Paleologus regains Conſtantinople,	1260
Otteman the founder of the preſent Turkiſh Empire,	1297
The *Mariner's* compaſs found out,	1302
The oppoſers of the Romiſh ſuperſtition burnt at *Auſtria*,	1315
Edward III. king of England, beat the French, and took the French king priſoner,	1327
A. Peleologus III. Emperor at Conſtantinople,	1326
Gunpowder invented in *Germany*,	1344
Charles IV. (K. Bohemia) Emperor of Germany,	1347
The *Turks* take Adrianople,	1359
Richard II. king of England,	1377
Tamerlane the Tartar reigns,	1387
Bajazet the Turk reigns,	1388
Tamerlane defeats Bajazet, conquers Babylon, Perſia, China, India,	1399
The great *Mogul* deſcended from him.	
Richard II. depoſed,	1399
Henry IV. king of England, the firſt of the line of Lancaſter,	1399
Civil wars in *Germany*,	1404
Sigiſmond (king of Hungary) Emperor,	1410
Henry V. king of England,	1413
——Beats the French at the battle of Agincourt,	1415
Henry VI. king of England,	1422
J. Huniades, Emperor of Conſtantinople,	1442
The Art of Printing invented,	1449
Mahomet II. ſultan of the Turks, takes Conſtantinople,	1453

	ye. aft. Chriſt.
The *Turks* conquer *Peloponneſus* or *Morea*,	1459
Henry VI. depoſed, and *Edward* IV. made king,	1460
Edward V. king of England,	1483
Richard III. king of England,	1483
Henry VII. king of England,	1485
C. Columbus (a Genoeſe) diſcovered America,	1493
Americus Veſpaſius diſcovers South America,	1497
Henry VIII. king of England,	1509
M. Luther begins the Reformation,	1517
Boetius, *Eraſmus*, *Copernicus*.	
Solyman Emperor of the Turks,	1520
Belgrade taken by the Turks,	1521
The Streights of *Magellan* diſcovered by *F. Magellan*,	1521
The Council of *Trent*,	1540
Edward VI. king of England,	1547
Mary, queen of England, reſtores Popery,	1553
Eliſabeth, queen of England, reſtores the Reformation,	1558
Charles IX. king of France,	1560
The Maſſacre of the Proteſtants at *Paris*,	1572
Mercator.	
The *Venetians* defeat the Turks at ſea,	1572
Henry III. king of France,	1574
Rodulphus II. Emperor,	1576
The holy League in *France*,	1576
Holland revolts from Spain,	1579
Sir *Francis Drake* ſails round the Globe,	1580
Tycho Brahe, *Scaliger*.	
Pope *Gregory* XIII. corrects the Calendar,	1582
Mary queen of Scots beheaded,	1587
The *Spaniſh Armado* deſtroyed by the Engliſh	1588
Henry III. of France, ſtabbed by Clement, a friar,	1589
Philip III. king of Spain,	1598

Vieta

	ye. aft. Chriſt.
Vieta.	
James I. king of England, —	1603
Gunpowder Plot, — —	1605
Henry IV. of France ſtabb'd by Ravillac,	1610
Lewis XIII. king of France, —	1610
Guſtavus Adolphus, king of Sweden,	1611
The *New River* brought to London,	1614
Frederic II. Emperor, — —	1619
Des Cartes.	
Philip IV. king of Spain, — —	1621
Charles I. king of England, —	1625
G. *Adolphus* ſlain in battle, —	1632
Ferdinand III. Emperor, — —	1637
Portugal revolts from Spain, —	1640
Lewis XIV. king of France, —	1643
King *Charles* I. beheaded, — —	1649
Oliver Cromwell died, —	1658
King *Charles* II. reſtored, —	1660
The great Plague at *London*, —	1665
London burnt down, — —	1666
Ricciolus.	
Peace concluded with the *Dutch*,	1674
St. *Paul's* church begun to rebuild, —	1675
The *Czar of Muſcovy* dies, and his ſons rule jointly, — —	1682
The Emperor at war with the *Turks*,	1682
The *Turks* defeated before Vienna,	1683
The great Froſt,	1684
King *Charles* II. died, and is ſucceeded by James II. — —	1685
James II. depoſed, — — —	1688
William and *Mary* king and queen of England, — — —	1689
The battle of *Boyne* in Ireland, —	1690
An Earthquake in England,	1692
Queen *Mary* died, — — —	1694

	ye. aft. Chriſt.
Queen *Anne* came in,	1702
Battle of *Blenheim*,	1704
Battle of *Ramalies*,	1706
England and *Scotland* united,	1707
St. *Paul's* church finiſhed,	1708
Queen Anne died,	1714
George I. king,	1714
A total eclipſe of the ſun at London, April 22d,	1715
Rebellion at *Preſton*,	1715
The *Aurora Borealis* firſt obſerved,	1716
The *Spaniſh* fleet deſtroyed near Sicily,	1718
Sir *I. Newton* died,	1727
George II. king,	1727
A great Froſt 1739, and	1740
Battle of *Dettingen*,	1743
A bright Comet, in the ſpring,	1744
Rebellion at *Culloden*,	1746
Correction of the Calendar in *England*,	1752
George III. king,	1760
Peace with *France* and Spain,	1763

FINIS.

CALCULATION, LIBRATION, AND MENSURATION;

Or the ARTS of

RECKONING, WEIGHING, AND MEASURING.

BEING A

MECHANICAL WORK,

ADAPTED

To the Buſineſs and Practice of TRADESMEN and ARTIFICERS, in the ſhorteſt Method poſſible; and deſigned purely for common Uſe.

Pondere, menſura, et numero, Deus omnia fecit.

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MDCCLXX.

THE

PREFACE.

I Believe there never was a courſe of the Mathematicks printed, but what included the practical parts thereof, and ſuch things as are of conſtant and daily uſe among mankind. Therefore, leſt ours ſhould appear defective in this point, we ſhall comply with this cuſtom; and to oblige the lower ſort of readers (for every one is not deſtined to ſtudy the ſublime parts of the Mathematicks) I have compoſed the following Treatiſe.

In the firſt part I have put together a great number of rules for the reckoning up of ſuch things as commonly occur in trade and buſineſs; wherein I have endeavoured to ſuit the capacities of the loweſt claſs of tradeſmen and artificers, by laying down ſuch ſhort rules, for computing the values of things, as may with eaſe be retained in the memory, are readily and eaſily wrought, or rather performed in the mind, which are to be uſed as occaſion offers; and therefore muſt be very ſerviceable to ſuch as never had an opportunity of learning Arithmetic. And even to ſuch people as underſtand numbers, theſe rules will ſave them ſo much trouble and time. For men cannot always allow time to ſit down and compute every trifling matter, which may be done at once by this way of reckoning. For few of theſe rules need the uſe of Arithmetic; and thoſe that do require but an eaſy calculation. But where there is any danger of making miſtakes, it is proper, for thoſe that underſtand it, to uſe a little Arithmetic. And that no doubt or obſcurity may ap-

pear in theſe rules, I have given examples thereof, in the common caſes of trade; ſo that the meaneſt reader cannot be at a loſs for underſtanding them. And before this part, the common definitions are prefixt, alſo an account of weights and meaſures, tables of ſpecific gravity, the practice of workmen in meaſuring their work, and ſuch other things as are uſeful in a Treatiſe of this kind.

In the ſecond part are delivered ſuch common problems, as are requiſite in the art of Meaſuring, being ſuch as are introductory to what is to follow. Here we treat of the meaſuring of lines; and how, by having one line in a figure given, to find another. To this part is prefixt the deſcription and uſe of the ſliding rule, an inſtrument of great diſpatch in the art of meaſuring.

The third part teaches how to meaſure the areas of plane figures, and the ſurfaces of ſolids, and this both arithmetically with the pen, and inſtrumentally with the ſliding rule. In this part it is alſo ſhewn, how theſe fixt numbers are found, which are made uſe of in the different methods of finding the area of the circle, by the ſliding rule.

In part the fourth, we have the menſuration of ſolids, eſpecially the moſt common; and particularly the meaſuring of timber; and this both by Arithmetic with the pen, and alſo by the ſliding rule, when not too troubleſome. In this part it is alſo ſhewn how all the fixt numbers are found, which are made uſe of on the ſliding rule, for meaſuring cylinders and ſpheres.

The fifth and laſt part is gauging, contained in a narrow compaſs. Here it is ſhewn how to find the areas of plain figures, and the contents of ſolids, according to the common method of gauging; that is in ale or wine gallons, inſtead of inches or feet. How to inch any tun; and a general rule for gauging all caſks; and how to find the ullage; alſo how to gauge malt. To this part is prefixt a ſhort account of the *ſliding rule adapted to this buſineſs; an inſtrument which very much ſhortens all calculations in gauging. And in this part is ſhewn likewiſe, how to find the fixt numbers and gauge points, made uſe of on the rule, and how to work with them. In all theſe articles are given proper examples, to make the rules plain, which are wrought in the ſhorteſt method, both by the pen, and alſo with the ſliding rule, when it can conveniently be done.*

I have given no demonſtrations here, of any of theſe rules; for the practical Mechanic does not want them, and a mathematical reader may have them elſewhere, as in the Geometry, Conic Sections, Algebra, &c.

There is no ſtate of life which does not require the uſe of ſome ſort of meaſures, either in length or capacity, dry or liquid; on theſe moſt trades depend, whether it be thoſe that procure food for us, or thoſe that furniſh us cloathing. Menſuration is the ground of all arts, and of all things depending on trade, without which they could not ſubſiſt.

In meaſuring and gauging, the practitioner muſt underſtand common Arithmetic, without which theſe rules cannot be put in practice; and alſo decimals, as all ſorts of computations are eaſieſt made thereby; and particularly contractions in multiplication and diviſion, which often ſave long calculations. Tho' in ſuch caſes, I would adviſe to make uſe of logarithms, which in moſt caſes will be ſhorter.

As geometrical rules are made uſe of for ſuch purpoſes as require calculations to be made to the utmoſt exactneſs; ſo the ſhort rules in this Treatiſe, are moſt of them only approximations, being contrived to compute, or meaſure things for common uſe with expedition, and which require not that degree of exactneſs. They are adapted for preſent extemporary buſineſs, and for ſuch things as require to be done off hand.

When any plane figure or ſolid body is propoſed to be meaſured, we have no rule to determine it to be any particular geometrical figure or ſolid, but by gueſs. And therefore it is to be looked on as a mechanical figure or ſolid; and muſt be meaſured by ſome ſhort mechanical rule, ſuch as ſuits it, and its ſhortneſs will make amends for other defects. For by ſuch a rule the content may be found as truly, as the dimenſions can be taken, and that is ſufficient. And all ſuch rules muſt be eſteemed good rules; when they come very near, at a medium; as if in one caſe they give a ſmall matter too much, and in another caſe a ſmall matter ſhort of the truth, we may admit them as true rules; ſince the figure we meaſure may not have that geometrical exactneſs that we ſuppoſe; and it is mere chance if it has.

W. Emerſon.

Marks and Abbreviations.

$+$ to be added.
$-$ to be ſubſtracted.
$\times$ to be multiplied by.
$\div$ to be divided by.
$\surd$ the ſquare root of.
a^2 the ſquare of a.
a^3 the cube of a, &c.
$=$ is equal to.
$a : b :: c : d$, as a to b, ſo is c to d.
AG, ale gallons.
WG, wine gallons.
MB, malt buſhels.
GP, gauge point.

Calcu-

Calculation, Libration, &c.

DEFINITIONS.

1. *COMPUTATION* or Numbering is the Art of Reckoning; which is ſometimes performed barely by the memory; but generally by the help of theſe figures, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

2. *Libration* is the Art of Weighing any ſort of bodies. This is performed by ſome proper inſtrument, as a pair of ſcales, a ſteelyard, &c. and by the help of certain fixt weights, agreed on to be the ſtandard, for weighing all other things by.

3. *Menſuration* is a mechanical art, teaching how to find the content of any ſurface, or ſolid body, from the dimenſions thereof, taken by actually meaſuring them. The content of any ſurface is commonly called the *Area* of it.

4. *A Line* is the length or diſtance between any two points; and this length is eſtimated by the number it contains, of ſome known meaſure, as yards, feet, inches, &c.

5. *A Superficies* is that which hath length and breadth, and its quantity is meaſured by the number of ſquares it contains, whoſe ſides are linear units, of that meaſure it is meaſured by.

6. *A Solid* is that which hath three dimenſions, length, breadth, and thickneſs. And its content is eſtimated by the number of cubes it contains, whoſe ſides are linear units of that ſort of meaſure it is meaſured by.

7. *A right Angle* is that made by one line DC 14.
ſtanding upright upon another BA, ſo that CD lean

Fig. lean no more to one ſide than to another; thus CDA
is a right angle, and alſo CDB. If CD is not upright,
it is an *oblique angle*; an *acute angle* is leſs,
and an *obtuſe* one greater, than a right angle.

1. 8. *Parallel lines* are thoſe that are every where
equidiſtant, as AB and CD.

14. 9. *A Perpendicular*, is a line which ſtands at right
15. angles upon a line or a plane, and inclines to no
ſide. This is the neareſt diſtance from any point
to a line or a plane, as CD.

14. 10. *A Triangle* is a figure contained under three
15. right lines. If one angle be right, it is a *right angled triangle*.

16. 11. *A Square* is a figure contained by 4 equal
ſtreight lines and 4 right angles, as ABCD.

17. 12. *A Parallelogram* is a figure contained by
18. four right lines, ſo that the oppoſite be equal and
parallel.

18. 13. *A Trapezoid* is a four ſided figure, whereof
ſome two oppoſite ones are parallel, as ABCD.

19. 14. *A Trapezium* is any figure included by four
lines, as ABCD. A line drawn thro' the oppoſite
angles is called a *Diagonal*, as AC.

21. 15. *A Polygon* is a figure conſiſting of any num-
22. ber of ſides. If all the ſides and angles are equal,
it is a *regular Polygon*; if it have three ſides, it is
called an *equilateral Triangle*; if four, a *Square*; if
five, a *Pentagon*; if ſix, a *Hexagon*, &c.

23. 16. *A Circle* is a figure bounded by the curve
line ABCD, called the Circumference, in which all
lines drawn from the *Center* C to the circumference
are equal. Any line AD paſſing thro' the center
C is called a *Diameter*; and any line CD, drawn
from the center C to the circumference, is called
the *Radius*. Any right line drawn through it is
called a *Chord*.

17. *A Cube* is a ſolid contained under 6 equal Fig.
ſquares ſtanding at right angles to one another; 38.
like a die.

18. *A Parallelepipedon* is a ſolid contained under 40.
6 planes, whereof every two oppoſite ones are parallel.

19. *A Priſm* is a ſolid whoſe two ends are equal 41.
and like, and parallel to one another. 42.

20. *A Cylinder* is a ſolid whoſe ends are two e- 39.
qual circles, parallel to one another.

21. *A Pyramid* is a ſolid whoſe baſe is any plane 44.
figure, and the ſides all triangles meeting in a point called the Vertex, as D.

22. *A Cone* is a ſolid ABC contained between a 43.
circular baſe, and a convex ſurface ending in a point A called the Vertex.

23. *A Poliedron* is a ſolid contained under ſeveral planes, if all the planes be any regular poligon, it is called a *regular body*, of which there are but five. 1. The *Tetraedron*, has four faces equilateral triangles. 2. The *Cube* has ſix ſquares. 3. The *Octaedron*, 8 triangles. 4. The *Dodecaedron*, 12 pentagons. 5. The *Icoſaedron*, 20 triangles.

24. *A Sphere* ABCD, is a ſolid contained under 33.
one curve ſurface; within which there is a point called the *Center*, from which all lines drawn to the ſurface are equal, and called the *Radius*. Any line paſſing thro' the center is a *diameter*.

25. *A Segment* is any part of a figure or ſolid cut off by a line or a plane.

26. *A Fruſtum* is the thick end of a ſolid cut off by a plane parallel to the baſe.

27. *Vaults* are curved arches, that ſpring from the oppoſite ſide walls; and meet in a right line over the middle of the building. If they meet in an angle, they are called Gothic arches. The arches of a bridge are in this form.

28. *Dome's* or *Cupolo's* are arched roofs that meet in a point at the top; the baſe being a circle or poligon.

29. *A Saloon* is a flat roof or cieling, joined to the ſide walls by arches.

30. *A Groin* is when a vaulted roof is interſected by ſome other vaults.

31. *A Meaſure* is a certain length made uſe of to meaſure another greater length by, in order to know how many of theſe the greater contains. When a length is to be meaſured, it is meaſured by a line, and is called *lineal meaſure.* When a ſurface is to be meaſured, it is done by the ſquare of that line, and is called *ſuperficial meaſure.* If a ſolid is to be meaſured, the meaſure is the cube of that lineal meaſure, and then it is called *ſolid meaſure.* Thus a foot in length meaſures any other length; and a ſquare foot, any ſurface; and a cube foot any ſolid; and ſhews how many feet each contains. And the like for inches, yards, &c.

In meaſuring any ſurface or ſolid, all the dimenſions muſt be taken in one and the ſame kind of meaſure, except it be otherwiſe expreſſed.

A Table of Numbers.

12 Units	make	a Dozen,
12 Dozen		a Groſs,
20 Units		a Score,
5 Score		a ſhort Hundred (C)
6 Score		a long Hundred (C)
10 Hundred		a Thouſand (M)

A Table of Money.

4 Farthings	make	1 Penny (d)
4 Pence		1 Groat,
12 Pence		1 Shilling (s)
5 Shillings		1 Crown,
6 Shil. 8 d.		1 Noble,
10 Shillings		1 Angel,
13 Shill. 4 d.		1 Mark,
20 Shillings		1 Pound (l)
21 Shillings		1 Guinea.

A Table of Time.

60 Seconds (″)	make	1 Minute (′)
60 Minutes		1 Hour,
24 Hours		1 Day natural,
7 Days		1 Week,
4 Weeks		1 Month,
365¼ Days		1 Year.

Goods ſold by the Dozen are,

Buttons, candles, hats, ſtockings, lint, bottles, bruſhes, Chinaware, pigeons, apricots, and peaches.

Goods ſold by the Groſs are,

Buttons, beads, tobacco-pipes, combs, comb-caſes, playing cards, knives, thread, ſilk hair, tobacco-boxes, buckles, ink-horns, bodkins, pepper-boxes, tinder-boxes, waſh-balls, and ſpectacle-caſes.

Goods ſold by the Score are,

Paper, walking-ſticks, marbles.

Goods ſold by the ſhort Hundred are,

Capers, thrums, ginger, cloves, indigo, pack-thread, quick-ſilver, cotton-wool, braſs and tin-work, ſeveral ſorts of ſkins, braſs and tin implements.

Goods ſold by the long Hundred are,

Herring, ling, cod, and all ſtock-fiſh, balks, clap-boards, nails, fiſh-hooks, pipe-ſtaves, bow-ſtaves, deals, ſpurs, outlandiſh cloth, lamb-ſkins, ſeveral ſorts of outlandiſh cloth, and eggs.

Goods ſold by the Thouſand are,

Paſte-boards, tennis-balls, lemons, oranges, boards, quills, teazles, tiles, bricks, tacks, tenter-hooks, thimbles, pins and needles, billets, lanthorn-horns, barrel-hoops, ox-horns, ſquirrel-ſkins, ſlates, &c.

Standard gold is 11^{oz} fine, and 1^{oz} alloy.

Standard ſilver is 11^{oz} 2^{dw} fine, and 18^{dw} alloy.

A pennyweight of gold coin is worth 3 s. 10d$\frac{1}{2}$, which is near 2 d. a grain.

44$\frac{1}{2}$ Guineas make 1 pound Troy; and 1 guinea, 129 grains.

An ounce of ſilver with alloy is worth 5 ſhillings.

An ounce of gold is worth 3 l. 18 s.

22 Pence in copper is about 1 l. Averdupoiſe.

A Table of Averdupoiſe weight.

16 Drams	make	1 Ounce (oz)
16 Ounces	make	1 Pound (lb)
14 Pounds	make	1 Stone (St)
8 Stone	make	1 Hundred weight (C)
20 Hundred	make	1 Tun. (T)

8 lb make a ſtone among the London butchers.

A Table

A Table of Troy Weight.

20 Mites	make	1 grain, (gr)
24 Grains		1 Penny weight, (dw)
32 gr. wheat		1 Penny weight,
20 Penny weight		1 Ounce, (oz)
12 Ounces		1 Pound. (lb)

The Apothecaries divide the Troy ounce thus, 20 grains make a ſcruple (℈), 3 ſcruples a dram (ʒ), 8 drams an ounce (℥).

By Averdupoiſe weight, is weighed mercery ware, and groceries, and all things that can waſte, as pitch, tar, roſin, wax, tallow, flax, hemp, fleſh, butter, cheeſe, ſalt, candles, ſoap, ſtarch, ſpices, coffee, tea, chocolate, ſilk, bread in the country, and ſometimes oil; alſo copper, tin, iron, ſteal, lead, &c.

By Troy weight is weighed gold, ſilver, jewels, amber, apothecaries drugs, bread, liquors, precious ſtones.

Weights compared.

The Averdupoiſe pound is 7000 grains, the Troy pound 5760 grains.

The Averdupoiſe ounce is 437½ grains, the Troy ounce 480 grains.

The Troy pound is to the Averdupoiſe pound, as 9 to 11, or rather, as 14 to 17.

The Averdupoiſe ounce is to the Troy ounce, as 10 to 11, or more exactly, as 73 to 80.

The London pound is to the *Paris* pound, as 100 to 109.

The Troy ounce is to the *Paris* ounce, as 64 to 63.

And the Averdupoiſe ounce is to the *Paris* ounce, as 63 to 68.

A Table

A Table of long Meaſures.

12 Lines		1 Inch,
3 Barley corns		1 Inch,
12 Inches		1 Foot,
3 Feet		1 Yard,
5 Feet		1 Pace,
2 Yards	make	1 Fathom,
5½ Yards		1 Pole or Rod,
4 Poles, 100 links		1 Chain,
40 Poles		1 Furlong,
8 Furlongs		1 Mile,
69½ Miles		1 Degree on the earth.

Alſo 1 finger or thumb is an inch; 4 inches a hand; 9 inches a ſpan; 2 ſpans a cubit; 5 ſpans an ell; 16 nails a yard; 7 yards 1 rod in the north; a geographical mile 2030 yards; the *Engliſh*, *French* and *Swediſh* feet are to one another, as 1 to 1.066, and 0.97; a toiſe is 6 *French* feet.

A Table of ſquare Meaſures.

144 Inches		1 Foot,
9 Feet		1 Yard,
30¼ Yards		1 Pole or Perch,
160 Feet		1 Square,
16 Poles	make	1 Chain,
40 Poles		1 Rood,
10 Chains		1 Acre,
4 Roods		1 Acre,
640 Acres		1 Mile.

A Table of ſolid Meaſure.

1728 Inches	make	1 Foot,
27 Feet		1 Yard.

A Table

A Table of dry Meaſure.

268.8 cub. Inches, or 4 Quarts,	make	1 Gallon,
2 Gallons		1 Peck,
4 Pecks, or 2150.4 cub. Inches		1 Buſhel,
8 Buſhels		1 Quarter,
4 Quarters		1 Cauldron,
10 Quarters		1 Laſt.

Alſo 2 pints make a quart; 2 pottles 1 gallon; 2 buſhels a ſtrike; 4 buſhels a comb; 2 weys a laſt.

Water meaſure is upheap'd meaſure, or elſe allows 5 ſtriped pecks to the buſhel.

By this, grain, coals, ſalt, lime, are meaſured.

A Table of liquid Meaſure.

4 Jacks	make	1 Pint,
2 Pints		1 Quart,
4 Pints		1 Gallon,
231 cub. Inches		1 Gallon of wine,
282 cub. Inches		1 Gallon of ale or beer,
8½ Gallons		1 Firkin or Anchor,
6 Firkins, or 51 Gallons		1 Hogſh. of ale or beer,
63 Gallons		1 Hogſhead of wine,
4 Hogſheads		1 Tun.

Alſo 2 gills make a pint; 2 quarts, a pottle; 18 gallons, a runlet; 42 gallons, a terce; 2 terces, a puncheon; 2 firkins, a kilderkin; 4 firkins, a barrel; 2 hogſheads, a pipe or butt.

Note, 8 gallons make a firkin of ale, and 9 of beer, in London; and the hogſhead proportionable. Alſo 4 gills make a pint there.

A *Paris* pint contains 49 cubic inches.

Oil, honey, &c. are meaſured by wine meaſure, and ſometimes by weight Averd. Alſo brandy, ſpirits, mead, perry, cyder, vinegar, verjuice, ſweets, mum, &c.

A Wincheſter buſhel ought to be 8 inches deep, and 18$\frac{1}{2}$ inches diameter.

A ſtraked buſhel is to the ſame unheaped as 3 to 4; therefore the heap is $\frac{1}{3}$ the buſhel; or, as ſome ſay, $\frac{1}{4}$ thereof.

The old wine gallon was 224 cubic inches.

The old corn gallon 272 cubic inches.

The corn gallon is to the ale gallon, as 20 to 21.

A hogſhead of water (wine meaſure) is about 8$\frac{1}{2}$ ſolid feet; the ſame by ale meaſure at 51 gallons per hogſhead. And wine meaſure to ale meaſure, as 9 to 11.

A cubic foot is to the Wincheſter buſhel, as 4 to 5; or exactly, as 45 to 56.

The Engliſh foot is to the *Paris* foot, as 107 to 114.

SCHOLIUM.

However theſe things may have varied; yet it is very probable that, originally, a hogſhead of wine and ale was the ſame in quantity; and a tun of water juſt a tun weight.

Hence we may ſuppoſe, that anciently 8 cubic feet of water made juſt a hogſhead, and 4 hogſheads a tun. And conſequently that a tun weighed 2000 lb.

Likewiſe that a hundred weight was then but 100 lb; for this multiplied by 20, makes 2000 lb. alſo for a tun; and was increaſed afterwards by merchandizing, upon account of the waſte in goods.

Alſo if a buſhel of good wheat weighs 62$\frac{1}{2}$ lb, then the quarter would be 500 lb, and the chaulder 2000 lb. which is alſo a ton. So that originally, what was a ton in meaſure was alſo a ton in weight. A ton in weight has been the common ſtandard of all things.

Weight and Meaſure compared.

1 lb. Troy of water is $22\frac{3}{4}$ inches ſolid.

1 lb. Averdupoiſe of water is $27\frac{1}{2}$ ſolid inches.

1 inch of water weighs .58 ounces Averdupoiſe.

A cubic foot of water weighs 76 lb. Troy, or $62\frac{1}{2}$ lb. Averdupoiſe, which is 1000 ounces, and contains $6\frac{1}{8}$ ale gallons.

A wine gallon of water weighs 8.35 lb. Averdupoiſe.

An ale gallon of water weighs 10.2 lb. Averdupoiſe.

A ton of water weighs $18\frac{3}{4}$ hundred weight.

A cylinder of water of *d* inches diameter, and a yard high, contains $\frac{1}{10}dd$ ale gallons; and weighs $1\frac{1}{5}dd$ pounds Averdupoiſe.

A Table of ſpecific Gravities, ſhewing the weight of a cubic Inch, of ſeveral bodies, in ounces Averdupoiſe.

	oz.		oz.
Fine gold	11.366	Caſt iron	4.090
Standard gold	10.930	Marble	1.569
Quickſilver	8.102	Common glaſs	1.493
Lead	6.554	Flint	1.490
Fine ſilver	6.418	Portland ſtone	1.488
Standard ſilver	6.096	Free ſtone	1.465
Copper	5.208	Common ſtone	1.446
Plate braſs	4.832	Brick	1.157
Caſt braſs	4.630	Alabaſter	1.084
Steel	4.544	Ivory	1.055
Iron	4.423	Horn	1.055
Pewter	4.323	Water	0.578
Tin	4.236		

A Table ſhewing the weight of a cubic foot of ſeveral ſorts of Bodies, Averdupoiſe weight.

	lb.	oz.		lb.	oz.
Wheat	48	0	Lignum vitæ	82	15
Oats	29	8	Butter	58	12
White peaſe	50	8	Pitch	71	14
Barley	41	2	Mahogany	66	7
Malt	30	4	Box	64	6
Field beans	50	8	Milk, Urine, and Sea water	63	2
Wheat meal unſifted	31	0			
Rye meal unſifted	28	4	Rain water	62	8
Bay ſalt	54	1	Red wine	61	14
White ſea ſalt	43	12	Bees wax	59	11
Sand	85	4	Linſeed oil	58	4
Newcaſtle coal	67	12	Brandy	57	15
Pit coal (Wedneſbury)	63	0	Oak	57	8
			Oil olive	57	1
Gravel	109	5	Beech	53	6
Wood aſhes	58	5	Elm Aſh	49	1
Pump water	62	8			
Chalk	112	10	Cedar	38	7
Allum	107	2	Fir	36	4
Clay	107	0	Cork	14	8
Honey	90	10	Air	0	1¼

REMARKS.

20 cubic foot of hay is about a hundred weight.

56 lb. of butter make a firkin, which is 4 ſtone, Averdupoiſe.

When wheat is at 5 ſhillings the buſhel, the penny white loaf is to weigh 7¼ oz. out of London.

If a miller carries and recarries his corn, he has a quarter of a peck per buſhel, for toll and waſte. But if the owner carry it, he has but half ſo much.

40 feet of unhewn timber, or 50 feet of hewn timber, is reckoned a load or ton.

60 ſolid feet of Newcaſtle coals is a chaldron, or 36 buſhels.

A buſhel of wheat weighs about 60 lb. Averdupoiſe, which is 4 ſtone 4 lb; but ſome weighs more.

A load of hay is 36 truſſes, and a truſs four ſtone, or half a hundred.

The ſeveral ways of meaſuring different ſorts of Work.

Different work is computed by different meaſures. In meaſuring boards, timber, glaſs, ſtone; the dimenſions are taken in feet and inches; and the content caſt up in feet.

In caſks, the dimenſions are taken in inches, and the content caſt up in ſolid inches, or elſe in gallons.

In painting, plaiſtering, paving, flooring, wainſcot; the dimenſions are taken in feet or yards; and the content caſt up in yards.

In roofing, tyling, ſlating, raftering, partitioning, flooring; the dimenſions are taken in feet, and the content given in ſquares of 100 ſquare feet.

Walling is meaſured in feet or yards, and the content given in rods, containing 272 ſquare feet; or, in the North country, the rod contains 7 ſquare yards.

Land is meaſured by Gunter's chain, and caſt up in acres, at 10 chains an acre.

Earth works, as cellars, vaults, &c. are meaſured by the yard ſolid.

In any rule, &c. to meaſure with, it is the beſt way to have it divided into decimal parts; for the content of any thing is ſooneſt caſt up, when the dimenſions are taken in decimals. And the ſhorteſt way is to work by logarithms; adding the logarithms, inſtead of multiplying the numbers; and ſubtracting inſtead of dividing.

The Practice of Workmen.

Glaziers take dimenſions to a quarter of an inch, and never make any allowance for round or oval windows; but meaſure the circumſcribing parallelograms.

In plaiſtering, deductions muſt be made for doors and windows; but ſome make no deductions.

Painters, in taking the height, gird the ſtring into all the hollows, and round all the ſwellings of the pannels, &c. And in taking the height of a door, or the breadth, they add the thickneſs. Their price is according to the number of times the colour is laid on. Deductions are made for chimnies, caſements, &c.

Joiners, in meaſuring the height of a room, put the ſtring into all the ſwellings and mouldings. But in taking the length, they meaſure in a ſtrait line. Deductions are made for all vacancies, as windows, chimneys, &c. In partitioning, deductions muſt be made for doors and windows.

In flooring, the hearth-ſtone is deducted, except there be a border round it. Window ſhutters and doors are commonly reckoned meaſure and half; becauſe they are worked on both ſides.

In roofing, tiling, ſlating, it is cuſtomary to reckon the flat and half of building within the walls, for the meaſure of the roof, when it is a true pitch; that is, when the rafters are $\frac{3}{4}$ the breadth of the building. For workmanſhip alone, ſky-lights, chimney ſhafts, &c. are deducted. But finding materials and work, takes in all, except places of 9 or 10 feet area. Some never deduct for window ſhafts and ſky-lights at all; they are ſo troubleſome to the workmen.

In tiling and ſlating, ſky-lights and chimney ſhafts are deducted. The angle of the roof which bends inwards is called a *Valley*, and the angle bending outwards is called a *Hip*.

In walling a houſe, deductions muſt be made for doors and windows.

In meaſuring chimnies, gird round the middle for the breadth, and the height of the ſtory is the length.

In meaſuring timber, it is cuſtomary for the buyer to take the girt any where he pleaſes, between the greater end and the middle of the tree. Branches of two feet compaſs are reckoned timber; any thing leſs is not timber. When the bark is on, deduct $\frac{1}{10}$ or $\frac{1}{12}$ the circumference.

PART I.

Computing and Weighing.

PROB. I.

To reduce Guineas into Pounds.

1 RULE.

TAKE the tenth part of the Guineas, and add the half thereof to the guineas, for the pounds.

What remains is ſhillings.

Examp.

In 674 Guineas, how many Pounds?

The tenth of 674 (dropping 4) is 67, half of 67 (dropping 1) is 33. Then 33 added to 674, gives 707 pounds, to which put the 14 ſhillings remaining.

2 RULE.

Call the guineas ſhillings, which reduce into pounds, and add to the guineas.

Example.

How many Pounds in 168 Guineas?

Here 168 ſhillings is 8 l. 8 s. which added to 168, gives 176 l. 8 s.

Cor. *To reduce pounds to guineas, ſubtract the 21ſt part thereof.*

PROB. II.

Having the price of 1, *to know what* 100 *comes to?*

1 RULE.

For every farthing in the price of one, reckon twice as many ſhillings and once as many pence.

Examp.

If 1 *coſts* 9½ *pence, what will* 100 *come to?*

9 pence halfpenny is 38 farthings.
Then twice 38 is 76 s. 0 d.
And 38 pence 3 2

79 s. 2 d. = 3 l. 19 s. 2 d.

2 RULE.

For every ſhilling of the price reckon 5 pounds.
And for every 3d. beſides reckon 25 ſhillings.
And for a penny 8 s. 4 d.

Examp.

What comes 100 *to, at* 7 s. 10 d. *a piece?*

The ſhillings	—	35	0	0
The 3 pences	—	3	15	0
A penny	—		8	4
		39	3	4

PROB. III.

Having the price of 100 given, to find the price of 1.

RULE.

Take half as many farthings as there are ſhillings in the price; abating the 50th part of the whole number of farthings.

Examp.

If 100 *coſt* 33 l. *what is that a piece?*

Whole num. farthings	660	
Half the number	330	
$\frac{1}{50}$ the whole, ſub.	13	
	317	farthings.
	79$\frac{1}{4}$	pence.
Anſ.	6 s. 7 d$\frac{1}{4}$.	

PROB. IV.

Having the price of 1, to know what the long hundred will coſt.

1 RULE.

How many pence you give for each one thing,
So many angels the long hundreds bring.

Examp.

If one thing coſt 3$\frac{1}{4}$ d. *what will* 120 *be?*

3$\frac{1}{4}$ angels 32$\frac{1}{2}$ s. = 1 l. 12 s. 6 d.

2 RULE.

2 RULE.

Take half as many pounds as 1 coſts pence; adding 2 s. 6 d. for every odd farthing.

Ex.

If 1 *coſts* 13¾ d. *what is the charge of* 120?

Half of 13 is 6½	6 l.	10 s.	0 d.
3 half crowns for ¾		7	6
Anſ.	6	17	6

PROB. V.

By knowing the price of the long hundred, to know what 1 *coſts.*

1 RULE.

For every pound reckon 2 pence, and for every half crown more a farthing.

Ex.

If the long hundred (120) *coſts* 7 l. 15 s. *what does* 1 *coſt?*

7 two pences	1 s.	2 d.
6 farthings		1½
Anſ.	1	3½

2 RULE.

Double the price, then the number of pounds call pence, and the number of crowns over, call farthings.

Ex.

If deals be 18 l. 16 s. 3 d. *for the long hundred, what does* 1 *coſt?*

double, 37 l. 12 6 = 3 s.—$1\frac{1}{2}$ d. Anſ.

PROB. VI.

Having the price of 1, *to know what the thouſand comes to.*

1 RULE.

For every 6 pence of the price, reckon 25 pounds; and for every penny more, 4 l. 3 s. 4 d. or $4\frac{1}{6}$ pounds.

Or ten times the price of 100, is the price of a thouſand.

Ex.

Suppoſe 1 *coſts* 13 s. 4 d. *what does* 1000 *coſt?*

26 ſix pences	650		
4 times 4 l. 3 s. 4 d.	16	13	4
Anſ.	666	13	4

2 RULE.

Take as many pounds as there are farthings in the whole price. Then add as many pounds as there are ſix pences in the price. Alſo for every $1\frac{1}{2}$ d. above, add 5 ſhillings, and 10 d. for each farthing.

Examp.

If 1 *coſts* $14\frac{3}{4}$ *pence, what will* 1000 *coſt?*

59 farthings	59 l.	s.	d.
2 ſix pences	2		
$1\frac{1}{2}$ d.	0	5	0
5 farthings		4	2
Anſ.	61	9	2

PROB.

PROB. VII.

Having the price of a thouſand, to find the price of one.

RULE.

Take as many farthings as pounds in the price, abating the 25th part thereof.

Ex.

At 564 l. 18 s. 9 d. *per thouſand, what does* 1 *coſt?*

565 farthings 565

$\frac{1}{25}$ part ſub. 22

Anſ. 543 farthings = 11 s. 3$\frac{3}{4}$ d.

PROB. VIII.

Having the price of 1, *to find that of a dozen.*

RULE.

For every penny in the price reckon a ſhilling, and 3 pence for each odd farthing.

Ex.

If 1 *coſts* 16$\frac{1}{2}$ *pence, what will* 12 *be?*

Anſ. 16 s. 6 d.

Cor. *The price of a dozen being given; the price of* 1 *is found, by reckoning ſo many pence as there are ſhillings; or by taking* $\frac{1}{12}$ *the price.*

PROB.

PROB. IX.

Having the price of 1 thing; to find the price of a groſs.

RULE.

Find the price in pence, and take $\frac{6}{10}$ of it, for the pounds.

Ex.

If one coſts 3 s. $11\frac{1}{2}$d. *what is a groſs?*

3 s. $11\frac{1}{2}$ $47\frac{1}{2}$ pence
.6

Anſ. 28.50 or 28 l. 10 s.

PROB. X.

Having the price of a groſs, to find the price of one.

RULE.

Take the third part of the price in ſhillings, for the price of 1 in farthings.

Ex.

At 7 l. 9 s. 6 d. *the groſs, what is the price of* 1 ?

7 l. 9 s. 6 d. is $149\frac{1}{2}$ ſhillings, or 150 s.
3)150(50 far. = $12\frac{1}{2}$ d. Anſ.

PROB. XI.

Having the price of a gallon of liquors; to find the price of a ton.

RULE.

Put the price into pence, and call them ſo many guineas; adding quarter guineas for farthings.

Ex.

If brandy be ſold for 6 s. 10¾ d. *a gallon, what is that a ton?*

6 s. 10¾ d. is	82¾ d.		
82 guin.	82	0	0
	4	2	0
¾ d.		15	9
Anſ.	86	17	9

Cor. *The price of a hogſhead is had, by taking a quarter of this ſum.*

PROB. XII.

Having the price of a ton of liquors; to find that of a gallon.

RULE.

Reduce the price to guineas; then for every guinea reckon a penny, and a farthing for a quarter guinea.

Ex.

At 97 l. *per ton, what is that a gallon?*

97 l. is	92 g. 8 s.		
92 pence	—	7	8
8 ſhillings	—		0¼
	Anſ.	7	8¼

PROB. XIII.

Knowing the price of 1 *pound, to find what the hundred weight coſts.*

1 RULE.

For every penny which a pound denotes,
Count once the angels, 'bating twice the groats.

Ex.

If a pound be 7½ d. *what is the hundred weight?*

7½ angels	3 l. 15 s.
15 groats, ſub.	5
Anſ.	3 10

2 RULE.

Number the farthings that each pound denotes,
Take twice the ſhillings, once as many groats.

Ex.

If 1 *coſt* 3 s. 6¾d. *what will* 100 *weight coſt?*

3 s. 6¾ d. is 171 farthings.

twice 171 ſhillings	17 2 0
171 groats, add	2 17 0
	19 19 0

3 RULE.

Find the pence for 1; take 9 times as many ſhillings, and once as many groats.

Ex.

At 10 d. *per pound, what is the hundred weight?*

9 ten pences	90
ten groats	3 4
	93 4
Anſ.	4 13 4

PROB.

PROB. XIV.

The price of a hundred weight being given, to find that of a pound?

1 RULE.

For every 7 ſhillings in the price of a hundred weight, reckon 3 farthings for a pound.

Ex.

At 8 l. a hundred weight, what will one coſt?

8 pounds are	160 ſhillings
the 7th part	23
3 times 23 farth^s.	69 = 17¼ pence, Anſw.

2 RULE.

Take the tenth part of the price in ſhillings, and the fourteenth part of that; the ſum is the pence for a pound.

Ex.

At 29 l. 17 s. 6 d. the hundred, how much is a pound?

29 l. 17 s. 6 d. is	597½ ſhillings.
the tenth part	59.7
$\frac{1}{14}$ of this —	4.2
ſum	64 pence.
Anſ.	5 s. 4 d.

PROB. XV.

In Troy weight, having the price of a grain, to find the price of an ounce, or a pound?

RULE.

Multiply the farthings in the price, by 6; gives the price of a pound, in pounds.

Or double the pence in the price; gives the price of an ounce in pounds ſterling.

Ex. 1.

At $1\frac{1}{4}$ d. *a grain, how much it a pound?*
$1\frac{1}{4}$ or $5^{f} \times 6 = 30$ l. Anſ.

Ex. 2.

At 2 *d. a grain, how much is an ounce of gold?*
2 d. doubled is 4 pounds. Anſ.

PROB. XVI.

In Troy weight, the price of a penny weight being given, to find that of a pound.

RULE.

The number of pence in the price, call pounds.

Ex.

And $10\frac{1}{2}$ *d. the penny weight, how much is a pound?*
$10\frac{1}{2}$ pounds 10 l. 10 s. Anſw.

PROB. XVII.

The price of a buſhel of coals being given, to find that of a chaldron, or 36 buſhels.

RULE.

Thrice the number of pence in the price, call ſhillings.

Ex.

At $4\frac{1}{2}$ *d. the buſhel, how much is a chaldron of coals?*
Thrice $4\frac{1}{2}$ is $13\frac{1}{2}$ ſhillings, or 13 s. 6 d. Anſ.

PROB.

PROB. XVIII.

Given the price of a ſtone, to find the price of a ton.

RULE.

Find the pence, and take $\frac{2}{3}$ that number of pounds.

Ex.

At 3 *s.* $4\frac{1}{4}$ *d. a ſtone, what is that a ton?*

3 *s.* $4\frac{1}{4}$ *d.* is $40\frac{1}{4}$.

and $\frac{2}{3}$ of $40\frac{1}{4}$ is $\frac{80}{3}+\frac{1}{6}$ or $26\frac{5}{6}$ l.

and $26\frac{5}{6}$ l. is 26 l. 16 s. 8 d. Anſ.

PROB. XIX.

The price of one thing being given in pence, to find what any number will come to.

1 RULE.

For 6 pence reckon half ſo many ſhillings;
For 3 pence, half of the laſt ſum; and
For a penny, $\frac{1}{3}$ of the laſt.

Example.

What will 57 *coſt at* 8 d. *a piece?*

	s.	d.	l.	s.	d.
half of 57	28	6	1	8	6
half of this	14	3		0	0
a third of that	4	9		4	9
ditto —	4	9		4	9
Anſ.			1	18	0

2 RULE. *By a Table.*

Againſt the quantity on the ſide, and under the price of 1 at top, you have the price of your quantity. This is to be taken out at ſeveral times as occaſion requires.

num	1 farth.	2 farth.	3 farth.	1pen.	2 pen.	3 pen.	4 pen.
	l. s. d.	l. s. d.	l. s. d.				
1	0¼	0½	0¾	1	2	3	4
2	0½	1	1½	2	4	6	8
3	0¾	1½	2¼	3	6	9	1 0
4	1	2	3	4	8	1 0	1 4
5	1¼	2½	3¾	5	10	1 3	1 8
6	1½	3	4½	6	1 0	1 6	2 0
7	1¾	3½	5¼	7	1 2	1 9	2 4
8	2	4	6	8	1 4	2 0	2 8
9	2¼	4½	6¾	9	1 6	2 3	3 0
10	2½	5	7½	10	1 8	2 6	3 4
20	5	10	1 3	1 8	3 4	5 0	6 8
30	7½	1 3	1 10½	2 6	5 0	7 6	10 0
40	10	1 8	2 6	3 4	6 8	10 0	13 4
50	1 0½	2 1	3 1½	4 2	8 4	12 6	16 8
60	1 3	2 6	3 9	5 0	10 0	15 0	1 0 0
70	1 5½	2 11	4 4½	5 10	11 8	17 6	1 3 4
80	1 8	3 4	5 0	6 8	13 4	1 0 0	1 6 8
90	1 10½	3 9	5 7½	7 6	15 0	1 2 6	1 10 0
100	2 1	4 2	6 3	8 4	16 8	1 5 0	1 13 4
200	4 2	8 4	12 6	16 8	1 13 4	2 10 0	3 6 8
300	6 3	12 6	18 9	1 5 0	2 10 0	3 15 0	5 0 0
400	8 4	16 8	1 5 0	1 13 4	3 6 8	5 0 0	6 13 4
500	10 5	1 0 10	1 11 3	2 1 8	4 3 4	6 5 0	8 6 8
600	12 6	1 5 0	1 17 6	2 10 0	5 0 0	7 10 0	10 0 0
700	14 7	1 9 2	2 3 9	2 18 4	5 16 8	8 15 0	11 13 4
800	16 8	1 13 4	2 10 0	3 6 8	6 13 4	10 0 0	13 6 8
900	18 9	1 17 6	2 16 3	3 15 0	7 10 0	11 5 0	15 0 0
1000	1 0 10	2 1 8	3 2 6	4 3 4	8 6 8	12 10 0	16 13 4

Examp.

	5 pen.			6 pen.		
1			5			6
2			10		1	0
3		1	3		1	6
4		1	8		2	0
5		2	1		2	6
6		2	6		3	0
7		2	11		3	6
8		3	4		4	0
9		3	9		4	6
10		4	2		5	0
20		8	4		10	0
30		12	6		15	0
40		16	8	1	0	0
50	1	0	10	1	5	0
60	1	5	0	1	10	0
70	1	9	2	1	15	0
80	1	13	4	2	0	0
90	1	17	6	2	5	0
100	2	1	8	2	10	0
200	4	3	4	5	0	0
300	6	5	0	7	10	0
400	8	6	8	10	0	0
500	10	8	4	12	10	0
600	12	10	0	15	0	0
700	14	11	8	17	10	0
800	16	13	4	20	0	0
900	18	15	0	22	10	0
1000	20	16	8	25	0	0

Examp.

What comes 1453 to, at 9½ d. a piece?

	l.	s.	d.
1000 (at 6d.)	25	0	0
at 3d.	12	10	0
at ½	2	1	8
400 (at 6 d.)	10	0	0
at 3 d.	5	0	0
at ½		16	8
50 (at 6 d.)	1	5	0
at 3 d.		12	6
at ½		2	1
3 (at 6 d.)		1	6
at 3 d.			9
at ½.			1½
Anſ.	57	10	3½

PROB. XX.

The price of 1 being given in ſhillings, to find what any number will come to.

1 RULE.

In the number of things propoſed, double the right hand figure for ſhillings, and reckon the reſt of it pounds. Then take this ſum of pounds and ſhillings as oft as 2 is contained in the price; and if there is an odd ſhilling, take half the ſaid ſum.

Examp.

What is 2344 ounces at 13 ſhillings an ounce?

234 l. 8 s. taken 6 times is 1406 l. 8 s.
half of it — 117 4

Anſ. 1523 12

2 RULE.

Multiply the number of things by half the price; then double the right hand figure for ſhillings, and the reſt is pounds.

In the former Example.

2344 × by 6½ is as follows,
6 × 2344 is 14064
half of it is 1172

15236

Anſ. 1523 l. 12 s.

PROB.

PROB. XXI.

Having the price of 1 *in pounds*; *to find the price of any number of them.*

RULE.

Multiply the price ſeparately by every figure of the propoſed number; putting each product in its proper place.

Examp.

What comes 5307 *to at* 57 l. *a piece?*

5000 × 57 — 285000
300 × 57 — 17100
7 × 57 — 399

Anſ. 302499 l.

PROB. XXII.

To caſt up ſums of money by Counters.

RULE.

Make four ſeparate partitions upon a table, and mark them with chalk, pounds, ſhillings, pence, farthings; then write down the ſeveral ſums, or elſe put as many counters into each partition as there are pounds, ſhillings, &c. in each ſum. Then begin to reckon at the loweſt denomination, and proceed till you amount to 1 of the next greater; then throw away that number of counters in the leſſer, and inſtead of them, put 1 counter in the place of the greater. Proceed thus throughout, leaving only ſo many counters in each partition, as is the overplus at laſt.

Note, pieces of paper may ſerve inſtead of counters, and brown paper may ſerve for pence, and white, for ſhillings.

Example.

Suppoſe you have the following ſums,

l.	*s.*	*d.*
3	7	8
105	12	$2\frac{1}{2}$
17	11	$6\frac{3}{4}$
9	0	11

	poun.	ſhill.	penc.	far.
	o	o	o	
		o		
tot.	135	oooo	oo	o
	ooo	oooo	oo	
	oo	oooo		
	oo			
	o			
	o			

Firſt, I ſay, 3 farthings and a halfpenny are 5 farthings, leave 1 counter in the place of farthings; and carry 1 to the place of pence. Then 1 and 11 is 12, which throw away, and put 1 counter in the place of ſhillings; then 6 and 2 is 8 and 8 is 16, throw away 12, and put 1 in the place of ſhillings, and put in 4 counters at pence. Then 2 and 11 is 13 and 12 is 25, throw away 20, and but 1 in the place of pounds. Then 5 and 7 is 12, ſo put 12 counters in the place of ſhillings. Laſtly, 1 and 9 is 10 and 7 is 17, and 5 is 22, and 3 is 25, and 10 is 35, and 100 is 135; ſo put 135 counters in the place of pounds. So examining each partition, you will find 135 l. 12 s. $4\frac{1}{4}$ d. to be the ſum required.

PROB. XXIII.

To find the intereſt of a ſum of money for any number of days, at any rate per cent.

RULE.

Multiply the rate per cent. and the principal together, and take the third part of it, from which cut off 3 places to the right hand. From this ſubtract the 73d part of itſelf (or for ſhortneſs the 70th part); and the remainder is the intereſt for 12 days.

Then 2, 5, 10 times this, &c. gives the intereſt for 24, 60, 120 days, &c. Alſo $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$ the ſaid intereſt gives the intereſt for 1, 2, 3, 4, 6 days; and conſequently it will be had for any number of days.

Examp.

What is the intereſt of 311 l. 13 s. 4 d. for 315 days, at 4½ per cent.

311 l. 13 s. 4 d. = 311⅔ = 311.66666

4½ × 311.666 is —	1402.5
the third part is —	467.5
three places cut off —	.4675
the 70th part, ſubſt.	66
the intereſt for 12 days	.4609
Intereſt for 120 days —	4.609
—— for 120 days more	4.609
—— for 60 days more	2.3045
—— for 12 days more	.461
—— for 3 days more	.1152
315	12.0987
	20
	1.9740
	12
	11.6880

Anſ. 12 l. 1 s. 11¾ d.

PROB. XXIV.

Having your weekly expences; to know what it coſts you in the year.

1 RULE.

Each ſix-pence in the week does make appear,
The guineas with the crowns ſpent in a year.

Examp.

At 5 s. 6 d. *per week, or* 11 *ſix-pences per week.*

		l.	s.	d.
11 guineas	—	11	11	0
11 crowns	—	2	15	0
	Anſ.	14	6	0

2 RULE.

The farthings in each week do make appear,
The ſhillings with the pence ſpent in a year.
Add one day's expence.

Ex.

At 15¼*d. in a week, what is it in a year?*
15¼ is 61 farthings.

		l.	s.	d.
61 ſhillings		3	1	0
61 pence			5	1
One day	—			2¼
		3	6	3¼

PROB. XXV.

Knowing your daily expences; to find what it coſts in a year.

RULE.

Number the pennies of each day's expence;
So many pounds, and angels, groats, and pence,
You ſpend within the year's circumference.

Examp.

At 9d. *per day, what for the year?*

		l.	s.	d.
9 pounds	—	9	0	0
9 angels	—	4	10	0
9 groats	—		3	0
9 pence	— —			9
	Anſw.	13	13	9

PROB. XXVI.

By knowing your yearly income; to find what you can ſpend a day.

RULE.

The third part of the pounds you yearly have,
So many two-pences afford to ſpend or ſave. nearly.

Examp.

At 40 l. *a year, quere what a day?*

$\frac{1}{3}$ of 40 is $13\frac{1}{3}$
and $13\frac{1}{3}$ two-pences is $26\frac{2}{3}$ pence, or 2 s. $2\frac{1}{2}$ d.

PROB.

PROB. XXVII.

The money gained in an hour being given; to find the produce in a week.

RULE.

Reduce the price to pence, and take $\frac{7}{10}$ thereof for the pounds.

Examp.

If an hour produces 9$\frac{1}{2}$d. what will be the product in a week?

Here $9\frac{1}{2} \times \frac{7}{10}$ is 6.65 = 6 l. 13 s. Anſw.

PROB. XXVIII.

Having the product of any thing for a minute, to find it for a day.

RULE.

Take ſo many pounds, and ſo many angels as there are farthings in the product of one minute.

Examp.

At 5$\frac{1}{4}$ d. a minute, what will a day come to?

5$\frac{1}{4}$ d. is 21 farthings.

		l.	s.	d.
21 pounds		21	0	0
21 angels	—	10	10	0
	Anſ.	31	10	0

PROB. XXIX.

To diſcover a deceitful Ballance.

RULE.

After the weights are made even in the two ſcales, change them into the oppoſite ones, Then if they ſtill remain equal, the ballance is true; if not, 'tis falſe.

But a ſteelyard cannot be tried this way; yet it may be tried, by weighing any thing with it, and then weighing that thing in a true pair of ſcales.

Cor. *If one arm of a ballance be longer than the other it is a deceitful ballance; though the ſcales, when empty, may be in equilibrio.*

PROB. XXX.

To weigh all things with the feweſt weights.

RULE.

Take any number of pound weights in this progreſſion 1, 3, 9, 27, 81, &c. and they will weigh any number of pounds, not exceeding the ſum of them all. But ſometimes ſome of the weights muſt be put in the contrary ſcale, to be ſubtracted.

The ſame thing may alſo be done with the weights in this progreſſion, 1, 2, 4, 8, 16, 32, &c. where none need be put in the oppoſite ſcale, which is better.

PROB. XXXI.

To find the content of any plane figure by weighing it.

RULE.

Having the figure drawn upon paper, cut it out in paſteboard, which is eaſily done, by laying the paper ſcheme upon it; and weigh your paſteboard figure in grains. Then mark upon it any ſquare you can meaſure, and cut it all away to that ſquare; and then weigh the ſquare. And ſay, as the weight of the ſquare, to its meaſure; ſo is the weight of the figure, to its content in the ſame meaſure.

PROB.

PROB. XXXII.

The ſolid content of a body being known, to find its weight.

RULE.

Find what an inch or a foot of that ſort of body weighs, either by trial, or by the tables of ſpecific gravity. Then multiply the content of the body, by this weight, and you have the weight of the whole.

Ex.

How much does a piece of elm weigh, which contains 20 *feet?*

By the table of ſpecific gravities, a foot of elm weighs 49 l. 1 oz. This multiplied by 20, produces 981$\frac{1}{4}$ lb. that is, 70 ſt. 1$\frac{1}{4}$ lb. or 8 hundred 6 ſtone, 1$\frac{1}{4}$ lb.

PART

PART II.

Geometrical Problems, and the Meaſuring of Lines.

SINCE meaſuring the contents of ſuperficies and ſolids by the Sliding rule, is very eaſy and expeditious; and we ſhall have occaſion to make frequent uſe of it, in what follows; therefore I ſhall begin with a ſhort deſcription thereof.

The deſcription and uſe of the Sliding rule.

This Rule (commonly called *Cogſhal's* Sliding rule) has four lines upon it, adapted to meaſuring; they are marked A, B, C, D. Three of theſe, A, B, C, are double lines, becauſe they proceed from 1 to 10 twice over. The fourth line D is a ſingle line, proceeding from 4 to 40, and is called the *Girt line.*

Two of theſe lines B, C, are upon the ſlider, and the other two A, D, upon the rule. The three ſingle lines are all exactly alike, and alike numbered, and divided; the diviſions being numbered 1, 2, 3, 4, to 1 again, from the beginning to the middle; and again, 1, 2, 3, 4, to 10, from the middle to the end. The fourth line D, is numbered 4, 5, 6, 7 to 40; but figures are not ſet to all the diviſions, for towards the end they grow very ſmall. The number 4 in the ſingle line ſtands againſt 1 in the double lines, and the number 40 againſt 10. In all the lines the diviſions are large at the beginning, and continually decreaſe to the laſt.

For the numeration upon the lines; 1 at the beginning may be counted 1, then the 1 in the middle will be 10, and 10 at the end 100. If 1 at the beginning be called 10, 1 in the middle is 100, and 10 at the end 1000. Or if the firſt 1 be reckoned 100, the ſecond 1 will be 1000, and the laſt 1 or the 10 will be 10,000; and ſo on. And as the grand diviſions are numbered or altered, ſo are all the ſmaller diviſions altered in value accordingly.

The uſe of the double lines A and B, is for working the rule of proportion; as alſo for finding the areas of plain figures. And the uſe of the girt line D, and the other double line C, is for meaſuring ſolids.

Beſides theſe lines, there are ſeveral other lines and ſcales, as a line of inches with halves, quarters, and half quarters. A line of inches, each divided into ten parts; this is alſo a foot divided decimally, and ſerves to turn inches into decimal parts of a foot. The backſide of the ſliding piece is divided into inches, halves, quarters, and half quarters; and ſerves to meaſure any length, by ſliding it out.

There are upon ſome of theſe rules other ſcales and tables, as a table of board meaſure, one of timber meaſure. A line ſhewing what length, for any breadth, will make a foot ſquare: this is for compleating the table of board meaſure. Alſo a line, ſhewing what length will make a ſolid foot, for any thickneſs. This alſo compleats the table of ſolid meaſure.

If 1 on A be ſet to any number on B, theſe two lines will be a table of multiplication. For the numbers in A, will be the ſeveral multiplicands, and the correſponding numbers in B, the products.

Alſo if the lines C, D, be ſet together, 10 againſt 10; they will be a table of ſquares. The line D being the roots, and C the ſquares; the numbers at 10 and 10 being reckoned 10 and 100.

In a Gauger's rule, the line D is numbered from 1 at the beginning to 10 at the end; which is more commodious for extracting roots.

As the ſhorteſt and eaſieſt way of meaſuring all ſuperficies and ſolids, is by the ſliding rule; it will be neceſſary to lay down a few Problems to ſhew the uſe of it.

The Uſe of the SLIDING RULE.

PROB. 1.

To multiply two numbers together, as 7 and 26.

RULE.

Set 1 upon A, to one of the numbers (26) upon B; then againſt the other number (7) on A, is found the product (182) upon B.

Note, If the third term on A runs beyond the end of the line B, ſeek that third term on the other radius (or part) of the line A; then the product will be found on B, which muſt be increaſed ten times as much.

PROB. 2.

To divide one number by another, as 510 by 12.

RULE.

Set the diviſor (12) on A, to 1 on B; then againſt the dividend (510) on A, is the quotient $(42\frac{1}{2})$ on B.

Note, If the dividend runs beyond the end of the line A, diminiſh it 10 or 100 times, to make it fall on A; then increaſe the quotient 10 or 100 times, reſpectively.

PROB. 3.

To ſquare any number, as 27.

RULE.

Set 1 (or 10) upon D to 1 upon C, then againſt the number (27) upon D, is the ſquare upon C (729). And to reckon right, obſerve that as the 1 upon D is counted 1, 10, or 100, &c. the 1 upon C (againſt it) will be 1, 100, or 10,000; that is the ſquare of the other.

If you would ſquare 270, reckon the 1 on D to be 100, then that on C will be 10,000, and the product 72,900.

PROB. 4.

To extract the ſquare root of any number, as 4268.

RULE.

Set 1 upon C, to 1 (or 10) upon D; then againſt the number (4268) on C, is the root on D (65.3). Which to value right, you muſt ſuppoſe the 1 on C to be ſome of theſe ſquares 1, 100, 10,000, &c. which is near the given number; and then the root correſponding, will be the value of the 1 on D.

PROB. 5.

To find a mean proportional between any two numbers, as 27 and 450.

RULE.

Set one of the numbers (27) on C, to the ſame on D; then againſt the other numbers (450) on C, you have the mean (112) on D.

Note, if one of the numbers overruns the line, take the 100th part of it, and then take a mean, which multiply by 10.

PROB. 6.

Three numbers being given; to find a fourth proportional: ſuppoſe 12, 28, and 57.

RULE.

Set the firſt (12) upon A, to the ſecond (28) upon B; and againſt the third (57) on A, is the fourth (133) on B.

Note, if one of the middle numbers runs off the line, take the 10th part of it only, and then take 10 times the fourth proportional found. The finding a third proportional is the ſame, the ſecond number being twice repeated.

PROB. 7.

If the breadth of a board be given (8 *inches*); *to find how much in length will make a foot ſquare.*

RULE.

If the board be narrow, you will find it in the table of under board meaſure in the table on the rule; if not, ſhut the rule, and ſeek the breadth (8) in the line of board meaſure running along the rule, from that table; then over againſt it, on the oppoſite ſide, in the ſcale of inches, is 18 inches, the length required.

PROB. 8.

The ſide of the ſquare of a piece of timber being given (16 inches); to find how much in length will make a foot ſolid.

RULE.

If the timber be ſmall, look into the table of under timber meaſure on the rule, and you'll find the feet and inches. Otherwiſe look for the ſide of the ſquare (16) in the line of timber meaſure running along the rule from that table: and againſt it, in the line of inches, is the length $6\frac{3}{4}$ inches.

PROB. 9.

To find the content of any ſurface.

RULE.

Reduce the ſurface to a rectangle, then (by Prob. I.) multiply the length and breadth together, for the area or content of the ſurface, in the ſame ſort of meaſure.

Otherwiſe.

If the length be given in feet, and the breadth in inches. Set 12 on A to the length on B; and againſt the breadth on A, is the content on B, in feet.

Otherwiſe.

If you would have the dimenſions taken in one ſort of meaſure, and the area given in another ſort. See how many ſquares of the firſt, make one of the other; and make it a diviſor.

Then ſet this diviſor on A to the length on B, and againſt the breadth on A, is the area on B.

PROB. 10.

To find the content of the ſolid body.

RULE.

Reduce it to a long body, or parallelopipedon, ſquare at the ends; meaſure the length in feet, and the ſide of the ſquare in inches. Then ſet 12 on D, to the length on C; and againſt the ſide of the ſquare on D, is the content on C, in feet.

PROB. 11.

If two fixt numbers are given on the lines D *and* C, *as* p *and* q; *to change them to others that will perform the ſame operations.*

RULE.

Inſtead of p and q, take 1 and $\frac{q}{pp}$, or $\sqrt{\frac{pp}{q}}$ and 1, or r and $\frac{qrr}{pp}$; which may be expreſſed thus,

D	C
p	q
1	$\frac{q}{pp}$
$\sqrt{\frac{pp}{q}}$	1
r	$\frac{qrr}{pp}$.

That is, inſtead of ſetting p upon D, to q upon C; one may ſet 1 on D, to $\frac{q}{pp}$ on C; or $\sqrt{\frac{pp}{q}}$ on D, to 1 on C; or r on D to $\frac{qrr}{pp}$ on C.

For, p, q, being two fixt numbers, and r take at pleaſure; and ſince the numbers on C are as th ſquares of thoſe on D; therefore we muſt ſquare thoſe on D, and make them proportional, that they may perform the ſame thing; then we ſhall have $pp : q :: rr : s$, and $s = \frac{qrr}{pp}$, and $r = \sqrt{\frac{pps}{q}}$.

PROB. 12.

To find a proper fixt number for the line D, *for meaſuring planes or ſolids.*

RULE.

Here you muſt have given the content of ſome particular plane or ſolid of that ſort, whoſe diameter, circumference, &c. is 1, put this content $=$ S.

Then if the dimenſions be taken in one meaſure, and the content given in another; put $n =$ number of leſſer ſquares or cubes contained in 1 of the greater, $x =$ fixt number. Then $x = \sqrt{\frac{n}{S}}$.

For in areas it will be $x^2 : 1 :: 1 : S$, and $x = \sqrt{\frac{1}{S}}$. And in ſolids, $x^2 \times$ height : height :: 1 : S. And $x = \sqrt{\frac{1}{S}}$.

Examp.

In ſquares $x = 1$. Or for inches and feet, $x = 12$.

In circles $x = \sqrt{\frac{1}{.7854}}$. Or for inches and feet, $x = \sqrt{\frac{144}{.7854}}$.

In circles, when the circumference is 1, $x = \sqrt{\frac{1}{.0796}}$.

In cylinders for inches and feet, $x = \sqrt{\frac{144}{.7854}}$.

In a cube, $x = 1$. For inches and feet, $x = \sqrt{\frac{144}{1}} = 12$.

In a ſphere, circumference $= 1$, for inches and feet, $x = \sqrt{\frac{1728}{.0169}}$, &c.

PROB. I.

To draw a right line from one place to another.

RULE.

If it be a ſmall diſtance, apply a ruler, to the two points the line is to paſs through, and by the edge of it draw a line with a pin, or pencil, or with pen and ink.

If the diſtance be great, ſtretch a line from one place to the other. And if it be a plane chalk the line, and cauſe it to ſpring againſt that plane, and it will leave the mark of a right line. If ſtakes be placed all the way between the eye and any diſtant point, they will ſtand in a right line.

PROB. II.

To meaſure the length of any right line.

RULE.

If the diſtance be ſhort, open a pair of compaſſes to the extent of the line, and apply it to a ſcale of equal parts, and you will have the length in the ſame ſort of parts.

If the diſtance be great, apply any meaſure, as a foot, yard, &c. to one end of the line, and extend it as far as it will reach directly forward; do thus as often as you can, till you come to the other end;

then

Fig then ſo many times as you applied that meaſure, ſo many feet or yards your line is of.

Or extend a line from one end to the other; and meaſure the length of that line, by applying any meaſure to it, as oft as you can; and you will find how many of theſe meaſures it contains.

In taking the dimenſions of any ſurface or ſolid, they muſt all be taken by one ſort of meaſure; all feet, all yards, &c.

PROB. III.

To draw one line parallel to another, thro' any point.

RULE.

1. Let AB be the line and D the point; from the point D take the neareſt diſtance to the line AB, with a pair of compaſſes, or with a thread. And from ſome other point A in the line AB, ſet off the ſame diſtance to C, and deſcribe a ſmall arch at C. Draw a line from D to touch the arch at C; then CD is parallel to AB.

SCHOLIUM.

Theſe parallels are ſuppoſed both of them to be drawn upon ſome plane ſurface. But if one of them, as CD, be ſuppoſed to be in the air; beſides the equal diſtances, care muſt be taken, that they both be in the ſame plane; and that may be known, by applying the eye to one of them, as CD, for if it cover the other AB, then they are in one plane, otherwiſe not.

PROB.

PROB. IV.

To raiſe a perpendicular to a right line, to paſs thro' a given point.

RULE.

Apply a ruler to that line, then apply one ſide of a ſquare to the ruler, ſo that the other ſide may paſs thro' the point; or elſe that another rule applied thereto may paſs thro' the point; then draw a line by the ſide of the ſquare or the ruler.

Or draw a thread from that point, ſo that it may coincide with the ſide of the ſquare, while the other ſide lies upon, or coincides with the given line; and the thread will be a perpendicular.

Otherwiſe.

Make the baſe AB, 4; the height BC, 3, and the ſide AC, 5, feet, yards, &c. and then CB is perpendicular to AB. 2.

PROB. V.

To raiſe a perpendicular upon a plane to paſs thro' any point.

RULE.

Extend a thread from the point to the plane, and put it in ſuch a poſition, that it may juſt coincide with one ſide of a ſquare, when the other ſide is laid on the plane, round about the thread on every ſide. Then the thread is a perpendicular to the plane.

Or extend a thread from the point to three diſtinct places on the plane. Find the centre of a circle paſſing thro' thoſe three points, which is ſoon done by a few trials: then a line, paſſing through that center and the given point, is a perpendicular.

Fig. 2. Or a circle may be deſcribed on the plane with a pencil, by extending the thread; and then the center of this circle may be found, by drawing two parallel lines thro' it, and drawing a third line thro' the middle of theſe two; for if that line be biſected, the middle point will be the center.

PROB. VI.

To meaſure the perpendicular height of any point above a line or plane.

RULE.

Take the neareſt diſtance from that point to the line or plane, with a ſtring; and then meaſure its length.

If the perpendicular falls within a ſolid body; you muſt lay a ruler from that point, parallel to the baſe, and take the neareſt diſtance from the ruler to the baſe; and this is eaſily done, if the body ſtands upon a plain floor; but if not, then a board muſt be applied to the plane of its baſe, and a line drawn parallel to the board through the top of the body, which muſt be held ſo as to be every where equidiſtant; and then that diſtance is the height required.

If it be ſo ſituated, that a line or ruler cannot be applied, ſo as to be made parallel to the baſe. Then if it has a ſtreight ſide, the angle that the ſide makes with the baſe, and the length of the ſide, muſt be found; from whence the perpendicular height may be had.

PROB. VII.

To divide a line AB *into any number of equal parts.*

RULE.

Draw any line *ab* parallel to AB, and ſet off your number of any equal parts from *a* to *b*. Draw the lines *a*A and *b*B, to interſect in C. From C draw C*f*, C*e*, C*d*, C*c*; to interſect AB in 1, 2, 3, 4; which are the diviſions required. 3.

The line AB may alſo be divided by trials, by running a pair of compaſſes from one end to the other; and increaſing or decreaſing the diviſions, till they juſt anſwer to the whole.

If the diſtance be large, this way cannot be practiſed; then the length of the line AB muſt be meaſured in feet, yards, &c. and the whole length divided by the number of parts, to get the length of one part; which done, that length muſt be ſet off from B to 1, from 1 to 2, from 2 to 3, &c. to A.

PROB. VIII.

To deſcribe a circle thro' three points given, A, B, F.

RULE.

Open the compaſſes, and ſetting one foot in C, try if the other turned about will reach all the three points; if not, ſhift the center, and try again, till it do: or you may uſe a thread; then deſcribe the circle ABF. 4.

Otherwiſe.

From two of the points draw lines to the third, as AB, BF. Raiſe two perpendiculars DC, EC, upon the middle of the lines AB, BF; to interſect in C. From the center C deſcribe the circle ABF.

PROB.

PROB. IX.

To find the center of a circle.

1 RULE.

4. The center C may be found by trials; or thus; draw any line BF, and on the middle point E, erect the perpendicular GEH. Biſect GH in C, and C is the center.

2 *Or thus.*

Thro' the circle, draw any two lines BF, *bf*, parallel to one another; biſect theſe lines at E and *e*, thro' E, *e*, draw the line GH, which will be a diameter; therefore if GH be biſected in C, then C will be the center.

PROB. X.

To divide the ſemicircle ADB *into any number of equal parts.*

RULE.

5. Make AC and BC equal to AB. Divide AB into the ſame number of equal parts, at *b*, *c*, *d*, *e*. And let the diviſions at the center rather be bigger than leſs than thoſe near the circumference at A and B. Then draw the lines C*b*, C*c*, C*d*, C*e*, to interſect the circle at 1, 2, 3, 4, which will divide the ſemicircumference into equal parts as required.

The ſemicircle, or any other arch, may alſo be divided into any number of equal parts by trials; running a pair of compaſſes from one end to the other, and increaſing or decreaſing the extent, as it is found to be too little or too big. Where note, the length of the radius is juſt the extent of the third part of the ſemicircle ADB.

PROB.

PROB. XI.

To make a ſquare equal to a given circle.

1 RULE.

Thro' the center C draw two diameters AB, DE 6.
perpendicular to one another. And divide each of the quadrants into three equal parts at 1, 2. So that the middle diviſion of each be rather bigger, than leſs, than the other two. Thro' 1, 1, draw the lines *ab*, *cd*; and thro 2, 2, the lines *ac*, *bd*; and the ſquare *abdc* is equal to the circle AEBD.

2 *Otherwiſe.*

Thro' the center draw two diameters AC, BD, 7.
perpendicular to each other. And divide one diameter AC into eight equal parts, and ſet one of theſe parts (rather more than leſs) from A to *a*, and from B to *b*, C to *c*, and D to *d*. Draw *ab*, *bc*, *cd*, *da*. And *abcd* is a ſquare equal to the circle ABCD.

3 *Otherwiſe.*

Make the diameter AB 14 parts, and AD 11; 9.
raiſe the perpendicular DE. Draw AE, and the ſquare AEFG is equal to the whole circle, whoſe diameter is AB.

By the Sliding Rule.

Set 1 on A to .282 on B, and againſt the circumference on A, is the ſide of the ſquare on B; whoſe area is equal to the circle.

For .282 is the ſide of a ſquare equal to the circle, whoſe circumference is 1.

PROB.

Fig.

PROB. XII.

To deſcribe an Oval.

RULE.

8. Draw the tranſverſe AB, and conjugate CD, at right angles to it. Deſcribe two circles AFEK, and EGBL; then take two centers H, I, any way in D, equidiſtant from E; and from theſe centers deſcribe the arches KL, MN, to touch the circles KF and GL.

The two circles may be drawn otherwiſe than to touch at E; for they may be made to paſs thro' each other's center, or at a diſtance from one another, &c.

An ellipſis is eaſily drawn, by ſetting up two pins in two centers *n*, *o*; about which putting a thread *nop*, and knotting it at *p*; the point *p* carried about, will deſcribe an ellipſis DPKAMCNBLD, by help of a pencil.

PROB. XIII.

To reduce any irregular figure, or any curve or part of a curve, into a rectangle.

RULE.

10. Let ACB be the figure. Erect any number of perpendiculars upon the baſe AB at equal diſtances; meaſure their lengths, and add them together, and divide the ſum by their number, the quotient is the mean breadth AD, which is the breadth of the parallelogram ADEB, equal to the fig. ACB: the length being the ſame; or both of them having the common baſe AB.

Examp.

Fig. 10.

Example.

Erect ſeven perpendiculars, whoſe lengths meaſured and marked in the figure, are 0, 2, 4, 4, 2, 3, 2. Their ſum is 17, and 17 divided by 7, gives $2\frac{3}{7}$ for the mean breadth, which is equal to BE or AD, the breadth of the parallelogram ABED, equal to the figure ACB.

Note, this Prob. is of uſe in meaſuring irregular planks or boards.

PROB. XIV.

To find the hypothenuſe of a right angled triangle.

RULE.

From any ſcale lay down the length of the baſe AB. Draw BC perp. to it, of the length given, by the ſame ſcale. Draw AC, which, meaſured on the ſcale, gives its length. 11.

Otherwiſe.

To the ſquare of the baſe add the ſquare of the perpendicular; extract the ſquare root of the ſum, and you have the hypothenuſe.

Examp.

Let the baſe AB *be* 22, *the perpendicular* BC 13.

22	13	653(25.55	the hypothe-
22	13	4	nuſe AC.
44	39	45)253	
44	13	225	
484	169	505) 2800	
	484	2525	
	653	275	

PROB.

PROB. XV.

To find either leg of a right angled triangle.

1 RULE.

11. Draw the given ſide AB, of it due length; raiſe BC perpend. to it; with the length of the hyp. in your compaſſes, ſet one foot in A, and with the other deſcribe an arch cutting CB in C; then CB is the other ſide, which meaſured on the ſcale, gives its length.

2 *Or thus.*

From the ſquare of the hyp. ſubtract the ſquare of the given ſide; extract the ſquare root of the remainder, and you have the other ſide.

Examp.

Let the hypothenuſe AC *be* 25.55, *the baſe* AB 22.

22	25.55	168(12.96, the perpen-	
22	25.55	1	dicular, CB.
44	5110	22)68	
44	1277	44	
	128		
484	13	249)2400	
		2241	
	652.8		
	484	159	
	168.8		

PROB.

PROB. XVI.

Having the three ſides of any triangle ACB, *to find the perpendicular* CD.

1 RULE.

Draw the baſe AB of a proper length, by any ſcale; and with the length of AC in your compaſſes, and one foot in A, deſcribe a ſmall arch; then with BC in your compaſſes, and one foot in B, croſs that arch at C. Then draw AC, BC; and the triangle is conſtructed. 14. 15.

Then from C let fall a perpendicular upon AB, as CD, for the perpendicular. Or take the neareſt diſtance from C to AB, which apply to your ſcale.

2 RULE.

Find one ſegment as $AD = \frac{AC^2 + AB^2 - CB^2}{2AB}$.

Then find $CD = \sqrt{CA^2 - AD^2}$.

3 RULE.

Take half the ſum of the three ſides, and ſubtract each ſide from it, and you have three remainders; then multiply that half ſum, and the three remainders together continually. Extract the ſquare root, and divide it by half the baſe, and it gives the perpendicular.

Cor. *Hence either ſegment of the baſe is had thus*; To the ſquare of the adjoining ſide add the ſquare of the baſe, from which ſubtract the ſquare of the oppoſite ſide; and then divide by twice the baſe.

PROB.

PROB. XVII.

To find the circumference of a circle, from the diameter.

1 RULE.

23. Say as 7 to 22, ſo the diameter to the circumference.

2 *Or more exactly.*

As 113 to 355, ſo the diameter to the circumference.

3 *Or thus.*

Multiply the diameter by 3.1416, the product is the circumference.

Examp.

Suppoſe the diameter be 32.

	Or thus.
7 : 22 : : 32 :	3.1416
22	32
64	62832
64	94248
7)704(100.6	100.5312

1 *By the Sliding Rule.*

Set 7 on A, to 22 on B, then againſt the diameter (32) on A, is the circumference (100.6) on B.

2 *Or thus.*

Set 1 on A, to 3.14 on B; then againſt the diameter (32) on A, ſtands (100.6) the circumference on B.

PROB.

Fig.

PROB. XVIII.

To find the diameter of a circle from the circumference.

1. RULE.

Say as 22 to 7, or as 355 to 113, or as 3.1416 to 1; ſo the circumference to the diameter. 23.

2. *Or thus.*

Multiply the circumference by .3183, the product is the diameter.

Examp.

Suppoſe the circumference be 50.

```
      50
       7
     ----
  22)250(15.91 the circumference.
     22
     ---
     130
     110
     ---
      200
      198
      ---
        2
```

By the Rule.

Set 22 on A, to 7 on B, and againſt the circumference (50) on A, is the diameter (15.9) on B.

PROB. XIX.

The baſe and height of the ſegment of a circle being given; to find the diameter.

RULE.

12: As height of the ſegment DF:
to half the chord or baſe AF or FB::
So the ſame half:
to a fourth GF;

then this fourth proportional GF, added to the height of the ſegment FD, gives the diameter GD.

Examp.

Suppoſe DF 2 feet; AF, 3. Then 2 : 3 :: 3 : $\frac{9}{2}$ = $4\frac{1}{2}$ = FG, then adding DF, we have GD $6\frac{1}{2}$.

PROB. XX.

Having the radius of a circle; to find the ſide of a regular poligon inſcribed in it.

RULE.

Multiply the given radius, by the multiplier, over againſt the number of ſides in this table; the product is the ſide of the poligon.

Numb. of ſides.	Multipliers.
3	1.732051
4	1.414214
5	1.175570
6	1.000000
7	0.867767
8	.765367
9	.684040
10	.618034
11	.563465
12	.517638

Ex. 1.

Suppoſe the radius of a circle 5, *what is the ſide of an inſcribed equilateral triangle?*

$$\begin{array}{r} 1.73205 \\ 5 \\ \hline 8.66025 \end{array}$$ Anſ.

Ex. 2.

If the radius of a circle be 315, *what is the ſide of an inſcribed octagon?*

$$\begin{array}{r} .76536 \\ 315 \\ \hline 382680 \\ 76536 \\ 229608 \\ \hline 231.08840 \end{array}$$ Anſwer.

PROB. XXI.

To find the length of the arch of a circle, ADB, *by having the chord* AB.

1 RULE

Divide the chord AB into four equal parts, and ſet one part from B to E, upon the arch; and from A to H upon the chord; and draw HE; then 2HE is the length of the arch ADB, the leſſer ſegment. 12.

2. *Otherwiſe.*

Make arch AD = DB, and draw the chord AD; then $\frac{8AD - AB}{3}$ = length of the arch ADB.

3. *Otherwiſe.*

Make AD = DB, and draw the diameter DFG. Then $\frac{DG - .16DF}{DG - .82DF} \times AB$ = length of the arch ADB.

Note, if the arch is greater than a ſemicircle; find the length of half of it by this rule, and double it.

Fig. 12. This Prop. may more eaſily be reſolved, by putting a ſtring round ADB, and then meaſuring the ſtring.

Examp.

Suppoſe AB *be* 7; DF, 2. Then by Prob. 18. DG = $8\frac{1}{8}$.

.16
2
.32 = .16DF

.82
2
1.64 = .82DF

8.125
.32
7.805
7
6.485)54635(8.42
51880
2751
2594
161

8.125
1.640
6.485

So the length of ADB is 8.42.

PROB. XXII.

To find the length of the quadrant of a circle, AB.

RULE.

13. Divide the ſemicircle AGC into three equal parts at G, H; which is done by ſetting CO from C to H, and from A to G; and draw BG, BH. Make DF = DE; then BF is equal to the quadrant BA or BC.

Or thus.

$1\frac{1}{9}$ the chord AB = quadrant AIB.

Examp.

Let the radius AO *be* 20;
Then by Prob. 14, AB is 4.24; these numbers is wrong

9)4.34(.47
36
64

4.24
.47
4.71 = quad. AIB.

should be then by prob. 14, AB is 28.284,
9) 28.284 (3.142, 28.284
3.142
31.426 =
quad AIB.

PROB.

Fig. 8

PROB. XXIII.

To find the periphery of an ellipſis.

1. RULE.

Multiply half the ſum of the tranſverſe and conjugate diameters, by 3.1416; the product is the periphery.

2. *Or thus.*

To twice the tranſverſe add $1\frac{1}{7}$ the conjugate for the periphery.

Examp.

Let the tranſverſe be 3, *and the conjugate* 2.

Then $1\frac{1}{7} \times 2 = 2\frac{2}{7} = 2.285$

$$\begin{array}{rr} & 2.285 \\ & 6. \\ \hline \text{Anſ.} & 8.28 \end{array}$$

Otherwiſe.

$$\begin{array}{l} 3 \\ \underline{2} \\ 2)\,5\,(2\frac{1}{2} \end{array} \qquad \begin{array}{rr} & 3.1416 \\ & 2\frac{1}{2} \\ \hline & 62832 \\ & 15708 \\ \hline \text{Anſ.} & 7.8540 \end{array}$$

PART III.

The Meaſuring of Areas and Surfaces.

PROB. I.

To find the area of a triangle ABC.

1 RULE.

14. 15. MULTIPLY the baſe AB, by the perpendicular CD, half the product is the area.

2. *Or thus.*

Take half the ſum of the three ſides, and ſubtract each ſide from it; then multiply that half ſum and theſe three remainders together; the ſquare root of the laſt product is the area.

3. *Or thus.*

Find the nat. ſine of the angle A, and multiply that ſine, and the ſides AC, and AB, together; half the laſt product is the area. Or elſe uſe logarithms.

If CD be not perpendicular, but drawn to make any angle you will at D. Then add the logarithms of AB, CD, and ſine <D, together; abate 10. Then find the number belonging; half of it is the content,

Examp. Fig. 14. 15.

The baſe AB *is* 24, *the perpendicular* CD, 13.

13
12
26
13
156

By the Rule.

Set 1 on A, to half the baſe (12) on B; then againſt the perpendicular (13) on A, is the content (156) on B.

PROB. II.

To find the area of a ſquare ABCD.

RULE.

Multiply any ſide by itſelf; the product is the area. This is called ſquaring the ſide AB. 16.

Examp.

Suppoſe the ſide of the ſquare 23 *inches.*

23
23
69
46
Anſ. 529

By the Rule.

Set 1 on A, to the ſide 23 on B, then againſt 23 on A, is the area 529 on B.

Fig.

PROB. III.

To find the area of a right angled parallelogram ABCD.

RULE.

17. Multiply the longer ſide by the ſhorter; the product is the area.

Examp.

Suppoſe the ſides 16 *and* 35.

```
 35
 16
----
210
35
----
560 area.
----
```

By the Rule.

Set 1 on A to the length (35) on B; and againſt the breadth on A (16), is the area (560) on B.

SCHOLIUM.

To find the number of tiles to cover a houſe.

Find the ſquare yards in the floor or baſe, which multiply by 23, if a ſquare roof. Or by 25 if ſharper; or by 32 if true pitch. Generally a quarter of the ſquare yards, ſhews how many hundred are required.

PROB. IV.

To find the area of a Rhomboides, or oblique Parallelogram, ABOD.

RULE.

18. Multiply the length DO or AB, by the breadth, or perpendicular AE; the product is the area.

Examp.

Fig. 18.

Examp.

The length AB 27 *feet, the breadth* AE 18.

27
18

216
27

486. area.

By the Rule.

Set 1 on A, to the length (27) on B; then againſt the breadth (18) on A, is the content (486) on B.

PROB. V.

To find the area of a Trapezoid ABCD.

RULE.

Multiply the ſum of the parallel ſides AB, DC, by the perpendicular diſtance between them AE; half the product is the area or content. 18.

Ex.

Suppoſe the length AB 30 *feet,* DC 37; *and the breadth* AE 18; *to find the area.*

30 87
57 18

87 696
87

2)1566.

783 area.

By

Fig.

By the Rule.

Set 1 on A, to half the ſum of the parallel ſides ($43\frac{1}{2}$) on B; and againſt the breadth (18) on A, is the area (783) on B.

PROB. VI.

To find the area of a Trapezium ABCD.

RULE.

19. 20. Meaſure the diagonal AC, and the perpendiculars thereon BE, DF. Then multiply the diagonal, by the ſum of theſe perpendiculars; half the product is the area.

Ex.

$$\begin{array}{rr} AC = & 28 \\ \hline BE = & 13 \\ DF = & 25 \\ \hline & 38 \\ & 28 \\ \hline & 304 \\ & 76 \\ \hline & 1064 \\ \text{Area,} & 532 \\ \hline \end{array}$$

By the Rule.

Set 1 on A to 28 on B, then againſt 19 (half of 38) on A, is the content (532) on B.

PROB. VII.

To find the content of any irregular plain figure.

RULE.

Divide it into triangles and trapezia, by diagonal lines. And meaſure all theſe triangles and trapezia ſeparately, by the former Rules; and the ſum of them all is the area or content. 21.

If any ſide be crooked, draw a ſtreight line, that ſhall leave as much out, as it takes in.

Examp.

In the figure annext, there are eight ſides, and it is divided into two trapeziums, ABGH, and BDEC; and two triangles BCD, GEF. In the trapezium ABGH, AG = 12, BI = 4, HK = 4.25; In the trapezium BDEG, GD = 167, BM = 8.6, EN = 5.7. In the triangle BCD, BD = $10\frac{1}{2}$, CO = 4.71. In the triangle GEF, GE = 12,36, LF = 3.25.

BI = 4. BM = 8.6
HK = 4.25. EN = 5.7
8.25. 14.3, then by Logarithms,

Fig. 21.

½AG,	6	0.77815	
Sum. per.	8.25	0.91645	
Area	49.5	1.69460	49.5
			119.4
½GD,	8.35	0.92168	24.72
Sum. per.	14.3	1.15533	20.08
Area	119.4,	2.07701	213.70
			whole area.
½BD	5.25,	0.72015	
CO	4.71	0.67302	
Area	24.72	1.39317	
½GE	6.18	0.79098	
LF	3.25	0.51188	
Area	20.08	1.30286	

PROB. VIII.

To find the area of a regular Poligon.

1. RULE.

22. Meaſure the perpendicular diſtance CP from the center of the figure to one ſide AG.

Then multiply the perpendicular CP by the circumference of the poligon ABDEFG; half the product is the area.

2. *Other-*

Fig. 22.

2. *Otherwiſe.*

Multiply the given ſide of the poligon by itſelf, and then by the multiplier over againſt the number of ſides, in this Table; the product is the area.

number of ſides.	Multipliers.
3	0.433013
4	1.000000
5	1.720477
6	2.598076
7	3.633912
8	4.828427
9	6.181824
10	7.694209
11	9.365640
12	11.196152

Examp.

Let the ſide of a hexagon be 12 *inches.*

12		2.598
12		144
24		10392
12		10392
		2598
144	area,	374.112

By the Rule.

Set 1 on A to 144 on B, and againſt 2.6 on A is the area 374 on B.

PROB. IX.

To find the area of a circle ABCD.

1. RULE.

Multiply half the circumference ABCD, by half the diameter AC; and the product is the area. 23.

2. *Or thus.*

Make as 14 to 11; ſo the ſquare of the diameter AC, to the area.

Fig. 23.

3. *Or thus.*

Multiply the ſquare of the diameter by .7854, the product is the area.

4. *Or thus.*

Say as 88 to 7, ſo the ſquare of the circumference, to the area.

5. *Or thus.*

Multiply the ſquare of the circumference by .07958; the product is the area.

You may uſe any one of theſe rules, according as the data happens to be.

Ex. 1.

Let the diameter be 26, *to find the area.*

26	.7854
26	676
156	47124
52	5498
	471
676	530.93 area.

1. *By the Rule.*

Set 14 on A, to 11 on B; then againſt the ſquare of the diameter on A (676), is the area on B (531).

2. *Or thus by the Rule.*

Set 1 on D, to .785 on C; } then againſt the
or ſet 1.13 on D, to 1 on C; } diameter (26)
or ſet 11 on D, to 95 on C; } on D,
or ſet 8 on D, to 50 on C; } is the area (531)
or ſet 12 on D, to 113 on C; } on C.

Ex.

Fig. 23.

Ex. 2.

Suppoſe the circumference be 5, what is the area?

5	.0796
5	25
25	3980
	1592
	1.9900, or 2 the area.

3. *By the Rule.*

Set 88 on A, to 7 on B, and againſt the ſquare of the circumference (25) on A, is the area (1.99) on B.

4. *Or thus by the Rule.*

Set 10 on D, to 7.96 on C; } then againſt the
or ſet 11.3 on D, to 10 on C; } circumference
or ſet 5 on D, to 2 on C; } (5) on D, is the
or ſet 11 on D, to 9.6 on C; } area (1.99) on C.

Cor. *To meaſure the area of a ring or annulus.*

1. RULE.

Find the areas of the outer and inner circles, and their difference is the area of the annulus.

2. *Or thus.*

Multiply the ſum of the diameters by their difference, and the product by .7854.

SCHOL.

The reaſon of theſe different proceſſes is founded on this, that the diameter is to the circumference, as 1 to 3.1416, or nearly as 7 to 22; and the ſquare of the diameter to the area, as 1 to 3.1416

Fig. 23. $\frac{3.1416}{4}$ or .7854; that is nearly as 14 to 11. Likewiſe the ſquare of the circumference, to the area; is as 1 to .07958, or as 22 × 22 to $\frac{7 \times 22}{4}$, or as 22 × 4 to 7, or 88 to 7. And upon the rule, the numbers on C are as the ſquares of the numbers on D. Therefore it will be, $1^2 : .785 :: \overline{1.13}^2 : 1 :: 11^2 : 95 :: 8^2 : 50$, which are all in the ſame proportion. For the ſame reaſon, $1^2 : .0796 :: \overline{11.3}^2 : 10 :: 5^2 : 2 :: 11^2 : 9.6$, are in the ſame proportion, nearly.

Likewiſe in the ſquare which is equal to the circle, whoſe diameter is 1, the ſide is $.886227 = \sqrt{.7854}$. And the ſide of the inſcribed ſquare is $.707107 = \sqrt{\frac{1}{2}}$.

Alſo in the circle whoſe circumference is 1, the diameter is $.318310 = \frac{1}{3.1416}$; and the area $.079577 = \frac{.7854}{3.1416}$; and the ſide of the equal ſquare, is $.282095 = \frac{.886227}{3.1416}$. And ſide of the inſcribed ſquare is $.225079 = \frac{707107}{3.1416}$.

Whence as 7 to 11, ſo the inſcribed ſquare to the circle's area.

PROB. X.

To find the area of the ſector of a circle ADC.

RULE.

24. Multiply the arch AD by the radius AC, and half the product is the area.

Ex.

Fig. 24.

Ex.

Let the radius AC *be* 10, *and the arch* AD 7.

10
7
2)70(35 area.

PROB. XI.

To find the area of the ſegment of a circle, ADB.

1 RULE.

Multiply the baſe AB by the height DE; and $\frac{2}{3}$ the product is the area, when the height is ſmall; or $\frac{3}{4}$ the product, if near a ſemicircle.

2. *Otherwiſe.*

Let DE be the height, and AE or EB half the baſe; then multiply the ſquare of DE, by .392, to which add the ſquare of AE. Extract the ſquare root of the ſum, which multiply by $\frac{4}{3}$ DE, for the area.

3. *Otherwiſe.*

Let D be the vertex, and draw the chord AD; to the ſquare of AB, add the ſquare of DE, extract the ſquare root of the ſum; to twice that ſquare root add AD, which multiply by $\frac{4}{15}$ DE for the area.

4. *Otherwiſe.*

Find *d* the diameter, then $\frac{80d - 39\text{DE}}{16d - 3\text{DE}} \times \text{AD} \times \frac{4}{15}\text{DE}$, is equal to the area.

Fig. 24.

5. *Otherwiſe.*

Find d the diameter, then $\frac{5d - 3\mathrm{DE}}{5d - 4\mathrm{DE}} \times \frac{2}{3}\mathrm{AB} \times \mathrm{DE} =$ the area of the ſegment.

Note, a zone adjoining to the center may be found by this Rule, by ſubtracting the ſegment from the ſemicircle.

6. *Otherwiſe.*

$\frac{4\mathrm{AD} + 3\mathrm{AB}}{5} \times \frac{2}{3}\mathrm{DE} =$ area of the ſegment ADB.

Note, the 1, and 5th Rules, will alſo find the area of an elliptic ſegment.

7. *Or thus.*

Let $\mathrm{DI} = \mathrm{IE}$, then $\frac{\mathrm{AD} + 4\mathrm{AI}}{3} \times \frac{2}{3}\mathrm{DE} =$ area of the ſegment.

8. *Or thus.*

Let arch $\mathrm{AD} + \mathrm{AE} = p$, arch $\mathrm{AD} - \mathrm{AE} = q$. Then $\frac{p \times \mathrm{DE}^2 + q \times \mathrm{AE}^2}{2\mathrm{DE}} =$ area of the ſegment.

Examp.

Let the baſe AB *be* 32, *chord* AD 20, *height* DE 12: *Then by the 6th way*,

```
 20     32      12
  4      3       2
 ---    ---     ---
 80     96     3)24(8
        80     -----
       ----      35.2
  5)176(35.2      8
                -----
                281.6 the area.
                -----
```

When the ſegment is greater than a ſemicircle, find the leſſer ſegment, which ſubtract from the whole circle.

PROB.

Fig.

PROB. XII.

To find the area of an ellipſis ACBD.

RULE.

Multiply the tranſverſe AB by the conjugate axis CD; and the product by .7854, for the area. 8.

For an elliptic ſegment, ſee the laſt Problem.

Example.

Suppoſe the tranſverſe be 32, *and the conjugate* 24.

32	.7854
24	768
128	62832
64	47124
	54978
768	
	603.1872 area.

By the Rule.

Find a mean 27.7 between 32 and 24. Then ſet 1 on D to .785 on C; and againſt 27.7 on D, is the content 603.. on C.

PROB. XIII.

To find the area of a Parabola ABC.

RULE.

A parabola is made by cutting a cone by a plane parallel to its ſide. 25.

Multiply the baſe AC by the height BD; and $\frac{2}{3}$ that product is the area.

Fig. 25.

Examp.

Suppoſe the height BD = 100, *and baſe* AC = 68.

68	6800
100	2
6800	3)13600(4533, the area.

By the Rule.

Set 1 on A, to $\frac{1}{3}$ the height (33.3) on B, and againſt twice the baſe (136) on A, is the content (4533) on B.

PROB. XIV.

To find the ſegment AEFC *of a Parabola.*

RULE.

EF is parallel to AC, then $\frac{AC^3 - EF^3}{AC^2 - EF^2} \times \frac{2}{3}GD$ = the area.

Examp.

Let AC *be* 12, EF 8, GD 5, *to find the area.*

12	8	144	80)1216(15.2	15.2
12	8	64	80	5
144	64	80	416	76.0
12	8		400	2
		1728		
1728	512	512	160	3)152.0
			160	50.66 area.
		1216		
			0	

PROB.

Fig. 25.

PROB. XV.

To meaſure irregular Parallelograms, ſuch as irregular planks, &c.

RULE.

Take the breadth in ſeveral equidiſtant places, the ſum of which divide by the number of places, for the mean breadth; multiply the mean breadth by the length, for the area.

Examp.

Suppoſe a plank 16 *feet long, and the breadth in ſeveral places* 6, 7, 9, 9, 11 *inches.*

6	16
7	8.4
9	———
9	64
11	128
———	———
5) 42 (8.4 mean breadth.	12)134.4(11.2 feet.

By the Rule.

Having found the mean breadth in inches; then ſet 12 on A, to the length 16 on B; and againſt the breadth 8.4 on A, is the content on B 11.2 feet.

PROB. XVI.

To meaſure any irregular figure, bounded by a curve.

1. RULE.

Draw AC for the baſe, then meaſure the three equidiſtant perpendiculars Aa, Bb, Cc; then 26.

$$\frac{Aa + Cc + 4Bb}{6} \times AC = \text{area } AacC.$$

2. *Otherwiſe.*

27. Meaſure the four equidiſtant perpendiculars A*a*, B*b*, C*c*, D*d*; then

$$\frac{Aa + Dd + 3Bb + 3Cc}{8} \times AD = \text{area } AadD.$$

3. *Or thus more exactly.*

28. Meaſure the five equidiſtant perpendiculars A*a*, B*b*, C*c*, D*d*, E*e*; then

$$\frac{7Aa + 7Ee + 32Bb + 32Dd + 12Cc}{90} \times AE = \text{area}$$

A*ae*E; but this is troubleſome to complete.

29. And the ſame holds, if the figure be curve on both ſides; drawing the baſe AC thro' it.

4. *Otherwiſe.*

30. Divide the baſe AI into any even number of equal parts, and

Let X equal ſum of the extreme ordinates A, I.

O = ſum of all the other odd ones, the 3d, 5th, 7th, &c.

E = ſum of all the even ones, 2d, 4th, 6th, &c.

L = length of two of theſe equal parts. Then

$$\frac{X + 2O + 4E}{6} \times L = \text{area of the curve.}$$

Where for every two ordinates, from the firſt; the curve ought to be concave the ſame way.

Examp.

26. *There is a curve line whoſe baſe is* 67 *feet, and three ordinates or perpendiculars erected,* A*a*, B*b*, C*c*, *are* 35, 40 *and* 28.

By the firſt Rule.		35	223
		28	67
	40	160	——
	4	——	1561
	——	223	1338
	160	——	——— area.

6)14941(2490$\frac{1}{6}$.

PROB.

PROB. XVII.

To find the ſurface of a vaulted roof, being like the arch of a bridge.

RULE.

Meaſure the length of the curve of the arch by a ſtring or otherwiſe; then multiply this length of the arch, by the length of the vault; the product is the ſurface.

Examp.

Suppoſe the arch be 30 *yards, and the length of the vault* 7½.

$30\frac{1}{2}$
$7\frac{1}{2}$

210
15

Surf. 225 yards.

Cor. *If the arch be a ſemicircle, or any ſegment of a circle; its length may be found by* Prob. II. *without meaſuring.*

PROB. XVIII.

To find the ſurface of a pyramid, parallelopipedon, or of any ſolid bounded by planes.

RULE.

Meaſure every particular plane ſeparately; and the ſum of all is the ſurface.

Cor. *If it be an upright priſm; meaſure the circumference of the baſe, which multiply by the height of the ſolid.*

PROB. XIX.

To meaſure the ſurface of a right cone, or a fruſtum of it.

RULE.

31. Multiply the circumference of the baſe by half the ſlant ſide, for the ſurface of the whole.

Or multiply half the ſum of the circumferences of the top and bottom, by the ſlant ſide, for the fruſtum.

Or thus.

If AB be the ſide, and BC the Diameter. Then as 7 : 11 : : AB × BC : ſurface of the cone.

Example.

Let the ſide AB *be* 20; *diameter* BC 8.

20
8
―――
160
11
―――
160
160
―――
7) 1760
251.4
Surface.

PROB. XX.

To find the ſurface of a right cylinder.

1. RULE.

32. Multiply the circumference of the baſe AB by the height AC, for the convex ſurface.

2. *Or thus.*

As 7 : 22 : : AB × AC : ſurface.

Examp. Fig.

Let the ſide CA *be* 20, *diameter* AB 8. 32.

20
8

160
22

320
320

ſurface.
7) 3520(502.8

PROB. XXI.

To find the ſurface of a ſphere.

1. RULE.

Multiply the circumference of the ſphere ABCD, by the diameter AC, for the ſurface.

2. *Or thus.*

Find the area of a great circle of the ſphere, which multiply by 4 for the ſurface. 33.

Or thus.

7 : 22 : : AC^2 : ſurface.

Examp.

Let AC = 9 *feet.*

9	81	
9	22	
81	162	=162
	162	162
	7)1782	1782
	254.6 ſurface.	

PROB. XXII.

To find the ſurface of the ſegment of a ſphere.

1. RULE.

34. Multiply the circumference of the whole ſphere by the height of the ſegment CD, for its ſurface.

2. *Or thus,*

To the ſquare of the diameter of the baſe add 4 times the ſquare of the height CD; the ſum multiplied by .7854, gives the ſurface; or $3.1416AC^2$ = ſurface.

3. *Or thus.*

As 7 to 22, ſo AC^2 to the ſurface ACB.

Cor. *Hence if the ſurface of any zone be required, work by the firſt rule, viz. multiply the circumference of the whole ſphere, by the height of the zone.*

Examp.

If AC *be* 25 *feet, what is the ſurface* ACB?

25	625
25	22
125	1250
50	1250
625	7)13750(1964.3 feet.

PROB.

Fig. 35.

PROB. XXIII.

To find the ſurface of a cylindric ring.

1. RULE.

Add the thickneſs AB to the inner diameter BD, which ſum multiply by the thickneſs AB, and the product by 3.1416^2, gives the ſurface.

2. Or rather thus.

The ſurface $= AB \times AD \times 9.87$.

3. Or thus.

From $10AB \times AD$, ſubtract $\frac{1}{77}$ part thereof, for the ſurface.

Examp.

Let AB *be* 1, AD 12,

12	9.87
1	12
12	1974
	987
	118.44 ſurface.

$\surd 3.1416^{2} = 9.8696$

PROB. XXIV.

To find the ſurface of a ſpheroid.

1. RULE.

Multiply the two axes AB CD together, and that product by 3.1416 for the ſurface of an oblong ſpheroid. 36.

2. *Or*

Fig. 36.

2. *Or thus.*

To twice the tranſverſe (or axis of revolution) add the conjugate diameter, multiply the ſum by $\frac{1}{2}$ the conjugate; and the product by 3.1416 gives the ſurface.

3. *Otherwiſe.*

Meaſure the circumferences about AB and CD, which call P and Q. Then as 22 : 7 : : P × Q : ſurface of the ſpheroid; which is a ſmall matter too big, when the diameters AB, CD, are very unequal.

Examp.

Let AD *be* 10, CD 7.

10	3.1416
7	70
70	219.9120 or 220 the ſurface.

PROB. XXV.

To find the ſurface of any ſolid CAD, *generated by revolving about the axis* AB.

RULE.

37. Take with a ſtring the length of the arch CA or AD; and divide the ſtring into 3 equal parts at *e*, *f*; apply the ſtring again to the curve, and mark where the points *e*, *f*, fall: take the diameters there, which call P and Q, then $\frac{CD + 3P + 3Q}{8} \times 3.1416\ AfeC =$ ſurface.

Note, If the curve does not come to the axis at A, you muſt take its diameter there, and add it to CD.

If the body be very long, divide it into two or more parts, and meaſure each part ſeparately, by this Rule. Fig. 37.

Examp.

Suppoſe the three diameters at C, *e and f*; *that is* CD, P *and* Q, *to be* 12, 10, 7; *and the length of the curve* AfeC, 15.

10	7	12	3.1416	
3	3	30	63	
—	—	21	——	197.92
30	21	—	94248	15
—	—	63	188496	——
		—	——	98960
			197.9208	19792
			——	——
			8)	2968.80
				371.10
				Surface.

SCHOL.

Any proportion on the lines A and B on the ſliding rule, may be wrought on Gunter's line, with a pair of compaſſes; by extending from the firſt term to the ſecond; and ſetting that extent the ſame way, from the third to the fourth.

Or, extending from the firſt to the third; and ſetting that extent from the ſecond to the fourth.

Alſo, any of theſe proportions, which are wrought on the lines D and C, may be wrought on Gunter's line with compaſſes, thus. Extend from the firſt term to the third; and ſet twice that extent (the ſame way) from the ſecond term, and it will reach to the fourth.

Examp. If the diameter of a circle be 26 (*Ex.* 1. Prob. IX,); extend from 11 to 26, and that extent

Fig. 37. tent (turned twice over) will reach from 95 to the area 531.

And the ſame rule holds good for ſolids, by the proportions laid down in the following parts of this book.

Alſo if the diameter or ſide of a ſolid is known, and its content; the content of any other ſimilar ſolid may be found; whoſe diameter or ſide is given. Thus

Extend from the known ſide to the given ſide, and thrice that extent will reach from the known content to the content required.

PART IV.

The Meaſuring of Solids.

PROB. I.

To find the ſolidity of a Cube ABCD.

RULE.

MULTIPLY the ſide AB by itſelf, and that 38.
product by the ſame ſide again; and the laſt product is the ſolid content, in the ſame ſort of meaſure.

This is called *cubing* the ſide AB.

Examp.

Suppoſe the ſide AB, *be two feet two inches.*

2 f. 2 in. = 2.166 feet.

```
     2.166          4.691
    ------          2.166
                   ------
     12996          28146
    12996           28146
    2166           4691
   4332           9382
  --------        --------
  4.691556        10.160706
  --------
              or  10.16 feet.
                  ---------
```

Fig. 38.

1. *By the Rule.*

Set 1 upon D, to the ſide (2.166) on C; and againſt the ſide (2.166) on D, is the content on C (10.16) feet, being the ſame ſort of meaſure.

2. *Or thus, by the Rule.*

Set 12 upon D, to the height in feet (2.16) on C; and againſt the ſide in inches (26) on D, is the content in feet (10.16) on C.

PROB. II.

To find the content of a ſquare priſm, or parallelopipedon with a ſquare baſe, ABCD.

RULE.

40. Square the ſide AB for the area of the baſe; which multiply by the height, gives the ſolidity.

Examp.

Suppoſe the baſe 2 feet 6 inches, and the height 19 feet.

2 f. 6 in. =	2.5	6.25
	2.5	19
	125	5625
	50	625
baſe	6.25	118.75 content.

1. *By the Rule.*

Set 1 upon D, to the height (19) upon C, and againſt the ſide of the baſe ($2\frac{1}{2}$) upon D, is the content (118.7) upon C, in the ſame meaſures.

2. *Or thus, by the Rule.*

Set 12 upon D, to the height in feet (19) upon C, and againſt the ſide of the baſe in inches (30) upon D, is the content in feet (118.7) upon C.

PROB. III.

To find the ſolidity of any ſort of priſm, ABCD.

RULE.

Find the area of the baſe AB, by ſome of the 41
Problems in Part III. then multiply that baſe by 42.
the height AC, gives the ſolid content.

Exam.

Let the baſe AB, *be* 4.65 *feet, and height* AC 25 *feet.*

$$\begin{array}{r} 4.65 \\ 25 \\ \hline 2325 \\ 930 \\ \hline 116.25 \text{ feet.} \end{array}$$

1. *By the Rule.*

Set 1 upon A, to the baſe (4.65) upon B; then againſt the length (25) on A, is the content on B (116), in the ſame ſort of meaſure.

2. *Or thus, by the Rule.*

Set 144 upon A, to the baſe in inches ($669\frac{1}{2}$) on B, then againſt the length (25) in feet on A, is the content in feet on B (116).

PROB. IV.

To find the ſolid content of a cylinder, ABCD.

RULE.

39. Find the area of the baſe AB or DC, by Prob. 9. Part III. then multiply that baſe by the height AD, for the content.

Examp. 1.

Let the diameter of a cylinder be 14 *inches, and the height* 5 *feet.*

14 in. =	1.166 +	.785	1.067
	1.167	1.36	5
	8162	4710	5.335 content.
	6996	2355	
	1166	785	
	1166	1.06760 base.	
	1.360722		

1. *By the Rule.*

Set 1.13 upon D, to the height (5) upon C, and againſt the diameter (1.166) on D, is the content (5.33) on C, in the ſame meaſure.

2. *Or thus, by the Rule.*

Take $\frac{1}{4}$ the circumference for the girt. Then ſet .886 on D, to the height (5) on C; then againſt the girt (.917) on D, is the content (5.33) on C; in the ſame meaſure.

For here the diameter being 14 inches the circumference is 44, and the girt 11 or .917 feet.

3. *Or*

Fig. 39.

3. *Or thus, by the Rule.*

Set 3.54 upon D, to the height (5) on C; and againſt the circumference (3.66) on D, is the content (5.33) on C; in the ſame meaſure.

Ex. 2.

There is a cylinder whoſe height is 40 *feet, and diameter* 30 *inches, or circumference* 94.25 *inches; to find the content in feet.*

30 in. = 2.5 f.	.7854	4.908
2.5	6.25	40
125	47124	196.320 content.
50	1571	
	392	
6.25	4.9087	

4. *By the Rule.*

Set 13.54 on D, to the height in feet (40) on C; and againſt the diameter in inches (30) on D, is the content in feet (196.3) on C.

5. *Or thus, by the Rule.*

Take $\frac{1}{4}$ the circumference in inches for the girt. Then ſet 10.63 on D, to the height in feet (40) on C; and againſt the girt in inches (23.5) on D, is the content in feet (196.3) on C.

6. *Or thus, by the Rule.*

Set 42.54 on D, to the height in feet (40) on C; and againſt the circumference in inches (94.25) on D, is the content in feet (196.3) on C.

Fig. 39.

SCHOLIUM.

Obſerve here, that .886227 is the ſquare root of the area of a circle whoſe diameter is 1 $= \sqrt{.7854}$.

And .282095 is the ſide of a ſquare, equal to a circle, whoſe circumference is 1.

And $3.5450 = \frac{1}{.282095} = \frac{3.1416}{.8862}$, and is the circumference of a circle, whoſe area is 1.

And 10.63472 is the ſide of a ſquare, equal to a circle, whoſe diameter is 12 inches $= \sqrt{.7854 \times 144} = 12 \times .886227$.

And $42.538896 = 4 \times 10.634724 = 12 \times 3.5450$.

And $13.54055 = \frac{42.538896}{3.1416}$, and is the diameter of a circle, whoſe area is 144, and therefore is $= \frac{12}{.8862}$.

And 1.128381 is the diameter of a circle, whoſe area is 1, $= \frac{1}{.8862}$.

PROB. V.

To find the content of a cone or pyramid ADB.

RULE.

43. 44. Meaſure the baſe, then multiply the baſe by one third of the height DC; or elſe multiply the baſe by the height, and take $\frac{1}{3}$ the product, for the content.

Fig.

Examp.

There is a cone or pyramid whoſe height DC *is* 60, *and baſe* 51, *to find the content.* 43. 44.

51 or 51
20 60

1020 content 3)3060
1020 content.

By the Rule.

Set 1 on A, to $\frac{1}{3}$ the height (20) on B; and againſt the baſe (51) on A, is the content (1020) on B.

PROB. VI.

To find the content of the fruſtum of a cone or pyramid AD.

1. RULE

Find the areas of the two baſes, at top and bottom AB, CD, and the height PQ; call the baſes B and *b*. Then $\frac{B+b+\sqrt{Bb}}{3} \times PQ =$ content. 45. 46.

That is, multiply the two baſes together, and extract the ſquare root; to the ſum of the baſes add this ſquare root, and multiply the third part thereof by the height.

2. *Or thus.*

For a ſquare pyramid, to the rectangle of the ſides of the two baſes, add $\frac{1}{3}$ the ſquare of the difference; the ſum multiplied by the height, gives the ſolidity of the fruſtum.

In a cone uſe the diameters inſtead of the ſides; and the laſt product muſt be multiplied by .7854, for the fruſtum.

3. *Otherwiſe.*

Fig. 45. 46. In a ſquare pyramid, let D, *d*, be the ſides of the baſes, or diameters in a cone. Then $\frac{D^3 - d^3}{D - d} \times \frac{1}{3}$ the height, = content of the ſquare fruſtum; which fruſtum muſt be multiplied by .7854, for the content of the conic fruſtum.

Examp.

Suppoſe the greater diameter 36, *the leſſer* 27, *and the height* 24.

36	36	972	23976 the ſquare fruſtum,
27	27	27	.7854
252	9	999	167832
72	9	24	19181
			1198
972	3)81(27	3996	95
		1998	
			18830.6 the
		23976	conic fruſtum.

PROB. VII.

To find the content of a Priſmoid or Pyramidoid GE.

47. This is a ſolid whoſe ſides are ſtreight, and the two baſes parallel, but not ſimilar figures; when they are poligons, 'tis called a *Priſmoid*, or *Pyramidoid*; but if they are ellipſes, 'tis called a *Cylindroid*. For this ſolid, when the baſes are rectangles, or ellipſes.

RULE.

To the bottom length (BE), add half the top length (AD); and multiply the ſum by the bottom breadth (BC), for the firſt product.

To the top length (AD), add half the bottom length (BE), and multiply the ſum by the top breadth (AG); for the ſecond product. Fig. 47.

Multiply the ſum of theſe products by $\frac{1}{3}$ the height (AL) for the content.

When the body is a cylindroid, with elliptic baſes; you muſt uſe the diameters inſtead of the ſides parallel to them; and compute the ſolidity as before; and then multiply the laſt product by .7854, and it gives the content of the inſcribed cylindroid.

When the baſes are poligons, you may uſe the firſt rule in the laſt Problem.

Examp.

There is a ſquare priſmoid, whoſe dimenſions are BE = 40, AD = 36, CB = 32, AG = 12; *and height* AL = 28.

40	36	1856
18	20	672
58	56	2528
32	12	$9\frac{1}{3}$
116	112	22752
174	56	$842\frac{2}{3}$
1856	672	$23594\frac{2}{3}$ content.

But for the inſcribed cylindroid, multiply by .7854.

23594.6
.7854

1651626
188757
11797
943

Content, 18531.23

Fig. 48. 49.

PROB. VIII.

To find the content of any ungula, or hoof of a cone.

This is a ſolid made by cutting the fruſtum of a cone, by a plane paſſing through it diagonally, or touching the two oppoſite baſes, on contrary ſides.

1. RULE.

Meaſure the diameter of the baſe AB, and the conjugate diameter IO, of the ellipſis AIBO; alſo the height CD. Then divide the difference of the cubes of AB, IO, by the difference of their ſquares; and multiply the quotient by .7854AB × $\frac{1}{3}$ CD, for the content of either ungula.

Note, if d be = the diameter that is wanting; then you may take $\sqrt{AB \times d}$ inſtead of IO.

2. *Or thus.*

Let diameter AB = b, conjug. diameter IO = c. Then $\frac{bb + bc + cc}{b + c} b \times \frac{.7854}{3} \times \cancel{AB}$ CD is the content.

By the 1 RULE

$16^3 = 4096$, $9^3 = 729$, $4096 - 729 = 3367$, and $16^2 = 256$, $9^2 = 81$ then $256 - 81 = 175$ and $3367 \div 175 = 19.24$, then $.7854 \times 16$ ~~Examp~~ $4 = 50.2656 \times 19.24 = 967.0436$ the content nearly as on the other side. $967.04 - 966.97 = 000.07$ = diff, Fig. 48. 49.

Examp.

Suppoſe AB 16, IO 9, CD 12; *to find the content.*

```
   16    9    16          481
   16    9     9           16
   --   --   ---          ---
   96   81   144         2886
  16    --    81         481
  --         256         ---
  256        ---      25)7696(307.8
  ---        481         75..
                         --
  307.8                   196
     12   1231.2          175
  -----    .7854          ---
   6156   ------           21
  3078    86184            --
  -----    9849
3)3693.6    615
  1231.2     49
  ------  ------
          966.97 the content.
          ------
```

PROB. IX.

To meaſure irregular parallelopipedons, ſuch as pieces of timber, &c.

1. RULE.

Girt it about in the middle with a ſtring, and meaſure it, and take a quarter of the girt in inches for the ſide of the ſquare; therefore ſquare it, and then multiply it by the length of the body; the product is the content.

2. *Otherwiſe.*

If the body is very irregular, divide it into two or more parts, and meaſure each part ſeparately, by this rule,

Or

Fig. Or elſe, gird it in ſeveral equidiſtant places, and add all the girths together, and divide the ſum by the number of places, for the girt; then proceed as before.

Note, if the girt be taken in inches, and the length in feet, the laſt product, divided by 144, gives the content in feet.

Examp.

A piece of wood is 18 *feet long, and the girt round about the middle, is* 66 *inches; to find the content in feet.*

A quarter of the girt is 16½ for the ſide of the ſquare.

16.5
16.5
825
990
165
272.25

272.25
18
217800
27225
144)4900.50(34.02 feet.
432...
580
576
400

3. *Otherwiſe by a Table.*

From this table, take the area correſponding to the ſide of the ſquare in inches; and multiply it by the length in feet for the content. If the ſide of the ſquare exceed the table, take half of it; and at laſt, take four times the content found.

A Table for meaſuring Wood.

Side ſquare	Area.	Side ſquare	Area.	Side ſquare	Area.
Inch.	Feet.	Inch.	Feet.	Inch.	Feet.
6	.250	12	1.000	18	2.250
6¼	.272	12¼	1.042	18½	2.376
6½	.294	12½	1.085	19	2.506
6¾	.317	12¾	1.129	19½	2.640
7	.340	13	1.174	20	2.777
7¼	.364	13¼	1.219	20½	2.917
7½	.390	13½	1.265	21	3.062
7¾	.417	13¾	1.313	21½	3.209
8	.444	14	1.361	22	3.362
8¼	.472	14¼	1.410	22½	3.516
8½	.501	14½	1.460	23	3.673
8¾	.531	14¾	1.511	23½	3.835
9	.562	15	1.562	24	4.000
9¼	.594	15¼	1.615	24½	4.168
9½	.626	15½	1.668	25	4.340
9¾	.659	15¾	1.722	25½	4.516
10	.694	16	1.777	26	4.694
10¼	.730	16¼	1.833	26½	4.876
10½	.766	16½	1.890	27	5.062
10¾	.803	16¾	1.948	27½	5.252
11	.840	17	2.006	28	5.444
11¼	.878	17¼	2.066	28½	5.640
11½	.918	17½	2.126	29	5.840
11¾	.959	17¾	2.187	29½	6.044
				30	6.250
				30½	6.460
				31	6.673
				31½	6.890
				32	7.111
				32½	7.335
				33	7.562
				33½	7.793
				34	8.027
				34½	8.265

Side square	Area
Inch.	feet
35	8.506
35½	8.752
36	9.000

By

Fig. 1

Examp.

If a piece of wood be 18 *feet long, and* 16½ *inches a quarter of the girt, as before.*

$$\begin{array}{r} 1.890 \\ 18 \\ \hline 15120 \\ 1890 \\ \hline 34.020 \end{array}$$

34.020 the content as before.

1. *By the Sliding Rule.*

Meaſure the circumference in the middle, and take a quarter of it in inches, and call it the girt.

Then ſet 12 on D, to the length in feet (18) on C; then againſt the girt in inches (16½) on D, is the content on C (34) feet.

2. *Or thus, by the Rule.*

Take a quarter of the circumference, or the girt in feet, which here is 1.375; then,

Set 1 upon D, to the length (18) upon C; and againſt the girt (1.37) in feet on D, is the content (34) upon C.

Note, theſe are the ſame rules, as given in Prob. II.

SCHOL.

This is the common method of meaſuring wood, as practiſed in all raff yards, taking a quarter of the circumference in the middle, for the ſide of the ſquare. But it is objected, that it makes the content too little in proportion as 14 to 11; and that it ought to be meaſured as a cylinder. But to this it is anſwered, that before the wood can be ſquared and made fit for uſe, a great part of it goes to waſte in chips; and therefore the quantity of round timber ought to be reckoned no more than what the inſcribed ſquare will amount to, which ſhould be reduced in that proportion, that is as 11 to 7, which is more than 14 to 7. Another objection is, that in tapering timber, taking the ſide of the ſquare in the middle of the piece, makes the content alſo too little. But to this it is anſwered, that in moſt caſes, the great end is to be cut away, till it be of the ſame dimenſions as the leſſer end; or elſe it cannot be ſawn into proper ſtuff; and upon that account taking the ſquare in the middle makes too much of it.

On the other hand, when ſquare timber is meaſured, if one ſide of the ſquare is greater than the other, a quarter of the girt will make too much of it; ſo that this Rule will in ſome caſes give too much, and in others too little, by an inconſiderable quantity; and therefore may paſs well enough in common uſe. But if a round piece of wood is to be meaſured, every inch of which may be made uſe of, then I think it ought to be meaſured as a cylinder, or elſe as the fruſtum of a cone, if it be tapering, according to the Rules in Prob. 4th and 6th.

PROB. X.

To meaſure a heap of ſand, or earth; a pond of water, &c.

RULE.

Take the depth of it, in ſeveral places, at equal diſtances, as near as you can; add all theſe depths together, and divide by the number of them; the quotient is the mean depth. Then find the area of the baſe on which it ſtands, which multiply by the mean depth, and the product is the content.

If it happen, that the heap is ſo ſolid, that the depth cannot be taken, draw a ſtring over the top of it, parallel to the baſe, and meaſure the height

Fig. height of it above the baſe, for the greateſt depth of the heap. Then take ſeveral equidiſtant places upon the ſtring, and meaſure from the ſtring plumb down to the heap; then theſe being ſubtracted from the greateſt depth, give the depth in theſe ſeveral places. Then put the ſtring into a new poſition (or elſe draw another ſtring acroſs it), and take equal diſtances, and meaſure as before; and repeat the ſame, as oft as need be; till you get a ſufficient number of depths thro' the whole heap; all which muſt be ſet down and ſubtracted from the greateſt depth as before. And then you may apply the rule at firſt given.

Note, you may ſometimes reduce the heap into ſome ſort of a regular body, which will facilitate the meaſuring.

Examp.

There is a heap of ſtones, whoſe baſe is 15 yards; the greateſt depth 3, and the depth from lines drawn over it, taken in 9 places, are as in the table.

0	3.
2½	0.5
2	1
1	2
1½	1.5
2	1
1	2
2½	0.5
1½	1.5
	13.0

9)13(1.44
9
—
40
36
—
4

1.44
15
—
720
144
—
content, 21.60 yards.

PROB.

PROB. XI.

To find the ſolidity of any of the five regular ſolids, from a ſide being given.

1. RULE.

Multiply the cube of the given ſide, by the number ſtanding againſt the name of the body in this Table; the product is the ſolid content.

Tetraedron or Pyramid.	.1178511
Hexedron, or Cube.	1.000000
Octaedron.	.4714045
Dodecaedron.	7.6631188
Icoſaedron.	2.1816951

2. *Or thus.*

Multiply the ſurface of the body by $\frac{1}{6}$ of the diſtance of two oppoſite faces, when they are parallel.

Examp.

The ſide of an Octaedron is 27.

$$\begin{array}{r} .4714 \\ 27 \\ \hline 32998 \\ 9428 \\ \hline 12.7278 \end{array}$$

content.

PROB.

Fig.

PROB. XII.

To find the content of any irregular body bounded by planes.

RULE.

Suppoſe it divided into priſms and pyramids; then meaſure all theſe ſeparately, and the ſum of all is the content.

Examp.

50. The ſolid AHFD is divided into five parts, by planes drawn parallel to the ſides. There is 1. The parallelopipedon KHGLNFIO. 2. The triangular priſm AHKOIB. 3. The triangular priſm EFNOIC. 4. The ſquare pyramid IBDCO. 5. The priſm GPF. And each of theſe may be computed by the foregoing Problems, and the ſum will be the whole content. Part of this ſolid may be meaſured by Prob. VII. But when they are compounded they cannot, but muſt be cut in pieces, like this.

PROB. XIII.

To find the content of a cylindric ring.

RULE.

35. To the thickneſs of the ring, AB, add the inner diameter BD; multiply the ſum by the area of the ſection AB, and the product by 3.1416 gives the ſolidity.

Examp. Fig.

Let AB *the thickneſs be* 13, BD *the inner diameter* 35.
157, *to find the ſolid content* BAED.

```
 157                 .7854
  13                   169
 ---             ---------
 170                 70686
 ---                47124
  13                7854
  13             ---------
 ---        area, 132.7326
  39                   170
 13              ---------
 ---              92912820
 169              1327326
 ---             ---------
                 22564.54|20
                   3.1416
                 ---------
                 6769362
                  225645
                   90258
                    2256
                    1353
                 ---------
       Content, 70888.74
                 ---------
```

PROB. XIV.

To find the ſolidity of a ſphere or globe, ABCD.

1. RULE.

Multiply its ſurface by the radius AO; $\frac{1}{3}$ the 33.
product is the ſolidity.

2. *Otherwiſe.*

Say as 21 to 11, ſo the cube of the diameter, to the content: which is nearly as 2 to 1.

Fig. 33.

3. *Otherwiſe.*

Multiply the cube of the diameter by .5236; the product is the ſolidity.

4. *Or thus.*

Multiply the cube of the circumference by .0169; and you have the content.

5. *Or thus.*

Find the area of a great circle of the ſphere, and multiply it by ⅔ of the diameter.

Examp.

Suppoſe the diameter of a globe is 16 inches, or circumference 50.3.

```
 16    256    4096   21)45056(2145½ content.
 16     16      11      42...
 ---   ----   -----     --
 96   1536    4096      30
16     256   4096       21
 ---   ----   -----     --
256   4096   45056      95
 ---   ----   -----     84
                        ---
                        116  116
                        105  105
                        ---  ---
                         11   11
```

1. *By the Rule.*

Set 1.38 on D, to the diameter (16) on C, and againſt the diameter (16) on D, is the content (2140) on C, in the ſame meaſure.

$16^3 = 4096 \times .5236 = 2144.6656$

= content

2. Or

Fig. 33.

2. *Or thus, by the Rule.*

Set 57.4 on D, to the diameter in inches (16) on C; and againſt the diameter (16) in inches, is the content (1.24) in feet.

3. *Or thus, by the Rule.*

Set 7.7 on D, to the circumference (50.3) on C, and againſt (50.3) on D, is the content (2140) on C, in the ſame meaſure.

4. *Or thus, by the Rule.*

Set 320 on D, to the circumference in inches (50.3) on C, and againſt the circumference (50.3) on D, is the content in feet (1.24) on C.

Cor. *To find the ſolidity of an Orb.*

This is only finding the ſolidities of the two ſpheres, and ſubtracting the inner from the outer.

SCHOLIUM.

The number .52359 is the ſolid content of a ſphere, whoſe diameter is 1.

And .0169 is the content, when the circumference is 1.

And $1.3819 = \sqrt{\frac{1}{.52359}}$.

And $57.444 = \sqrt{\frac{1728}{.52359}}$.

And $7.6923 = \sqrt{\frac{1}{.0169}}$.

And $319.76 = \sqrt{\frac{1728}{.0169}}$.

Fig. 51.

PROB. XV.

To find the content of the ſegment of a ſphere ADB,

1. RULE.

To thrice the ſquare of the baſe AB, add four times the ſquare of the height DC, divide the ſum by 6; multiply the quotient by the height DC, and the product by .7854 for the ſolidity. This holds for either ſegment ADB or ANB.

2. *Otherwiſe.*

From ſix times the diameter DN, ſubtract four times the height DC; multiply $\frac{1}{3}$ of the remainder by the ſquare of the height DC, and that product by .7854, for the content; for either ſegment, ADB or ANB.

3. *Otherwiſe.*

$\frac{9}{16}$ baſe × height = content, but leſs exact. This ſerves only for the leſſer ſegment.

Examp.

Let AB *be* 28 *inches, and the height* CD, 6½.

By Logarithms.

28	1.447158	2352	
28	1.447158	169	
3	0.477121		
		2521	3.401573
2352	3.371437	6.5	0.812913
		.7854,	—1.895091
6.5	0.812913		
6.5	0.812913		4.109577
4	0.602060	6,	0.778151
169	2.227886	2145,	3.331426
		Content.	

PROB.

Fig. 52.

PROB. XVI.

To find the ſolidity of any ſpherical Fruſtum or Zone, BACD.

1. RULE.

To the ſum of the ſquares of the diameters of the baſes, AB, CD; add $\frac{4}{3}$ the ſquare of the height AL; multiply the ſum by half the height AL, and the product by .7854 gives the content.

2. *Or thus.*

Multiply half the ſum of the ſquares of baſes AB, CD, by the height AL, to which add $\frac{2}{3}$ the cube of AL: and multiply the whole by .7854 for the content.

Examp.

Let AB *be* 24, CD 42, *and the height* AL 21.

24	42	21	441
24	42	21	4
96	84	21	3)1764
48	168	42	588
576	1764	441	

576
1764
588

Log: 2928,		3.466571
Log: $\frac{1}{2}$AL,	10.5	1.021189
Log: .7854	—	1.895091
Content, 24147	—	4.382851

Fig. 53.

PROB. XVII.

To find the ſolidity of a Spheroid ACBD.

RULE.

Multiply the ſquare of the diameter of the ſpheroid AB, by $\frac{2}{3}$ the axis CD; the product multiply'd by .7854 gives the ſolidity.

Examp.

Let the diameter AB *be* 34, *and axis* CD 60.

Log : AB	34	1.531479
Log :	34	1.531479
Log : $\frac{2}{3}$CD,	40	1.602060
Log :	.7854	1.895091
Content	36317,	— 4.560109

PROB. XVIII.

To find the ſolid content of the ſegment of a Spheroid ADB.

1. RULE.

54. Multiply twice the ſquare of the conjugate diameter, by the height of the ſegment, and divide the product by the tranſverſe diameter; to this quotient, add the ſquare of the diameter of the baſe; and multiply the ſum by $\frac{1}{3}$ the height, and then by .7854, for the content.

2. *Or thus.*

Let $2r$ be the tranſverſe, B the baſe of the ſegment, h the height; then $\frac{3r-h}{2r-h} \times \frac{1}{3}Bh =$ ſolid content.

That is, ſubtract the height of the ſegment from Fig. $\frac{2}{3}$ the tranſverſe axis, by this multiply the ſquare of the diameter of the baſe, for a dividend. Subtract the height from the tranſverſe, for a diviſor. Multiply the quotient by $\frac{1}{3}$ the height, and then by .7854.

3. *Otherwiſe.*

When you have not the tranſverſe and conjugate 53. diameters; multiply the area of the baſe by $\frac{9}{16}$ of the height, for the content of the leſſer ſegment.

Examp.

Let the axis be $36\frac{2}{3}$, the conjugate 18, the baſe AB 14, and height CD $6\frac{1}{2}$.

18	36.66)4212.0(114.9		673.4
18	36666		.7854
		14	
144	5454	14	47138
18	3666		5387
		56	337
324	1788	14	27
2	1466		
		196	528.89
648	322	114.9	content.
$6\frac{1}{2}$	324	3)310.9	
		103.6	
3888		$6\frac{1}{2}$	
324			
		6216	
4212		518	
		673.4	

PROB. XIX.

To find the content of the fruſtum or middle zone of a Spheroid, adjoining to the center.

RULE.

55. To twice the ſquare of the diameter of the greater end CD, add the ſquare of the diameter of the leſſer end AB; multiply the ſum by $\frac{1}{3}$ the height AF; and the product by .7854 for the content.

Examp.

Suppoſe the baſe CD 18, AB 14, *height* EF 12.

```
 18    14   844
 18    14     4 = 1/3 height.
 ---   ---  ----
144    56   3376
 18    14   ----
 ---   ---
324   196
  2   648
 ---   ---
648   844
      ---
```

Log : 3376 — 3.528402
Log : .7854 — 1.895091

Content $2651\frac{1}{2}$, —3.423493

Fig. 56.

PROB. XX.

To meaſure a Paraboloid, or parabolic Conoid ACB.

1. RULE.

This is generated by a parabola revolving round its axis CD.

Multiply the ſquare of the diameter of the baſe AB, by half the height CD; and the product by .7854 for the content.

2. *Or thus.*

Multiply the area of the baſe by half the height.

Example.

Let the diameter of the baſe AB *be* 32, *and height* CD 22.85.

Log : 32 — 1.505150
Log : 32 — 1.505150
Log : $\frac{1}{2}$ height 11.42, 1.057666
.7894 — 1.895091

Content 9184$\frac{1}{2}$, 3.963057

By the Rule.

Set 1.13 on D, to half the height (11.4) on C; and againſt the diameter (32) on D, is the content (9180) on C.

PROB. XXI.

To meaſure the fruſtum of a Paraboloid, AEFB.

RULE.

Add the ſquares of the diameters of the baſes together; multiply the ſum by half the height, and the product by .7854 for the content.

Examp.

Fig. 56.

Examp.

Suppoſe the greater diameter 32, *the leſſer* EF 26, *and height* OE 8.

32	26	1024		.7854
32	26	676		6800
64	156	1700		47124
96	52	4	= ½ height.	6283
1024	676	6800		5340.7 content.

PROB. XXII.

To meaſure a parabolic Spindle ABD.

RULE.

Multiply the ſquare of the diameter AB, by $\frac{8}{15}$ 7. the length CD, and the product by .7854 for the ſolidity.

Note, This is formed by revolving about the ordinate CD of a parabola.

Examp.

Let the baſe AB *be* 24, *the height* CD 37.52.

Log : AB 24	1.380211
Log : AB 24	1.380211
Log : .7854	—1.895091
Log : 8.	0.903090
Log : 37.52	1.574263
	5.132866
Log : 15	1.176091
Content 9053;	3.956775

PROB.

Fig. 57.

PROB. XXIII.

To meaſure the fruſtum of a parabolic Spindle AEFB.

RULE.

To 8 times the ſquare of the greater diameter AB, add 3 times the ſquare of the leſſer diameter EF, and 4 times the product of the two diameters AB, EF: Multiply the ſum by $\frac{1}{15}$ the length EL; and that product by .7854, for the content.

Examp.

Let AB *be* 24, EF 17, *and height* EL 20.

24	17	24	4608
24	17	17	867
			1632
96	119	168	
48	17	24	7107
576	289	408	
8	3	4	
4608	867	1632	

L : 7107	3.85168
L : 20	1.30103
L : .7854	— 1.89509
	5.04780
L : 15	1.17609
Content, 7442$\frac{1}{2}$,	3.87171

PROB.

Fig. 58.

PROB. XXIV.

To find the content of a Hyperboloid, or hyperbolical Conoid, ACB.

A hyperboloid is generated by a hyperbola ACB, revolving about its axis CD; and the hyperbola is one of the Conic Sections; to meaſure it.

1. RULE.

Let a = tranſverſe axis, round which it revolves; h = height; B = area of the baſe.

Then $\frac{3a + 2h}{6a + 6h} \times Bh$ = ſolid content. That is,

To 3 times the tranſverſe, add twice the height of the ſolid; multiply this into the ſquare of the diameter of the baſe, which multiply by the height, and then by .7854; reſerve the product for a dividend. To 6 times the tranſverſe add 6 times the height, for a diviſor; the quotient is the content.

2. *Or thus.*

When you know not the tranſverſe; multiply the ſquare of the diameter of the baſe by $\frac{5}{12}$ the height, and then by .7854, for the content, nearly.

Examp.

Fig. 58.

Examp.

Let the tranſverſe be 20, *the height* CD *of the ſolid* 6, *the diameter* AB *of the baſe* 9.

```
 20   60     9      5832
  3   12     9         6
 ——   ——    ——      ————
 60   72    81   156)34992(224.3
 ——         72       312   .7854
  6         ———      ———   ————
  2         162      379   15701
 ——         567      312    1794
 12         ———      ———     112
 ————————   5832     672       9
 20   6     ———      624   176.16
  6   6              ———   content
 ——   ——              48
120   36              ——
 36   ——
 ———
156
 ———
```

If the content be found by the other rule, it comes out only 159.

Cor. *If the fruſtum of a hyperboloid was required, it may be meaſured as the fruſtum of a cone. For in moſt caſes it differs ſo little from it, that the one may be taken for the other, there being no ſenſible difference.*

SCHOL.

If both the tranſverſe and conjugate axes of the hyperboloid were given; the ſolidity may be found by the very ſame rule, as for that of the ſegment of a ſpheroid, in Prob. 18; which is Rule 1.

PROB.

Fig. 59.

PROB. XXV.

To find the ſolid content of a circular ſpindle ABD, *or its fruſtum; which is a ſolid generated by the ſegment of a circle, revolving about the baſe* CD.

RULE.

Let D = diameter of the circle AF; d = MG, the top diameter of the fruſtum; x = height PC; A = arch AM; b = BF, the difference between the diameter AF and baſe AB.

Then $x \times \overline{DD - \frac{4}{3}xx} \mp b \times \overline{dx + DA} : \times .7854$, is the ſolid content; according as AB is leſſer or greater than AF. That is,

From the ſquare of the diameter of the circle, ſubtract $\frac{4}{3}$ the ſquare of the fruſtum's height; which multiply by the height, and reſerve the product.

Then multiply the top diameter by the height; alſo multiply the diameter of the circle, by the arch; and add it to the former; and multiply the ſum, by the difference between the diameter and baſe; ſubtract this product from the firſt product, when the baſe of the fruſtum is leſs than the diameter of the circle; or add it, when greater; and multiply the reſult by .7854.

When the whole ſpindle is required, the part dx is nothing.

And the part MDG may be found, by ſubtracting AMGB from the whole.

Examp.

Fig. 59.

Examp.

Let the diameter AF *be* 20, AB 13, MG 10, CP 5.26;

From the radius and height CP being given, the length of MA will be found 5.54 = A.

$D = 20$
20
400

$x = 5.26$
5.26
3156
1052
2630
27.6676

27.667
$\frac{1}{3}$, 9.222
36.889

400
36.89
363.11
5.26
181555
7262
2178
1 Prod. 1909.95

$x = 5.26$,
$d = 10$
52.6

$A = 5.54$
$D = 20$
110.8
52.6
163.4
$BF = b = 7$
1143.8

1909.9
1143.8
766.1
.7854
53627
6129
383
30
Content, 601.69

SCHOL.

If you deſire to know the ſurface of this ſolid.

It is $\overline{Dx \mp b}\,A \times 3.1416 =$ ſurface of the fruſtum AMGB, according as AB is leſſer or greater than AF.

PROB.

Fig.
60.

PROB. XXVI.

To find the ſolidity of any body CH, *generated by revolving about an axis.*

1. RULE.

Take the diameters of the body, at both ends, and in the middle of the height, as at A, B and I; and let them be CD, GH and EF. Then $\frac{CD^2 + GH^2 + 4EF^2}{6} \times$ length AB $\times .7854$, is the ſolid content.

2. *Or thus.*

Find the area of the Section, at the top, bottom, and middle, which let be A, B, and I. Then $\frac{A + B + 4I}{6} \times$ AB the length, is the content.

If the body be long, divide it into two or more parts, and meaſure each part by the foregoing rules; and the ſum of them gives the ſolid content.

3. *Otherwiſe.*

61. If the body be ſuch as you can ſee it on all ſides, and at any diſtance; cut a paper, as *agbh*, into the apparent form of the body; and holding it directly between your eye and the body AGH; clip away any ſuperfluous parts, from the ſides of the paper, till it juſt cover the body, and their edges coincide. Then if the baſe or height GH or AB, be meaſured, and the baſe or height *gb* or *ab* of your paper be made the ſame, by a proper ſcale; then you may find as many diameters, on your paper, as you will, by the ſame ſcale. From whence, by the former rules, you may find the ſolidity of the body generated by your paper ſcheme, which will be the ſame as that of the great body;

as they are ſimilar ſolids. And thus ſuch a body Fig.
as a hay ſtack may be meaſured. 60.
61.

Examp.

Suppoſe the diameters CD, EF, *and* GH, *be found to be* 12, 24 *and* 30, *and height* AB 32.

```
   12     24     30        576      .7854
   12     24     30          4        630
  ----   ----   ----      -----   --------
  144     96    900       2304     235620
  ----    48    ----       900    47124
         ----              576    --------
         576              -----   494.8020
         ----           6)3780    --------
                           630

  494.8
     32
  -----
   9896
 14844
 -------
 15833.6  content.
 -------
```

PROB. XXVIII.

To find the content of a very irregular body.

1. RULE.

Procure a priſmatic or cylindric veſſel that will hold the body. Put in the body, and pour in water to cover it; make a mark at top of the water, and then take out the body; and obſerve how far the water has deſcended; meaſure that ſpace of the veſſel correſpondent to that deſcent; and you have the ſolidity of the body. If the veſſel will not hold water, you may uſe ſand inſtead of water.

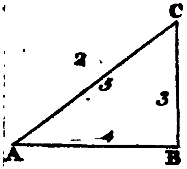
Fig. And thus a fragment or parcel of a body may be meaſured without dividing it from the whole; and that by immerſing only the fragment, and taking it out again, meaſure the deſcent of the water, as before.

Or if your veſſel be irregular, fill it with water, then put in your body; and weigh the water that runs over. Whence the content of the water, or of the body, will be found, by the tables of ſpecific gravity before given.

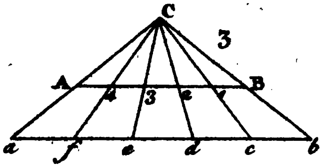
2. *Otherwiſe.*

Find what a cubic inch or a cubic foot of the ſame matter weighs, either by trials, or from the tables of ſpecific gravity. Then weigh the irregular body, and its content will be known, becauſe the ſolidities are as the weights. For if you divide the weight of the body by the weight of an inch or foot, the quotient will be the ſolidity in inches or feet.

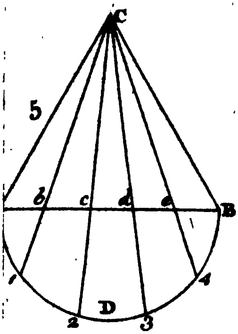
2


C
5
3
4
A
B

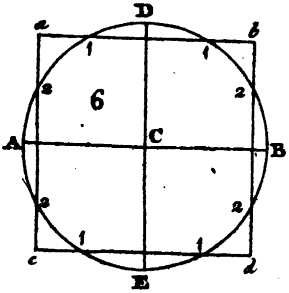
3


C
A
B
4
3
2
1
a
f
e
d
c
b

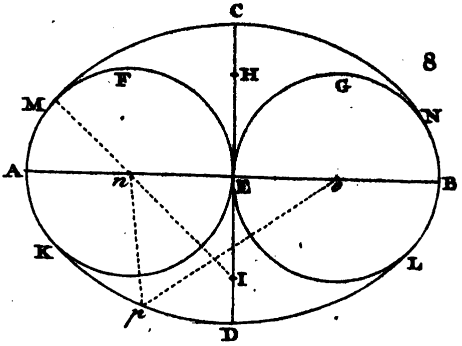
5


C
b
c
d
e
B
1
2
D
3
4

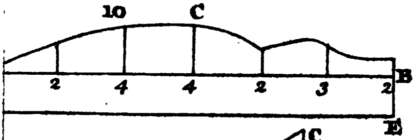
6


D
a
b
1
1
2
2
A
C
B
2
2
1
1
c
E
d

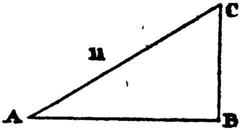
8


C
H
F
G
M
N
A
n
E
B
K
L
I
p
D

10


C
B
E

11


C
A
B

PART V.

GAUGING.

GAUGING is a branch of meaſuring ſolids, and teaches how to find the contents of all ſorts of veſſels, in gallons, or ſome meaſure of capacity. And this differs from other kinds of meaſuring, where the meaſures may be taken, and contents given in feet, yards, &c. But here the meaſures are always taken in inches, or at leaſt muſt be reduced to inches, and the contents given in gallons; for which purpoſe, certain fixt numbers are made uſe of to facilitate the computation, as will be ſhewn afterwards. But in the gauging of malt, the content muſt be had in buſhels.

And in finding the areas of all plane ſurfaces in gauging, their contents are alſo expreſſed in gallons; becauſe in the buſineſs of gauging, every area or ſurface is ſuppoſed to be one inch deep; and therefore in the Gauger's language, the area of ſuch a ſquare, or ſuch a circle, is ſo many gallons. For there will be the ſame number of ſuperficial inches in any plane figure, as there will be ſolid inches of one inch depth; ſo that what is called a ſurface in gauging, is in reality a ſolid, whoſe height is one inch.

As the ſliding rule is of great ſervice in the art of gauging, and very much uſed; I ſhall here deſcribe the principal parts of it, as it differs from the other ſort, deſcribed in the ſecond Part.

Of the SLIDING RULE.

The Gauger's Sliding Rule differs in many things from that before deſcribed, called Cogſhal's Sliding Rule. Upon this rule there are alſo three double lines of numbers, called A, B, C; and a ſingle line called D; this line is marked from the beginning 1, 2, 3, &c. to 10 at the end. And there is alſo a triple line of numbers called E, for extracting the cube root, &c. All theſe lines begin and end together. They work by the means of three ſliders, moving in three faces of the rule. And theſe lines are logarithmic lines, ſince they all anſwer to multiplication and diviſion. Each part of the line E is a third part of the correſpondent diſtance on D.

There is a line marked MD, (*malt depth*), the ſame with A or B, but inverted; it begins at 21.5, and ends at 2.15. This line is alſo logarithmical: Its uſe is for gauging malt. Here 2150.4 on MD is oppoſite to 1 on B; and 1 on MD is oppoſite to 2150.4 on B or A, at a point marked MB.

On the other ſide, by the line of inches, are 3 varieties of caſks; the uſe of theſe, is to find a mean diameter, to reduce the caſk to a cylinder. The practice is thus; take the difference of the head and bung diameters, and find it in the ſcale of inches, and againſt it (in the proper variety) you will have a number, to be added to the head diameter, for the mean diameter.

On the inſide of the ſliders are placed two lines, one numbered from 13 to 16, the other from .42 to 3.61 gallons; and ſerves to ſhew the area in ale gallons, for any diameter. The uſe is this; one ſlider is to be pinned faſt at the end towards the right hand; and the other ſlider is drawn out, till the two ends touch the oppoſite ſides of the tub;

then the diviſions, cut by the end of the rule, Fig. ſhew the diameter on one line; and the area in gallons on the other. When the diameter is more than $24\frac{1}{2}$, draw out the other ſlider, which will ſhew the like.

On another ſide of the rule is a ſlider, with a line of numbers C, like A and B, againſt which are two lines SL (*ſegments lying*), and SS (*ſegments ſtanding*); theſe are for finding the ullage of a caſk in ale. The uſe of theſe lines is this. Find the bung diameter when the caſt lies, or the length when it ſtands. Then

Set the bung diameter on C, to 100 on SL for a lying caſk; or ſet the length on C, to 100 on SS for a ſtanding caſk; and againſt the wet inches on C, is a ſegment on SL, or SS.

Then ſet 100 on A, to the caſks content on B; and againſt that ſegment on A, is the ullage (or liquor in the caſk) on B.

On the line D, are ſeveral marks, which ſhew the gauge points; as WG (*wine gallons*), AG (*ale gallons*), MS (*malt ſquare*), MR (*malt round*). What theſe are, and where to be placed, will be ſhewn in the firſt Prob.

On the plane ſide of the rule, there are ſeveral lines, as one marked FM (*foot meaſure*), where a foot is divided into 100 parts. And another marked IM (*inch meaſure*) being a ſcale of inches. And another marked FC (*fruſtum of a cone*), which ſerves to meaſure its content.

To extract the cube root by the rule. Set the lines D, E, even; that is, 1 againſt 1; then againſt the given number on E, is the cube root on D.

There is a line on the gauging rod, called the *diagonal line*, for finding the content of a caſk; which is thus. Put the rod into the bung hole, to either oppoſite angle; and mark the number at the middle of the bung hole, for that is the content of the caſk.

PROB. I.

To find proper multipliers, diviſors, and gauge points.

1. *For ſquare figures.*

Since 282 inches make an ale gallon, 231 inches a wine gallon, and 2150.4 inches a malt buſhel, therefore when the content of any figure or veſſel is had in inches; 282, 231, and 2150.4 will be the diviſors, for reducing it to *ale gallons*, *wine gallons*, or *malt buſhels* reſpectively.

But if inſtead of theſe diviſors, you would chuſe to have multipliers; then theſe will be had by dividing 1 with a number of cyphers, by 282, 231, and 2150.4 reſpectively. Thus

282)1.000000(.0035461 *for ale gallons.*
231)1.00000(.0043289 *for wine gallons.*
2150.4)1.00000(.0004651 *for malt buſhels.*

The gauge points for ſquare figures are no more than the ſquare roots of 282, 231, and 2150.4; which gauge points are therefore the ſide of a ſquare, whoſe area is a gallon or buſhel.

$\sqrt{282} = 16\ 79$ *ale gallon.*
$\sqrt{231} = 15.19$ *wine gallon.*
$\sqrt{2150.4} = 46.37$, *Wincheſter buſhel.*

For the ſecond gauge points for ſquares, multiply (or divide) the former gauge points by the ſquare root of 10, or 3.162, or thus,

$\sqrt{2820} = 53,10$ *ſecond* GP *for ale.*
$\sqrt{2310} = 48.06$ *ſecond* GP *for wine.*
$\sqrt{215.04} = 14.66$ *ſecond* GP *for malt buſhels.*

All gauge points are the firſt terms of any proportion upon the lines D and C; and theſe ſecond gauge points ſerve, when the terms run off the lines.

For the numbers on D, being as the ſquare roots of thoſe on C; the variation of 10 on C, will cauſe a varia variation of the gauge point on D, equal to the ſquare root of 10. And there muſt be ſuch a variation on C, to bring any number upon another radius.

2. *For circular figures.*

Since .7854, multiplied by the ſquare of the diameter, is the area of a circle in inches; this divided by 282, 231, or 2150.4, will give the content in ale gallons, wine gallons, or malt buſhels. That is $\frac{.7854}{282}$, or $\frac{.7854}{231}$, or $\frac{.7854}{2150.4}$, muſt be multiplied into the ſquares of the diameters. Whence the diviſors will be found by dividing by .7854;

.7854) 282 (359.05 *for ale gallons.*
.7854) 231 (294.12 *for wine gallons.*
.7854) 2150.4 (2738 *for malt buſhels.*

Then the ſquare of the diameter, being divided by any of theſe, gives the circle's area.

But to find the multipliers; theſe are had by dividing .7854 by 282, 231, and 2150.4 reſpectively.

282) .7854 (.002785 *for ale gallons.*
231) .7854 (.0034 *for wine gallons.*
2150.4) .7854 (.000365 *for buſhels.*

The gauge points for circles, are the diameters of ſuch circles as contain ſome known meaſure, as a gallon; as if d be the diameter, then $.7854dd =$ 282 for ale; whence $d = \sqrt{\frac{282}{.7854}}$ for ale, and ſo of the reſt. Or thus,

$\sqrt{359.05} = 18.95$ *the* GP *for ale.*
$\sqrt{294.12} = 17.14$ *the* GP *for wine.*
$\sqrt{2738} = 52.32$ *the* GP *for buſhels.*

The ſecond gauge points are found by multiplying (or dividing) the former by $\sqrt{10}$ or 3.162; or thus,

$\sqrt{3590.5} = 59.92$ *for ale gallons.*
$\sqrt{2941.2} = 54.22$ *for wine gallons.*
$\sqrt{273.8} = 16.54$ *for buſhels.*

Fig.

SCHOL.

The ſecond gauge points may alſo be found by the rule, thus. Set 1 upon C, to the firſt gauge point upon D; then againſt 10 on C, is the ſecond gauge point required; or elſe againſt $\frac{1}{10}$ upon C, is $\frac{1}{10}$ the gauge point, which multiply by 10.

Examp.

Let the firſt gauge point be 18.95. Set 1 upon C, to 18.95 on D; and againſt $\frac{1}{10}$ on C, is 5.99 on D, which multiply by 10, gives 59.9 the ſecond gauge point.

PROB. II.

To find the area of a ſquare.

RULE.

16. Multiply the ſide AB or AD in inches, by itſelf; and divide the product by the proper diviſor, (in Prob. I.); or multiply it by the proper multiplier.

Examp.

Let the ſide of the ſquare be 42 inches; to find the area in ale.

```
      42
      42
    ----
     84
   168
    ----
282) 1764(6.25 ale
          gallons.
```

or thus,

```
        .003546
           1764
        -------
           3546
          2482
          212
          14
        -------
ale gall. 6.254
        -------
```

By

Fig. 16.

By the Sliding Rule.

Set 282 upon A to 42 upon B; then againſt 42 on A is 6.25 on B.

PROB. III.

To find the area of any Parallelogram.

RULE.

Multiply the length by the breadth; and divide the product by the proper diviſor (Prob. I.) or multiply it by the proper multiplier. 18.

Examp.

There is an oblong whoſe length AB *is* 48, *and breadth* AE 32; *to find the area in wine.*

48
32
96
144
231) 1536(6.65 wine gallons.

or thus,

.004329
1536
4329
2164
130
26
wine gall. 6.649

By the Rule.

Set 231 on A to the length 48 on B; and againſt the breadth 32 on A, is 6.65 the area on B.

PROB.

PROB. IV.

To find the area of a Triangle, ACB.

RULE.

14. Multiply the baſe by half the perpendicular, and divide by the proper diviſor (Prob. I.), or multiply by the proper multiplier.

Examp.

Let the baſe AB *be* 154, *the perpendicular* CD 46.2; *to find the area in buſhels.*

154
23.1

154
462
308

2150.4)3557.4(1.65
21504 buſhels.

14070

or thus.

3557.4
.0004651

142296
21344
1779
35

buſhels 1.65454

By the Rule.

Set 2150 on A to the baſe 154 on B; and againſt 23.1 on A is the content 1.65 on B.

PROB. V.

To find the area of a Trapezium ABCD.

RULE.

19. Draw the diagonal AC, and the two perpendiculars upon it, BE, DF. Then multiply the diagonal by half the ſum of the perpendiculars, and divide by the proper diviſor (Prob. I.), or multiply by the correſpondent multiplier. *Ex.*

Fig. 19.

Examp.

Suppoſe the diagonal AC 28, *and the perp.* BE 13, *and* DF 24. *To find the area in ale gallons.*

13	28	*or thus,*
24	18.5	.003546
		518
2) 37	140	
18.5	224	17730
	28	354
		283
	282) 518.0(1.84 ale gallons.	1.8367

By the Rule.

Set 282 on A to 28 on B; and againſt 18.5 on A, is the content 1.84 on B.

Cor. *Hence the area of any irregular poligon will be found, by dividing it into trapeziums or triangles, and meaſuring each of them ſeparately, and taking the ſum of them all.*

And a regular poligon may be meaſured as a triangle, taking the circumference of it for the baſe; and the diſtance of the center from any ſide, for the perpendicular.

PROB. VI.

To find the area of a Circle.

RULE.

Square the diameter, and divide it by its proper diviſor (Prob. I.), or multiply by the correſpondent multiplier. 23.

Fig. 23.

Examp.

If the diameter AC *of a circle be* $54\frac{1}{2}$ *inches; what is the area in wine gallons?*

54.5	*or thus.*
54.5	2970.25
	.0034
2725	
2180	1188100
2725	891075
294.12)2970.25(10.10 w. gallons.	10.098850 w. gallons.

1. By the Rule.

Set 17.14 on D, to 1 on C; then againſt $54\frac{1}{2}$ the diameter on D, is the area 10.1 on C.

As the rule is thus ſet, it will be a table of circles areas, in wine gallons; for againſt any diameter on D, will be found its area on C. And the like for ale gallons, uſing the proper gauge point.

2. Or thus, by the Rule.

Set 294 on A, to the diameter $54\frac{1}{2}$ on B; and againſt $54\frac{1}{2}$ on A, is the area 10.1 on B.

Cor. *The ſector of a circle is meaſured as a triangle, taking the arch for the baſe, and the radius for the perpendicular.*

PROB. VII.

To find the area of an Ellipſis, or oval figure.

RULE.

Multiply the longeſt and ſhorteſt diameters together, and divide the product by the proper diviſor for circles (Prob. I.), or multiply by the proper multiplier.

Examp.

Fig. 23.

Examp.

If the greateſt diameter be 36.3, the leaſt 26.8; what is the area in ale gallons?

36.3	*or thus,*
26.8	972. | 84
	.002785
2904	
2178	1945
726	680
359.05)972.84(2.709	78
	4
	ale gall. 2.707

By the Rule.

Set 359 on A, to 36.3 on B; and againſt 26.8 on A, is 2.71 the content on B.

PROB. VIII.

To find the area of the ſegment of a Circle ADB.

RULE.

Let D be the vertex, and DE perp. to AB. Multiply the difference between the arch AD and ſine AE, by the radius; alſo multiply the ſine AE by the height DE; and divide the ſum of the products by the proper diviſor for ſquares (Prob. I.), or multiply by the proper multiplier. This ſerves either for the leſſer or greater ſegment. 24.

Note, if the diameter is not known, it may be found by Prob. XVIII. Part II.

Fig. 24.

Examp.

Suppoſe the radius CA 31.2, *height* DE 20, *arch* AD 37.55, *half the baſe* AE 29.12. *To find the area in ale gallons.*

37.55	29.12	*or thus.*
29.12	20	
8.43	582.40	845.4
31.2	263.01	.003546
1686	282)845.41(3.00 ale gall.	25362
843		4227
2529		338
		50
263.016		ale gall. 2.9977

PROB. IX.

To find the area of the ſegment of an Ellipſis.

RULE.

62. Multiply the baſe AB by $\frac{2}{3}$ the height DE, and reſerve the product. Then ſuppoſing C the center of the ellipſis, multiply the former product by the height DE, and then divide by ſeven times the ſemidiameter DC, and add the quotient to the firſt product. Then divide the ſum by the proper diviſor for ſquares, &c.

Examp.

Fig. 62.

Examp.

Suppoſe AB 30, DE 21, DC 40; *to find the area of* ADB *in wine gallons.*

21	420	420
2	21	31.5
3)42(14	420	231)451.5(1.95 wine gall.
30	840	
420	280)8820(31.5	
40		
7		
280		

PROB. X.

To find the area of a Parabola or Hyperbola.

RULE.

Multiply the baſe AB, by $\frac{2}{3}$ the height DE, in the parabola, or by $\frac{5}{8}$ the height in the hyperbola; and divide by the proper diviſor, for ſquares, &c,

Ex.

In a hyperbola let AB *be* 400, DE 280, *to find the area in malt buſhels.*

280	175
5	400
8)1400(175.	2150.4)70000(32.55 buſhels.

SCHOL.

If the area of any irregular curve is required, it muſt be found by the method laid down in Prob. XIII.

Fig. XIII. Part II. and then divided by the proper di-
62. viſor, or multiplied by the correſpondent multiplier, for ſquare figures.

PROB. XI.

To find the content of any Priſm CB.

RULE.

41. Find the area of the baſe AB, by ſome of the
42. foregoing Problems, multiply that by the perpendicular height, and you have the content.

Examp.

There is a priſm whoſe baſe contains 5.21 *ale gallons; and the height is* 20 *inches.*

$$\begin{array}{r} 5.21 \\ 20 \\ \hline 104.20 \text{ ale gallons.} \end{array}$$

By the Rule.

Set 1 on A to the baſe 5.21 on B; and againſt the height 20 on A, is 104 the content on B.

Or when the baſe is a ſquare.

Set the proper gauge point for ſquares on D, to the height on C; then againſt the ſide of the ſquare on D, is the content on C.

PROB. XII.

To find the content of a Cylinder.

RULE.

39. Find the area of the baſe by Prob. VI. and multiply it by the height.

Examp.

Fig. 39.

Example. 1.

The diameter of a cylinder is $54\frac{1}{2}$, *and height* 14 *inches; to find the content in wine gallons.*

The baſe contains 10.1 wine gallons, therefore,

$$\begin{array}{r} 10.1 \\ 14 \\ \hline 404 \\ 101 \\ \hline 141.4 \end{array} \text{ wine gallons.}$$

By the Rule.

Set 17.14 on D, to 14 the height on C; and againſt the diameter $54\frac{1}{2}$ on D, is the content on C 141 gallons.

Ex. 2.

Let the diameter of a cylinder be 40 *inches, and the depth* 30; *to find the content in ale gallons.*

By Prob. VI, the area of the baſe is 4.457 gal. Therefore,

$$\begin{array}{r} 4.457 \\ 30 \\ \hline 133.710 \end{array} \text{ ale gall.}$$

By the Rule.

Set 18.94 on D, to the height 30 on C; and againſt the diameter 40 on D, you have the content 134 on C.

Fig.

PROB. XIII.

To find the content of a Pyramid, ABD.

RULE.

44. Find the area of the baſe by the foregoing Problems, and multiply it by $\frac{1}{3}$ the height, gives the content.

Exam.

Suppoſe the area of the baſe 2025 *inches, or* .941 *buſhels, and height* 20; *to find the content in buſhels.*

3)20(6.66 .941

20

18.820 Buſhels.

By the Rule.

Set 1 upon A, to the height 20 upon B; and againſt the baſe .94 on A, is the content 18.8 buſh. on B.

Or when the baſe is a ſquare.

Set 46.37 on D, to the height 20 on C; and againſt the ſide of the baſe 45 on D, is the content 18.8 on C.

PROB. XIV.

33. *To find the content of a cone, ſphere, ſpheroid, pa-*
43. *raboloid, hyperboloid, or parabolic Spindle.*
53.
56.

RULE.

57. Find the area of the baſe or greateſt circle, by
58. Prob. VI. in gallons, &c. Then multiply this by ſuch a part of the altitude as here ſet down; for the cone $\frac{1}{3}$, for the ſphere and ſpheroid $\frac{2}{3}$, for the paraboloid $\frac{1}{2}$, hyperboloid $\frac{5}{12}$, parabolic ſpindle $\frac{8}{15}$ of the height.

Ex.

Fig. 38, &c.

Ex.

Suppoſe a parabolic conoid, the diameter of the baſe 30, the height 26; to find the content in ale gallons.

The area of the baſe is 2.506 gallons, then

	2.506
½ height	13
	7518
	2506
	32.578 ale gall.

By the Rule.

Set. 18.95 on D, to half the height 13 on C; and againſt the diameter 30 on D, is the content 32.5 on C.

PROB. XV.

To find the content of the fruſtum of a pyramid, cone, ſphere, ſpheroid, priſmoid, paraboloid, parabolic Spindle, or circular Spindle; or any ſegment thereof. 45. 46. 51, &c.

RULE.

Compute the contents thereof in inches, by the Problems in Part IV; and divide by 282, 231, or 2150.4, for ale gallons, wine gallons, or malt buſhels, reſpectively.

But in circular bodies, it is better to work by the diameters, and the fixt diviſors or multipliers for circles.

Fig. 45, &c.

Examp.

There is a fruſtum of a cone whoſe height is 28 inches, and the diameter of the leſſer baſe is 22 inches, of the greater 32; what is the content in ale gallons?

The area of the leſſer baſe is 379.7, and of the greater 804.25; then the mean area is 552.6; then

$$\begin{array}{r} 379.7 \\ 552.6 \\ 804.25 \\ \hline 1736.55 \\ \hline \end{array} \qquad \begin{array}{r} 1736.55 \\ 9\frac{1}{3} \\ \hline 1562895 \\ 57885 \\ \hline 282)16207.80(57.47 \text{ ale} \\ \text{gallons.} \\ \hline \end{array}$$

Or thus.

$$\frac{22^2 + 22 \times 32 + 32^2}{359} \times 9\tfrac{1}{3} = 57.46 \text{ the content.}$$

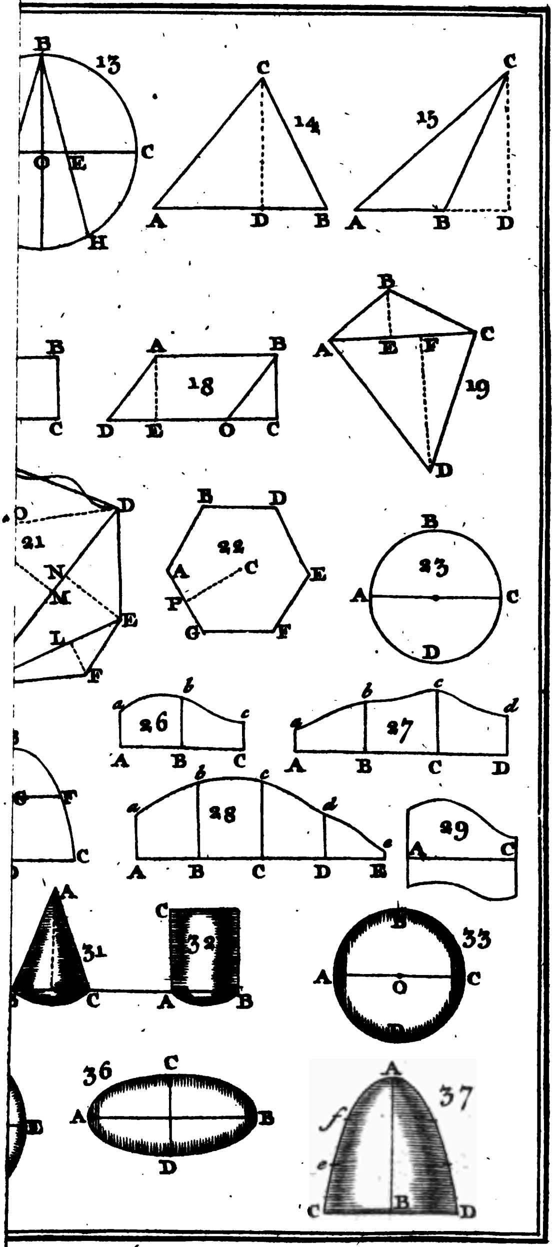
1. *By the Rule.*

Find a mean diameter between 22 and 32, which will be 26.5; then ſet 18.94 on D, to the height 28 on C; and againſt the mean diameter 26.5 on D, is the content 54.9 on C. But it will be nearer, if you take half the ſum of the diameters for the mean diameter.

2. *Or rather thus.*

Set 18.95 on D, to $9\frac{1}{3}$ ($\frac{1}{3}$ the height) on C.
and againſt 22 on D, is the content on C, 12.6
and againſt 26.5 on D, is the content on C, 18.3
and againſt 32 on D, is the content on C, 26.4

The ſum is the whole content 57.3 gall.



13
B
O
E
C
H
14
C
A
D
B
15
C
A
B
D
B
C
18
A
B
D
E
O
C
19
B
A
E
F
C
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21
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D
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29
A
C
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32
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B
33
B
A
O
C
D
B
36
C
A
B
D
37
A
f
e
C
B
D

Pl: II. pa: 148.

Fig. 45, &c. Cor. *If a ſquare fruſtum was propoſed, then the areas of the baſes would be two ſquares, and the content is eaſily had thus,* $\frac{22^2 + 22 \times 32 + 32^2}{282} \times 9\frac{1}{3}$ = content.

That is, to the ſquares of the baſes, add their product, and multiply the ſum by $\frac{1}{3}$ the height, which divide by 282 for the content.

And in working with the rule, 16.79 will be the gauge point for ale.

PROB. XVI.

To gauge any tun or ſolid body whatever, AD.

1. RULE.

63. Let AB be the diameter at top, CD the diameter at bottom, GHI the diameter in the middle of the depth EF. Then $\frac{AB^2 + CD^2 + 4GI^2}{6} \times$ depth EF = ſolid content; and that being divided by the proper diviſor (Prob. I.), will give the content in gallons.

If the veſſel or tun be high, divide it into two or more parts, and find a middle diameter for each part, and compute them ſeparately by this rule; and the ſum of all is the content.

2. *Or thus.*

64. Begin at the bottom of the tun CD, and divide the altitude FE in parts of 10 inches each; and in the middle of every ten inches, take the meaſures of the diameters OO, LL, II, HH; and from thence find all the areas in gallons, at theſe places; then the ſum of all theſe areas, multiplied by 10, will give the content.

Fig. 64. If any odd inches to ſpare at the top, then find likewiſe the area at top, and multiply it by theſe odd inches, to be added to the former.

If the ſections of the tun are not circular; then you muſt take croſs diameters; and take half the ſum, for the mean; or when the difference is great, multiply them together, and the product is the ſquare of a mean diameter, to be made uſe of.

Examp. I.

63. *Suppoſe the height of the tun* EF, *to be* 40 *inches; diameter* AB 24, CD 52, *and the middle diameter* GI 48; *to find the content in ale gallons.*

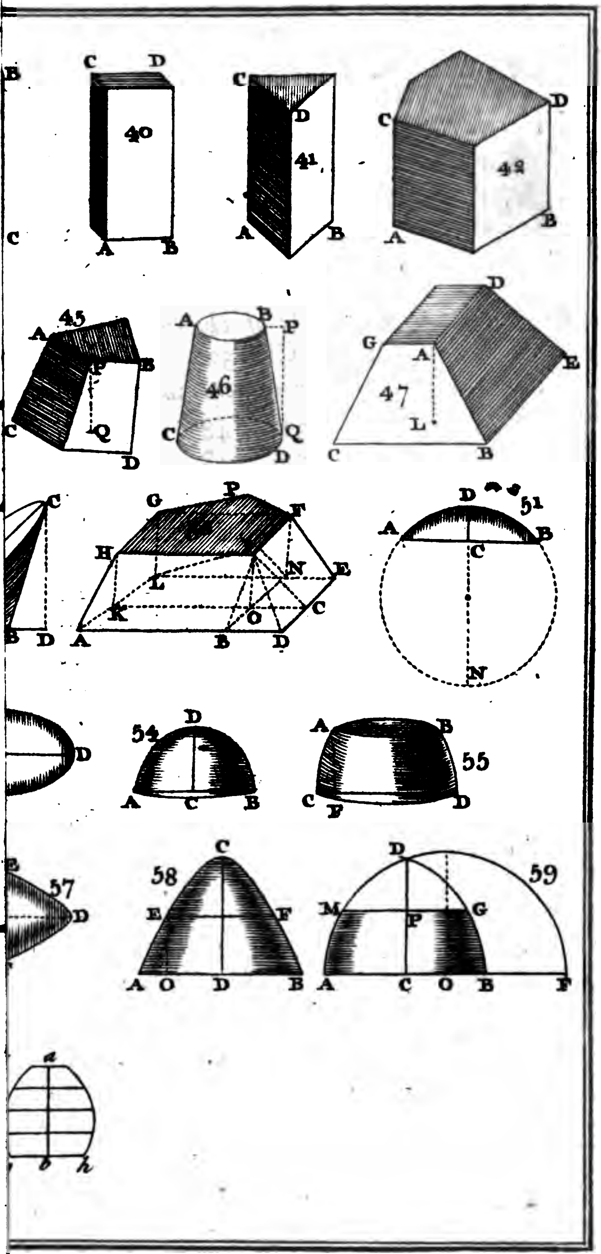
24	52	48	576
24	52	48	2704
			9216
96	104	384	
48	260	192	12496
			40
576	2704	2304	
		4	499840
		9216	

6)499840(83306.66

359.05)83306.66(232.02 gall.

A Tabl
1 foot th

Inches	
¼	.
½	.
¾	.
1	.
2	.
3	.
4	.
5	.
6	.
7	.
8	.
9	.
10	.
11	.
12	1.



Pl: III. pa: 150.

UNIV

.6 1666666666

.3 3333333

.25

.6666666666

.5

[illegible]

[illegible]

.25

[illegible]

.0

.06 [illegible]

[illegible]

Fig. 64.

Examp. 2.

Let EF *be* 40, CD 24, AB 52; *and the diameters taken at* 10 *inches diſtance,* OO 30, LL 46, II 50, HH 51; *to find the ale gallons in it.*

$$\begin{array}{rrrrr}
30 & 46 & 50 & 51 & 900 \\
30 & 46 & 50 & 51 & 2116 \\
\hline
 & & & & 2500 \\
900 & 276 & 2500 & 51 & 2601 \\
\cline{5-5}
 & 184 & & 255 & \\
\cline{2-2}\cline{4-4}
 & & & & 8117 \\
 & 2116 & & 2601 & \\
\cline{2-2}\cline{4-5}
\end{array}$$

$$\begin{array}{r}
359)8117(22.61 \\
10 \\
\hline
226.1 \text{ gallons.}
\end{array}$$

Cor. 1. *If it be a copper you are gauging, you muſt gauge the crown* CTD, *and ſubtract it from the former content.*

To do this, obſerve, when the copper is empty, the ſhape of the crown. If it be more curve at top than towards the bottom, meaſure it as a paraboloid; and its content will be had by multiplying the baſe CD by half the height FT.

If it have equal curvature at top and bottom, meaſure it as the ſegment of a ſphere, or take $\frac{9}{16}$ of the baſe, multiplied by the height, for the content.

The like operations may be performed in gauging a ſtill.

To find the diameter and height of the crown CD, FT; lay a ſtreight ruler AB over the top, and put down another NC into the hollow at C, and move it till it is perpendicular as at N. Then meaſure AN, and NC; in like manner put down

Fig. the rule at D, and find its diſtance from B, when 64. plumb. Theſe two diſtances from the ſides A and B, added together, and ſubtracted from AB, gives CD. Likewiſe put down a rule from E to T, and meaſure ET, which being ſubtracted from NC, gives TF.

When the copper is empty, if you meaſure as much liquor into it, as will juſt cover the crown, and note it down; then you need not afterward trouble yourſelf about the crown, but meaſure any part above OTO.

65. Cor. 2. *When any tun or veſſel ſtands not lever, the quantity of liquor* CDE, *which will juſt cover the bottom, muſt be computed ſeparately, and is called the drip or fall of the tun.*

Since the depth of the part, at D is but ſmall, it may be reckoned the hoof of a conic fruſtum; and ſo may be meaſured by Prob. VIII. Part IV. But an eaſier way will be to meaſure into the veſſel when empty, ſo much liquor as will riſe to the angle C, or the line CE; and note down the quantity. Alſo at the ſame time you will find the level CE, which you may mark. Alſo you will find the depth at D, which likewiſe ſet down, which will be of future ſervice. For at any time, by ſetting a rod upright at D, you may know how deep the liquor is above the level CE.

The quantity of the liquor CDE will be nearly equal to half the baſe CD × height DE, if DE be perp. to CD.

Cor. 3. *If* ABCD *be a cooler, you muſt find the true place of the dip, in order to get the depth of the liquor at any time.*

To do this, take ten dips in different places of the cooler, when there is but little liquor in it; divide the ſum of them by ten for the mean depth, If this is not exact enough, take ten more, for another mean depth; then half the ſum may be taken for the exact mean depth. Having the mean depth, find out a place on the ſide, and mark it, where the depth is juſt ſo much, and that will be the dipping place required. Fig. 65.

SCHOLIUM.

For taking the dimenſions of any veſſel; by help of the ſliding rule, take croſs diameters at the wideſt and ſtraiteſt places; and if they differ, take a mean between them; and that is the diameter in that place. This muſt be done at top, and bottom, and as many places as are neceſſary, if the tun be irregular. Set the rule to the mean diameter, and find where that diameter agrees with the tun at that depth. From theſe points, at the ends of the rule, draw chalk lines up and down. On theſe you may ſet any altitude, and draw croſs lines, through the points of diviſion.

In inching any tun, a chalk line is to be drawn through the place of the mean diameter, up and down; upon this every inch muſt be ſet, from the top or bottom. If the ſide of the tun is not perpendicular, allowance muſt be made in the inching.

When the top of a vat is covered with yeaſt; tye a cork (loaded with lead) to a ſtring, and let it down thro' the yeaſt, to ſwim upon the liquor. Then put down a rod to the bottom, cloſe to it; and taking them out together, you will ſee where the top of the cork touches the rod, for that place ſhews the depth.

PROB.

Fig. 66.

PROB. XVII.

To inch any Tun or Veſſel.

The *inching* of a tun or veſſel, is finding how much liquor it will hold at every inch of its depth. In order to this, take this,

1. RULE.

Divide the tun into ſeveral parts or fruſtums, each of ten inches deep; and find the content of every fruſtum, by taking diameters in the middle of them, as was ſhewn in the laſt Prob. And make a ſmall table, ſhewing the mean diameter, the area, and the content of each fruſtum; and the total ſum, or content of the tun.

Then by help of this, make another table, ſetting down the depth at top, againſt the content; and ſet down the depth of the liquor inch by inch, decreaſing from the whole depth to 0; againſt each of which the correſponding area is to be placed, which is found thus.

Take the area for the mean diameter of the top fruſtum, and write it on a ſcrip of paper, and ſubtract it from the whole content, and ſet down the remainder, and then ſubtract it from that remainder, and ſet down the next remainder, and ſo on; continuing to ſubtract as oft as there are inches in the top fruſtum; ſetting them down againſt their reſpective depths.

Then take the next mean area, and ſubtract continually as before, till the next fruſtum be done. And the like for all the fruſtums. And at laſt there will remain 0, if the work be right.

Fig. 66.

Examp.

Suppoſe ABDC *is a tun to be inched, whoſe height is* 23 *inches, the diameter* CD 24, OO 30, LL 46, AB 48; *to find what ale gallons it holds at every inch.*

Here the diameter OO is in the middle of the loweſt 10 inches, LL in the middle of the next 10 inches, &c. Then proceeding by the former directions, and the laſt Prob. we have,

Diameters.		Areas.	Content.
AB	48	6.417 × 3	19.251
LL	46	5.893 × 10	58.93
OO	30	2.506 × 10	25.06
CD	24		
		Cont.	103.24

Dep.	Content.
23	103.24
22	96.823
21	90.406
20	83.989
19	78.096
18	72.203
17	66.310
16	60.417
15	54.524
14	48.631
13	42.738
12	36.845
11	30.952
10	25.059

Dep.	Content.
10	25.059
9	22.553
8	20.047
7	17.541
6	15.035
5	12.529
4	10.023
3	7.517
2	5.011
1	2.505
0	0.000

Fig. 66. In this latter table, the mean area at top 6.417, is ſubtracted continually from 103.24 for 3 terms. And then the mean area 5.893 is ſubtracted continually from 83.989 for 10 terms; and laſtly, the mean area 2.506 is ſubtracted continually from 25.059, for ten terms, and at laſt 0 remains.

Then to find how much liquor there is in the caſk at any time; you come to the caſk or tun, and take the depth of the liquor, ſuppoſe 15 inches; look for it on the ſide of the table, and againſt it is 54.524 gal. and ſo much liquor there is in the tun.

If the tun be very crooked, you muſt take a diameter in the middle of every 6 inches. And if there be a crown or fall in it, remember to make allowance for it.

2. *Otherwiſe thus.*

Make a table with the depth in inches on the ſide, as before, and the diameters againſt them in the ſecond column; which are found thus.

Divide the difference of the diameters in any fruſtum, by the number of inches in the fruſtum's height, commonly ten; and ſubtract the quotient continually from the greateſt diameter, or add it to the leaſt, as oft as there are terms in the fruſtum; and you will get all the diameters in the fruſtum for every inch. Do the ſame with every fruſtum. Then putting them down, in the ſecond column; place their areas againſt them in the third column; which areas are moſt eaſily got from a table of areas. Then add up theſe areas one by one, beginning at the loweſt, and place them in a 4th column, which will then ſhew the liquor in the caſk at any depth.

Diame-

Fig. 66.

Diameters.		differences.	ad. num.
AB	48	2 ÷ 8	0.25
LL	46	16 ÷ 10	1.6
OO	30	6 ÷ 5	1.2
CD	24		

Dep.	Diameters.	Areas.	Content.
23	48.	6.417	103.249
22	47.75	6.350	96.832
21	47.5	6.284	90.482
20	47.25	6.218	84.198
19	47.	6.152	77.980
18	46.75	6.087	71.828
17	46.5	6.022	65.741
16	46.25	5.955	59.719
15	46.	5.893	53.764
14	44.4	5.490	47.871
13	42 8	5.102	42.381
12	41.2	4.727	37.279
11	39.6	4.368	32.552
10	38.	4.622	28.184
9	36.4	3.690	23.562
8	34.8	3.373	19.872
7	33.2	3.070	16.499
6	31.6	2.781	13.429
5	30.	2.506	10.648
4	28.8	2.310	8.142
3	27.6	2.122	5 832
2	26.4	1.941	3.710
1	25.2	1.769	1.769
0	24	0.	

Fig. 66. Here 1.2 is continually added to 24, till we come at 30; then 1.6 is continually added to 30, and ſo on till 46. And then .25 is continually added from 46 to 48. Then the next column contains the areas, correſpondent to theſe diameters. And the laſt contains the ſum of all.

This way is more exact than the former, but more troubleſome.

The diameters required, are had by meaſuring them; but if the form of the veſſel is known, they may be had more exactly by calculation.

Cor. *When the veſſel is very broad, as in a cooler; it is ſometimes neceſſary, to meaſure it to every tenth part of an inch.*

This will be done the ſame way as the inching was done, only inſtead of the number of inches in the height, we muſt take the number of tenths, which will be 10 times greater than before, or conſiſt of 10 times more terms. And the quantity to be added or ſubtracted, for finding each diameter or area, will be only the tenth part of what it is for inching: but all the reſt of the proceſs will be the ſame; for the area muſt be found at the middle of every inch in height; and then add (or ſubtract) continually the tenth part of every area, to the preceding.

Fig. 67.

PROB. XVIII.

To gauge a caſk ABHG.

1. RULE.

To $1\frac{1}{2}$ the ſum of the ſquares of the bung and head diameters CD, AB, add $\frac{5}{10}$, $\frac{3}{10}$ or $\frac{1}{10}$, or ſubtract $\frac{1}{10}$ their difference (according as the ſtaves are more or leſs crooked); multiply this ſum by $\frac{1}{3}$ the length EF, which divide by the proper diviſor, 359 for ale, or 294 for wine, gives the content in gallons.

2. *Or thus.*

To the head diameter AB, add $\frac{5}{10}$, $\frac{6}{10}$ or $\frac{7}{10}$, of the difference of the bung and head diameters, CD, AB (according as the ſtaves are more ſtreight or more crooked); the ſum is the mean diameter. Multiply the ſquare of the mean diameter by the length of the caſk EF, and divide the product by 359 or 294, gives the ale or wine gallons reſpectively, contained in it.

3. *Or thus.*

When the form of the caſk is known, which ſeldom happens; compute its content in inches, by its proper rule in the 4th part before going; and dividing by the proper diviſor (Prob. I.), you'll have the gallons it contains. Or proceed by Prob. 26, Part IV, by taking a diameter in the middle between AB and CD.

Ex.

Fig. 67.

Ex. 1.

There is a caſk whoſe bung diameter is 32 *inches, the head* 24, *and length* 40; *the ſtave pretty ſtreight, what is the content in ale gallons?*

$$\begin{array}{lrrrl}
 & 32 & 24 & 1024 & 3)40(13\frac{1}{3} \\
 & 32 & 24 & 576 & \\
\cline{2-4}
 & 64 & 96 & 2)1600 & \\
 & 96 & 48 & 800 & \\
\cline{2-4}
 & 1024 & 576 & 2400 & \\
\cline{2-3}
 & & & -44.8 & = \frac{1}{10}\text{ dif.} \\
\cline{4-4}
 & 1024 & & 2355.2 & \\
\text{diff.} & 576 & & 13\frac{1}{3} & = \frac{40}{3}. \\
\cline{2-2} \cline{4-4}
 & 448 & & 7065.6 & \\
 & & & 23552 & \\
 & & & 785.1 & \\
\cline{4-4}
 & & & 359)31402.7(87.4 & \text{ale gall.}
\end{array}$$

Examp.

Fig. 67.

Examp. 2.

There is a caſk, whoſe bung diameter is 40 *inches, the head* 30, *and length* 54; *the ſtaves crooked. How much wine does it hold?*

40 ſquare = 1600
30 ſquare = 900
ſum 2500
diff. 700
3
210.0

2500
1250
3750
210
3960
18 = $\frac{54}{3}$
31680
3960
294)71280(241.7 wine gall.

If the ſtaves were very crooked, you muſt have added $\frac{5}{10}$ of 700, or 350; and then the content would be 250.2 gall.

Or thus.

40
30
10
.6
6.0

30
6
36 = m. diam.
36
216
108
1296

1296
54
5184
6480
294)69984(238. wine gall.

Fig. 67.

By the Rule.

Set 17.14 on D, to the height 54 on C; and againſt the mean diameter 36 on D, is the content 238 on C.

Note, the mean diameter may be found on the rule, according to its variety.

This alſo may be done by the diagonal line on the gauge rod, for putting the rod in at the bung hole, to the oppoſite corner, at either end; note where the middle of the bung hole cuts the line, and there you have the number of gallons which the caſk contains.

Or thus, having the diagonal in inches; ſet 22.93 upon D, to 27 upon E, for ale; or ſet 15.9 upon D, to 11 upon E, for wine; and againſt the diagonal upon D, you have the content upon E.

SCHOLIUM.

It has been the common practice of Gaugers, to reduce all caſks to the conic ſections, ſuch as the ſpheroid, the paraboloid, the hyperboloid, the parabolic ſpindle, &c. and upon that ſuppoſition, to make ſo many caſes or varieties, for finding the contents of all ſorts of caſks, by reducing them to one or other of theſe. But it is impoſſible, by the mere ſight of a cask, to know what form to refer it to, and it muſt be mere chance if it happen to be any of them; therefore it is better to have regard only to the curvature of the ſtaves, ſince it is owing entirely to this, that a cask holds more or leſs; and therefore from this only a judgment may be formed how to proceed by theſe rules, in finding a mean area, or a mean diameter, for reducing a cask to a cylinder; or without truſting to the eye, to take a middle diameter (according to Prob. 26. Part 4.) which will determine the thing at once.

In order to get the dimenſions of any cloſe caſk; put a rod in at the bung hole, ſtreight to the oppoſite ſide, and note the length of the rod on the outſide of the caſk. From this, take the thickneſs of a ſtaff (which is eaſily taken with a notch'd ſtick) and the remainder is the bung diameter. Fig. 67.

The head diameter is had, by taking twice the thickneſs of a ſtaff (found at the bung hole) from the external diameter at the head. Or the diſtance between the inſide of the chine, and the outſide of the oppoſite ſtaff, will be nearly equal to the head diameter.

Take the diſtance between the outſides of the two heads, with a ſliding pair of calipers; from that ſubtract twice the thickneſs of a head, the remainder is the length of the caſk within.

The outſide diameters are eaſily found, by meaſuring round the caſk to get the circumferences; then from any circumference find the diameter.

To find the wet inches in a caſk, put the gauging rod into the bung hole, in a lying caſk; or into the end, in a ſtanding one; and thruſt it directly down, till it touch the oppoſite ſide or end; then taking it out, the wet part will ſhew how deep the liquor is, or the wet inches.

PROB. XIX.

To find the ullage of a caſk, whoſe axis lies parallel to the horizon.

The *ullage* is the quantity of liquor remaining in the caſk; or the vacuity, when the reſt is drawn out. 68.

RULE.

Here muſt be given the bung diameter, the wet or dry inches, and the content of the caſk.

Fig. 68. Divide the wet inches by the bung diameter, and note the quotient: take a quarter of the difference between this quotient and $\frac{5}{10}$, and add it to that quotient, if greater than .5, or ſubtract it if leſſer. By this, multiply the whole content of the caſk, and it gives the liquor remaining in it.

But when the bung diameter is far greater than the head, inſtead of $\frac{1}{4}$, take $\frac{1}{3}$ the difference aforeſaid.

Examp.

Suppoſe the bung diameter CD 32, *the wet part* LD 24. *Then by Prob.* 18, *the content of the caſk is* 97.46. *To find the liquor contained in* IKHB, *or the ullage.*

32)24.00(.75	.75		97.46
224	.5		.812
160	4).25(.062		77968
160	.75		975
			195
	.812		
		ullage	79.138

By the Rule.

Set the bung diameter 32 upon C, to 100 upon SL; and againſt the wet inches 24 on C, is a ſegment 82.25 on SL.

Then ſet 100 on A, to the caſk's content 97.46 on B; and againſt the ſegment on A, is the ullage 80 on B.

PROB. XX.

To find the ullage of a caſk, whoſe axis is perpendicular to the horizon.

1. RULE.

69. Here we muſt have the length of the caſk, the wet or dry inches, and the content of the caſk.

Divide the wet inches by the cask's length, and note the quotient; take $\frac{1}{10}$ the difference between this quotient and $\frac{5}{10}$, and add it to that quotient, if greater than .5, or ſubtract it if leſſer; by this, multiply the whole content of the cask, and you have the liquor remaining in the cask. Fig. 69.

2. *Or thus.*

Here you muſt have the diameters at the bung, head, and liquor's ſurface; alſo the wet inches, and whole length.

Then by Prob. 18. find the content of IKGH, by finding a mean diameter; and this is the ullage.

Or find the content of the half cask CDHG, and of the part CDKI; then the ſum or difference gives the quantity of liquor in the cask.

Or laſtly, find a middle diameter between IK and GH, and then find the content by Rule 1. Prob. 16.

Examp.

Suppoſe the bung diameter CD 32, *the head* GH 26, *the length* EF 40, *the wet inches* LF 12, *ſurface of the liquor* IK 31.5; *to find the quantity of liquor in* IGHK.

By Prob. 18. the content of the cask is 100.6 ale gallons.

40)12.0(.3 .5 .30 100.6

.3 .02 28

10).2(.02 .28 8048

2012

ullage 28.168

Or thus.

Fig. 69.

IK = 31.5
GH = 26.

diff. 5.5
.5

2.75
26.

m. diam. 28.750
28.75

57500
23000
2012
144

826.56

826.56
12

165312
82656
359)9918.72(27.6 ale gallons.

By the Rule.

Set the caſk's length 40 on C, to 100 on SS.; and againſt the wet inches 12 on C, is a ſegment 26.4 on SS.

Set 100 on A, to the caſk's content 100.6 on B; and againſt that ſegment 26.4 on A, is the ullage 28 on B.

PROB. XXI.

To gauge malt in any maſh tun, ciſtern or couch; or on any floor.

RULE.

In a *Tun*, *Ciſtern*, or *Couch*, if the malt does not lie level; take a ſufficient number of depths in different places; and divide the ſum of them by their number, and it will give the mean depth.

Then find the area at top or ſurface of the malt, and at the bottom; and take half the ſum, for the mean area; and multiply the mean area by the Fig. mean depth, and divide by 2150.4, gives the content in buſhels. See Prob. 16. 69.

Upon a *floor*, find the area of the baſe, and the mean depth (by ſeveral dippings); and multiply them together, and divide by 2150.4, as before. This work is the ſame as directed in Prob. 10. Part IV.

When the dimenſions are large, they are commonly taken with a tape divided into inches.

It has been obſerved, that when 2 or 3 worts have paſſed thro' malt of a middling ſort, it will be more compact by a ſixth or an eighth part. And therefore the quantity found in ſuch circumſtances, ſhould be increaſed ſo much.

Examp.

Suppoſe the length of a ciſtern 85, and breadth 54.4, and depth 18; what does it contain?

54.4
85
―――
2720
4352
―――
4624.0
18
―――
36992
4624
―――
2150.4(83232(38.7 content.

1. *By the Rule.*

Find a mean between 54.4 and 85, which will be 68; then ſet 46.37 on D, to the depth 18 on C; and againſt the mean 68 on D, is the content 38.7 on C.

2. *Or*

Fig. 69.

2. *Or thus, by the Rule.*

Set the depth 18 on MD, to the length 85 on B; and againſt the breadth 54.4 on A, is the content 38.7 on B.

SCHOL.

It may be obſerved, that many of the rules laid down here for gauging, are only approximations; being contrived for eaſe and expedition; and come pretty near the truth in moſt caſes. For to what purpoſe can it be to make uſe of long and laborious rules and calculations, ſuited to certain particular figures; when we do not know that any veſſels we deal with have theſe figures. If we ſhould gauge a cask as a paraboloid, which happens to be a parabolic ſpindle; we ſhould make a great miſtake. Therefore it is better in mechanical things to uſe mechanical rules; and judge from the apparent curvature of the veſſel, what rule to uſe, and how to apply it.

FINIS.

UNIV. OF

THE
ART
OF
SURVEYING,
OR
MEASURING LAND.

CONTAINING,

I. The Work to be Perform'd in the Field, as taking all neceſſary Dimenſions and Meaſures.

II. Concerning Protracting, Reducing, Caſting up, and Dividing.

III. Meaſuring inacceſſible Heights, and Diſtances.

Diſce ſed a doctis; indoctos ipſe doceto. Cato.

LONDON:
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MDCCLXX.

THE

PREFACE.

THE Art of Surveying is of great antiquity; and it has been ſaid, that it was firſt found out in Egypt. *For when the* Nile, *by its frequent inundations, overflowed the country, and deſtroyed all the boundaries of mens eſtates, or buried them under the ſlime and mud; they were neceſſitated to ſeek out ſome methods or other, whereby to ſet out to every man his due quantity of ground; that it might be reſtored to him without loſs. And as this happened very often, they were under a neceſſity of ſtudying this art, for preſerving the limits and extent of their lands and poſſeſſions.*

But though the knowledge of ſuch an art muſt be neceſſary on this occaſion, and accordingly made uſe of for that purpoſe; yet it might be found out ſooner. For without doubt, the Art of Surveying had been known, ever ſince there had been any ſuch thing as the dividing of lands. For as there could be no ſetting out, or making any diviſion of lands, without taking the meaſures of the whole, and of the ſeveral parts; ſo as ſoon as that happened, muſt the way of meaſuring be known; and thus we meet with a man meaſuring Jeruſalem, Zechariah, ch. 2. *And the ſame neceſſity that cauſed men to divide the land into ſhares, at the ſame time forced them to ſeek for methods to reduce them to an equality, and to find the proportion between one part and another. And thus the firſt dividing*

ding of the land of Egypt *among the people, gave riſe firſt to Surveying and Geometry.*

It has been objected, that the ſurveying of land makes the poor tanants to have dear bargains, and occaſions things to ſell dear and grow ſcarce; and therefore land ought not to be meaſured. But then a man muſt be kept ignorant of his property, in other caſes as well as this. Does a man's knowledge abridge his conſcience? We often ſee that men, by over-eſtimating their eſtates, let their tenants hard pennyworths, being ignorant of the true quantity of ground they let, for the want of ſurveying. But every man, in any ſort of buſineſs, will always make the beſt he can of his goods. And muſt every man have the liberty to make the beſt of his own, and only the owners of land tied from it? Now the Art of Surveying teaches how to give every man his due, and to do juſtice to every body, to the landlord as well as the tenant, to the buyer as well as the ſeller; by ſhewing the true meaſure and quantity of the land. Therefore Surveying is a neceſſary and uſeful Art, which not only gives the true quantity, but ſhews rules for laying down maps and drafts of an eſtate; which exhibits to view every field, and every object therein, in their proper ſituations; which is a uſeful invention.

Many large volumes have been writ concerning the Art of Surveying, by which it has been drawn out to an unneceſſary length. For the whole of it is founded upon the ſimpleſt principles of Geometry, which are eaſily learned, and cannot need ſuch long and tedious explications; for ſuch methods only confound the reader. For no ſubject ought to be treated more voluminouſly, than there is real difficulty in it; which is not to be found in this Art. It is a good old maxim, μένα Βίϐλιὸν μένα κακὸν, *and ought always to be obſerved. For by ſpinning out things in long diſcourſes, the reader is loſt in a miſt of words. But this perplexity is avoid-*

ed, by treating the ſubject in a conciſe way, and going no farther than the ſubject leads.

In the following Book, you have a verbal deſcription of the inſtruments uſed in Surveying. For it is not worth the while to give any Cuts of them; a ſight of them being more inſtructive than any figure can be. In Sect. I. *you have an account of all the work that is to be done out of doors; with proper Rules to proceed by.* Sect. II. *teaches how to plot, reduce, caſt up, and divide any parcel of land. And laſtly,* Sect. III. *ſhews the methods of taking heights and diſtances, both acceſſible and inacceſſible; which is a curious and uſeful ſubject, and very neceſſary to be known.*

I have omitted nothing belonging to this Art, and have treated of every thing in as conciſe a manner, and yet as clearly, as the ſubject would admit of. And ſo I take my leave of the reader, wiſhing he may meet with all the ſatisfaction he expects.

W. Emerſon.

SURVEYING.

SURVEYING, or *meaſuring Land*, is the art of finding the content of any field, or parcel of land; which is always expreſſed in acres and decimal parts; or perhaps in acres, roods and perches. This ſort of meaſuring differs from others, as it is concerned in nothing but meaſuring parcels of the earth's ſurface, which are large figures; and whoſe meaſures are taken in chains and decimal parts; whereas the dimenſions in other ways of meaſuring are taken, and caſt up, in inches, feet, yards, &c.

The inſtruments uſed in ſurveying are of ſeveral ſorts. 1. Thoſe for meaſuring angles; as the *Theodolite*, *plain table*, *ſemicircle*, *circumferentor*, *Geodetical ſtaff*, *infallible*, &c. 2. For meaſuring diſtances; as the *wheel*, or *perambulator*, the *chain*, &c. 3. For levelling; as the *water level*, *ſpirit level*, &c. 4. For making draughts upon paper; as *ſcales*, *line of chords*, *protractor*, *reducing ſcale*, *compaſſes*, *parallel ruler*, &c. 5. For marks, &c. as *arrows*, *ſtation ſtaves*, *ſtreamers*, &c.

The Theodolite.

This is a braſs circle, 12 or 14 inches diameter; its circumference is divided into 360 degrees; it has two ſights upon it, fixt at the beginning of the degrees; and alſo a moveable index, with two ſights, which moves about the center. The whole is to be fixt upon a three legged ſtaff, with a ball and ſocket. To ſome of theſe inſtruments are fitted teleſcopic ſights; which makes it one of the beſt inſtruments for taking angles. And ſome of them have a card and compaſs, with a needle playing round the center, which in ſome caſes is very uſeful.

The plain Table.

This is a flat board or table, the bigneſs of a ſheet of paper; fitted into a frame going round its edges, which frame is to keep the ſheet of paper faſt on the table, for uſing in the field. The edge is graduated into degrees, from a braſs center in the middle of the table; by means of which it may be uſed without a paper. It has a looſe index with two ſights, being ſighted both back and forward; and upon this index is a tangent line for taking heights. The index is made to turn upon any given point, againſt a pin, or the point of a pair of compaſſes placed therein, as a center; and in taking angles, this center muſt be the braſs center. The ſtreight edge of the index is called the *fiducial edge.*

The uſe of this inſtrument is to draw the plot or figure of any field or parcel of land upon a paper, in the field. For when the work in the field is finiſhed, you have the exact form of the ground upon the paper on the plain table. This is a proper inſtrument for ſurveying ſmall incloſures, and of great diſpatch; but not ſo good for large eſtates, by reaſon the paper upon the table will contract in dry weather, and expand in moiſt weather. And therefore if one part be drawn when it is dry, and another when it is moiſt, the field will be diſtorted, and the parts out of proportion.

The under ſide of the frame is divided into equal parts, and may be turned upſide down upon occaſion.

Circumferentor.

This is a parallelogram, eight inches long and four broad, made of box or braſs, its edge graduated or divided into degrees; to this belongs an index with ſights forward and backward. It may alſo be made in form of a circle eight inches diameter.

To this belongs a box and needle; and therefore it is the fitteſt inſtrument for mapping a harbour, and for drawing the poſition of rocks, ſands, ſhoals, ſoundings, &c. And proper for meaſuring a county, if it be made large.

Semicircle.

This is a large ſemicircle divided into degrees, having ſights placed at the beginning or edge. It has a moveable index with ſights back and forward, moving about the center. To this is fixed a box and needle; it is the ſame as half a Theodolite. It is fitted to a ball and ſocket, and ſet upon a three leg'd ſtaff, like all the reſt.

Geodetical Staff.

This is an inſtrument with two legs, having ſights upon them, opening upon a braſs center pin, like the legs of a pair of compaſſes; each leg is a foot and half long, or more. There is a long ruler, called the *Graduator*, which turns upon a pin in the end of one of the legs, and goes thro' the end of the other leg. This ruler is graduated with a line of chords, anſwerable to the length of the legs, which is the radius. As the inſtrument is opened, the moveable leg, running along the graduator, ſhews to what angle the inſtrument is opened. And when it is open, it appears in form of an iſoſceles triangle; and may be opened to a right angle, or ſomething more; and then the graduator will be the hypothenuſe. This is an excellent inſtrument for taking angles.

Infallible.

This is an inſtrument not very common; it is a flat board, or a plate of copper or braſs, a foot ſquare. On the middle or thereabouts is fixed a perpendicular pin, on which is ſcrew'd an index with two ſights. The table or plate is to be covered with a paper, which muſt be folded at the ends under the inſtrument, and ſcrew'd faſt at every corner, by four plates fixt underneath the table. Then the index muſt be put upon the pin, which will come thro' the paper.

This inſtrument is very ſimple, and very eaſily made, and differs nothing from the plain table, but only that it is ungraduated; and the index always turns upon the fixt pin. When it is uſed in the field, it muſt be placed on a three legged ſtaff, being a ſtaff with three legs opening outward, to ſtand on the ground in any poſition you will. All theſe inſtruments are for taking angles or draughts.

Wheel, or Perambulator.

The wheel to meaſure diſtances, is 2¾ yards in circumference, but ought rather to be 5½ yards about, or a ſtatute pole. It conſiſts of clockwork, and on the face has two hands or pointers, to ſhew the diſtance run. The wheel runs between two cheeks, in a frame of wood; by taking hold of the frame it is driven forward, and by the motions of the hands, and diviſions on the face, is ſhewn how many yards, poles, furlongs or miles, you have gone. One hand goes round in half a mile, and the other in 12 miles.

This inſtrument is very uſeful for meaſuring roads, rivers, and all level ground.

Chain.

Chain.

There are ſeveral ſorts of chains, but *Gunter's Chain* is the beſt for meaſuring land; its length is 4 poles or 22 yards; and is divided into 100 links; and at every 10 links there is a piece of braſs, with figures or tails for diſtinction; by theſe, the links are eaſily reckoned.

In meaſuring towns, a chain of 50 feet and 50 links is the moſt commodious.

Water Level.

This is a round tube made of wood or braſs, about three feet long; the ends of it are turned up ſquare; in which are cloſe cemented two glaſs tubes perpendicularly. The tube is to be almoſt quite filled with water. There are two ſights fixt to this machine; and often teleſcopic ſights, for great diſtances. When it is fixt level on a three legged ſtaff, the water will riſe to equal heights in both the glaſs tubes, where proper marks or diviſions are made to ſhew it.

The uſe of theſe is to find a true level, for conveying water, draining fens, &c.

Spirit Level.

A ſpirit level, or air level, is a cylindrical glaſs tube, very near filled with ſpirit of wine, wanting only a drop, and then ſealed. On this, ſights are fixt, which ſhould be teleſcopic ſights. When the ſights are placed level or horizontal, the bubble or drop of air will be in the middle; but when it is not level, the bubble will go to the higher end.

In the beſt ſort, this cylindric glaſs tube, with the bubble, is fixt upon the tube of an aſtronomical teleſcope, two or three feet long; and ſo adjuſted,

juſted, that when the bubble is in the middle, the object or mark, ſeen through the teleſcope, is juſt level with it. And there is a proper apparatus with ſcrews to ſet the whole level; which is known by the bubble being in the middle.

Scales.

The ſcales or rulers ought to have ſeveral lines of equal parts upon them; ſome greater, ſome leſſer, to ſuit all caſes, particularly diagonal ſcales, which ſhew the diviſion of 1 into a hundred parts. Thus, we muſt have a ſcale of a chain in an inch, a chain in $\frac{3}{4}$ of an inch, in $\frac{1}{2}$ an inch, &c. By theſe we draw a field, or a *map* or *plan*, called alſo a *plot* or *plat*.

There are alſo ſcales made that one may prick off the diſtances by the edge.

Line of Chords, Protractor, reducing Scale, Compaſſes, parallel Ruler.

Theſe are ſo well known, that they need no deſcription. The chords, and protractor, are for laying down angles; the reducing ſcale, for increaſing or diminiſhing a plan or map; the compaſſes, for ſetting off diſtances; and the parallel ruler, for drawing parallel lines. As to the reducing ſcale, it turns on a center pin, nearer one end than the other. On this are ſeveral lines of equal parts, running both ways from the center; and in certain given proportions.

There is alſo an inſtrument made with two drawing pens, ſo that tracing one of them over the old draught, the other will, on a clean paper, deſcribe it anew, in a leſs or bigger form; being a very expeditious inſtrument.

Arrows

Arrows.

Theſe are ten ſmall ſticks, about a foot or half a yard long, and ſharp at one end, to ſtick in the ground at the end of every chain, in meaſuring. They may be tipt or piled with iron at bottom; and coloured blue and red near the top, to make them viſible.

Station Staves, and Streamers.

Theſe are long poles to be ſet up in the angles of a field, or in other remarkable places. If they be far off, they have moveable vanes or ſtreamers placed on them, to be ſlid up and down; or tied to their tops. And theſe vanes or marks may be white, or coloured red or blue, to make them viſible at great diſtances.

Some of theſe poles are divided into links, for meaſuring offsets readily.

The Field Book.

As it is not poſſible to ſurvey any great quantity of ground without noting down the particulars; therefore you muſt have in the field a little book, called the *Field Book*, in which to write your ſtations, offsets, angles, lines, obſervations, &c. This book you may divide into columns, as you think convenient, as will beſt ſuit your occaſions: for there are hardly two perſons that have the ſame method for their field notes; however, it ought to be contrived in the eaſieſt and moſt intelligible manner; for any method, that lays a burthen upon the memory, is imperfect, and not fit to be practiſed.

This book may be contrived after the following manner. Make three columns, the middle one for the angles (mark'd <), ſtations (mark'd ⊙), the diſtances meaſured, and bearings, &c. The two columns on the left and right, are for the offsets on the left and right, againſt their reſpective diſtances in the ſecond column; and for ſuch remarks as occur. Draw a line under the work at the end of every ſtation; that the workings at the ſeveral ſtations may not confound one another.

Note, in ſurveying with the plain table, you need no field book; the whole work being deſcribed upon it, except offsets, which may be ſet thereon, as they happen.

The

The FIELD BOOK.

Offsets and remarks on the left,	Stations.	Offsets and remarks on the right.
	⊙ 1	
.12	< 113° 15	
	00	.26 Thomſon's hedge begins.
	56	touch the hedge
	4.13	.34
.32 a croſs hedge	6.09	.41 a tree
	8.00	.00
	⊙ 2	
	< 87 43	
	.00	.00
	1.80	Gutter changes
,87 houſe corner	3.15	
	4.82	.27 a pond
	⊙ 3	
	64 15	
a gate	0 . 84	.5
	2 . 75	.15
	4 . 11	.29 a gate
.88 a brook	5 . 28	
18 foot path	5 . 92	.15
	6 . 00	00
15 hedge corner	7 . 54	
	9 . 20	28 a ſpring.

Explanation.

⊙ 1 is the firſt ſtation, and the < there is 113 15. on the right an offset .26 or 26 links, where Thomſon's hedge begins; meaſuring to .56 we touch the hedge. At 4.13 an offset of 34. At 6.09 an offset .41 at a tree. At 8.00 no offset. On the left, at firſt an off-ſet .12; at 6.09 an offset .32 where there is a croſs hedge. And ſo for the reſt of the ſtations.

1. *A Table of Long Meaſures.*

7.92 inch	1 link						
12	$1\frac{17}{33}$	1 foot					
36	$4\frac{6}{11}$	3	1 yard				
198	25	$16\frac{1}{2}$	$5\frac{1}{2}$	1 pole			
792	100	66	22	4	1 chain		
7920	1000	660	220	40	10	1 furl.	
63360	8000	5280	1760	320	80	8	1 mile

2. *A Table of Square Meaſures.*

9 feet	1 yard					
$272\frac{1}{4}$	$30\frac{1}{4}$	1 perch				
4356	484	16	1 chain			
10890	1210	40	$2\frac{1}{2}$	1 rood		
43560	4840	160	10	4	1 acre	
27878400	3097600	102400	6400	2560	640	1 mile

In theſe Tables, each perpendicular column is of one denomination throughout. And all the lateral ones are equal, but of different denominations.

Note, a pole and perch is the ſame thing, and a ſquare chain is called a rape in the North, being the tenth part of an acre.

Fig.

SECT. I.

The Practice of SURVEYING, *containing the Work to be done in the Field.*

PROB. I.

To take an angle BAC, *or meaſure the quantity of it in degrees.* 1.

SET up two ſtation ſtaves perpendicular, in ſome parts of the lines AB, AC, as at B and C; and put thereon ſome viſible marks, large enough to be ſeen from A. Then plant the inſtrument in the angular point A, or as near it as poſſible. Then,

1. For an inſtrument with a paper upon it, as the plane table, and infallible. Screw the table faſt; then direct the index to one mark as B, and turning it about, till thro' the ſights you ſee B, then draw the line AB by the edge of the index. Then direct the index to the other mark C, and turning it about till you ſee it thro' the ſights, then draw the line AC by the edge of the index. Then you have the angle BAC upon the paper. Here the edge of the index muſt be kept all the while upon the point A.

2. For inſtruments with two fixt ſights, and a moveable index, as the theodelite, ſemicircle, geodetical ſtaff, &c. direct the fixt ſights along one of the lines AB, turning the inſtrument about, till thro' the ſights you ſee the mark B, there ſcrew faſt the inſtrument. Then direct the moveable index along the other line AC, and move it about,

Fig. 1. till thro' the ſights you ſee the mark C. Then the degrees cut by the index, upon the graduated limb of the inſtrument, ſhews the quantity of the angle; which ſet down in the field book. If the angle be obtuſe, you muſt take it by back ſights; cauſing *b*, *c* to lye on the other ſide of A.

3. By an inſtrument with a needle. Place the inſtrument ſo, that the north end of the needle may hang directly over the flower-de-luce. Then direct the ſights to one mark as B, and note the degrees cut by the needle. Then direct the ſights to the other mark C, and note again what degrees are cut by the needle. For their ſum or difference (as the caſe is) will be the quantity of the angle BAC; which ſet down.

If either mark cannot be ſeen from A, as in the caſe of hills, trees, &c. ſet up a new mark in ſome place between, from whence both A and the other may be ſeen. And to place it in the line, ſhift it to the right and left, till both A and the other mark appear through the ſights, both forward and backward; and that is the place of the new mark, which may be uſed inſtead of the other.

Note, if ſeveral angles are to be taken at one point as A; take them all from one and the ſame line as AB. For by this means, there will be leſs danger of committing any error.

SCHOL.

There is alſo a way to take an angle in the field by the chain only; which is this. Take a chain length along both ſides AB and AC, as A*b* and A*c*; then meaſure the diſtance *bc* between *b* and *c*, and write it down. Then the angle A may be found by plain trigonometry; for it will be as A*b* : radius : : ſo $\frac{1}{2}$ *bc* : S. $\frac{1}{2}$ the angle A. Or find $\frac{1}{2}$*bc* in the table of natural ſines, and it ſhews half the angle BAC.

But this method being ſubject to errors, ought not to be practiſed, but in caſe of neceſſity, or for want of proper inſtruments. Fig.

PROB. II.

To meaſure a right line, or any diſtance, upon the ground.

1. Let AB be the diſtance to be meaſured. Set up ſtaves with marks upon them, at A, B and C. Then the leader of the chain muſt take the ten arrows in his left hand, and take hold of the end of the chain with his right hand, by putting his hand into the ring. Then let him go from A towards B, drawing the chain after him; and as he goes, let him take an arrow into his right hand ready to ſtrike down. The follower muſt ſtand at A, having hold of the other end of the chain in his right hand; and cauſe the leader to go always along the right line AB, till he comes at the ſtretch of the chain; then the follower muſt hold his end cloſe to A, and the leader, ſtretching the chain ſtreight, muſt prick the arrow upright into the ground, at the very end of the chain. Then he muſt go forward, and take another arrow into his right hand; and when the follower comes to the arrow ſticking in the ground, he muſt ſet the end of the chain cloſe to it, and take it up with his right hand; whilſt the leader, at the ſame time, pulls the chain ſtreight, and pricks in another arrow; and then proceed as before; and whilſt the follower is going, he muſt put the arrow into his left hand. Thus at the end of every chain, the leader puts down an arrow, and the follower takes one up. And thus they go on, till they come at B, the end of the line; and then the follower muſt reckon his arrows, for there will be ſo many chains; 2.

and

Fig. 2. and if any thing to ſpare, ſee how many odd links reaches from the laſt arrow to B; and ſet all down, the chains in whole numbers, and the links in decimals.

If all the arrows are gone before the leader gets to the end at B, he muſt come back for the ten arrows; and the follower muſt ſet his toe at the place where the laſt ſtick or arrow was; till the leader, pulling the chain ſtreight, puts in another ſtick; and then proceed. In great diſtances, you will have the arrows to change ſeveral times; then you muſt note the number of changes, which will be eaſieſt done thus. Put 20 ſmall ſtones in your pocket, and at every change of the arrows (that is, at every 10 chains) remove a ſtone into another pocket. And at laſt, counting the ſtones removed, you will have the number of changes; where 8 changes make a mile. And then the whole length muſt be ſet down in the field book.

In going from A to B, the leader muſt always keep his eye on the mark at B, and keep in the line AB, ſo as to cover the mark at B from the follower.

At the end of the line AB, and at the end of every change, count all the arrows in both parcels, to ſee if they both make ten; if they do not, one has been dropt, and the diſtance muſt be meaſured over again.

2. In meaſuring to or from any hedge, you muſt account 5 links, from the roots of the quicks, on that ſide where the gutter is; and that will be the boundary; but in walls, the outſide is the boundary.

3. When the mark at B cannot be ſeen from A, then ſet up a mark in ſome place between, from which both A and B may be ſeen; and this place may be found, by the inſtrument, placing it there, and removing it, till A and B both appear through the

Fig. 2.

the ſights backward and forward. And if one will not do, ſet up two or three marks after the ſame manner.

4. If the diſtance be very great, as amounting to ſeveral miles, then the eaſieſt way is to meaſure it with the wheel; in which there are ſeveral indexes, ſhewing how many miles, furlongs, &c. have been paſſed over. But this is not ſo exact as the chain.

5. Great diſtances alſo may be meaſured expeditiouſly, by the firing of great guns in a dark night; and obſerving with a watch or a pundulum vibrating ſeconds, how many ſeconds there are between the flaſh, and the report of the gun. For that number of ſeconds multiplied into 1142, gives the number of feet in the diſtance. But this is not ſo exact as the former methods.

6. If the line you meaſure is not horizontal, but riſes or falls; and you want the horizontal length. Then with a quadrant, you muſt take the elevation or depreſſion of that line; and then the horizontal length may be found by trigonometry.

Or in deſcents, or going down a ſteep place, you may find the horizontal level thus. At the top, ſtretch out as much of the chain as you can horizontally, and let a ſtone fall from that point of the chain, and where it falls, ſet that point of the chain; and ſtretch out another part of it horizontally, from which let fall another ſtone, which will ſhew another place to hold at again horizontally; and proceed thus to the bottom. Then the ſum of all theſe parts of the chain is the horizontal line.

PROB. III.

To meaſure offſets, and find their ſituation upon any right line.

3. Let AB be any right line, and A*oegb* a crooked hedge; *co*, *de*, *fg*, B*b*, &c. ſeveral offſets, to be meaſured. From every angle or ſenſible bend of the hedge, as *o*, *e*, *g*, &c. let perpendiculars be ſuppoſed to fall on the line AB, as at *c*, *d*, *f*, &c. which points may be taken near enough, by the eye, if the offſets are not very large; but if they are, it muſt be done by the inſtrument. To this end, you muſt have a croſs line upon your index, perpendicular to the viſual ray. Then as you meaſure along the line AB, when you come at *c*, ſet down the length A*c*, and meaſure the offſet *co*; and to know when you are at the point *c*, place your inſtrument about *c*, and look thro' the ſights to A and B, and if you can ſee both A and B, looking back and forward, then you are in the line; if not, the inſtrument muſt be removed till you can ſee both. The inſtrument ſtanding thus, look along the croſs line on the index, and if it run directly to *o*, then *c* is the point where the offſet falls; but if it point not to *o*, the inſtrument muſt be removed back or forward in the line AB, till it does; whilſt the ſights ſtill point to A and B.

Proceeding forward, find the points *d* and *f*, &c. in the ſame manner, where the offſets *de*, *fg*, &c. fall; and meaſure the offſets as you go along. And care muſt be taken to go in the line AB; and that at each point *d* or *f*, the ſights may point to A and B, at the ſame time that the croſs line points to *e* or *g*. Then all muſt be ſet down in the field book, as you meaſured them; that is, both the diſtances along the ſtationary line, and the correſponding Fig. 3. offsets, which will be as follows.

	right hand.
⊙ A	0
Ac = 1.94	.87 = co
Ad = 3.42	.23 = de
Af = 4.17	.81 = fg
AB = 4.45	.47 = Bb

Cor. *After the ſame manner, if* A*oegb be an irregular curve; erect* 3, 4 *or* 5 *(equidiſtant) ordinates or offſets, at c, d, f,* &c. *and meaſure the baſe* AB, *and the offsets as before.*

PROB. IV.

To draw a ſtreight line on the ground, from one place A, *to another* B.

1. Take a long cord, and faſten one end at A, 4. and carry the other end towards B; if it will reach thither, ſtrain it tight, and faſten it at B. Then either pin down the cord, to the ground; or elſe, which is better, make a trench along the line with a ſharp ſpade, and that will be your line.

2. If the cord will not reach to B; let ſome body draw the cord ſtreight towards B, which ſuppoſe will reach to C; and there let him hold a ſtake upright; and moving it to and fro, ſet it at laſt, that A, C and B may hide one another, when ſeen from A or B; then A, C and B will be in a right line. And after this manner ſet one length after another, till you come at B.

3. If there happen to be any hill or riſing ground 5. in the way, that one place cannot be ſeen from the other; then perhaps, a very long pole may be ſet up in one of the places, and be ſeen at the other; and then proceed as before. But if this cannot be

Fig. done, ſet up two or more ſtakes in the way, as C,
5. D, &c. and let them be ſo placed by trials, that C ſeen from A, ſhall cover D; and D ſeen from C, ſhall cover B or the next ſtake, and ſo on to the laſt. *And note*, in directing one another, no words muſt be uſed, but only ſigns of the hand; for it is ſuppoſed to be too far off, to hear any directions.

But if you have an inſtrument with back ſights as well as fore ſights, fix your inſtrument ſo between A and B, that you may ſee both of them, through back ſight and fore ſight, and there you muſt place a ſtake as at C; but if A and B cannot both be ſeen from C, ſet it by gueſs, and another D alſo by gueſs, and ſo on. Then try them, and ſhift them, till A and D appears from C; and C and B from D, &c. by back ſight and fore ſight; and then they are all in a line.

PROB. V.

To raiſe a perpendicular upon a line AB, *at a given point* A, *upon the ground.*

6. Take a ſquare, or rather a graduated inſtrument with ſights, and placing it at A, direct the ſights of the inſtrument, or the ſide of the ſquare along AB, and fix the inſtrument. Then turn the ſights to 90 degrees, and look thro' them along the line AD, and let ſome perſon ſet a ſtake forward, as at D. Then DA is perpendicular to AB. If you uſe a ſquare, you muſt look along the other ſide of the ſquare, to find a point, as D.

Or thus.

Set off 40 links from A to B, and 50 links from B to D, ſo that 30 links may juſt reach from D to A. Then DA is perpendicular to AB.

For then the triangle BAD is right angled at A. Fig. For the ſquare of the hypothenuſe BD is equal to 6. the ſum of the ſquares of the two ſides, BA, AD.

PROB. VI.

To let fall a perpendicular from a given point D, *upon any line* AB, *upon the ground.*

Set an inſtrument ſomewhere about A, in the line 6. AB; and a mark at B; then directing the ſights along AB, fix the inſtrument. Then turn the index to 90 degrees, and if you ſee D thro' the ſights, you are in the perpendicular. If not, move the inſtrument back and forward in the line AB, till you can ſee D, with the index ſet to 90 degrees; the inſtrument having the poſition before deſcribed; that is, directed to B; then DA is a perpendicular. This may be done in a few trials.

A ſquare will perform the ſame thing; moving one ſide along AB, till the other ſide point to D, as at A.

PROB. VII.

To lay down a given angle on the ground, as CAB.

Set up a ſtake ſomewhere in the line AB, as at B; 7. then place an inſtrument at A, and direct the ſights at the beginning of the degrees, to B; then fixing the inſtrument, ſet the index to the degrees of the given angle; as DAE, and looking along the index AD, cauſe ſome aſſiſtant to ſet up a ſtake in the line ADC, as at C; then the angle CAB will be that required.

By an inſtrument with two legs; ſet it to the given angle, and placing it at A, direct one leg AE along the line AB, to the ſtake at B; and ſet up another

Fig. another ſtake in the line AC, as at C, in the di-
7. rection of the other leg AD.

PROB. VIII.

Thro' a given point C, *to drawn a line parallel to a given line* AB, *on the ground.*

8. Meaſure the neareſt diſtance of the given point C from the line AB; and at a convenient diſtance from A, as at B, raiſe the perpendicular BD to the line AB. Set the length AC from B along the perpendicular to D, and draw CD, which will be parallel to AB.

Or thus.

Take any ſtation B in the line AB; and with a proper inſtrument, meaſure the angle ABC. Then, by the laſt Prob. lay down the angle BCD equal to ABC; draw CD, which will be parallel to AB.

Or at any point A in the line AB, take the angle CAB; then going to C, ſet off the ſame angle ECD, to D; E being in the line CA.

Otherwiſe thus.

Set up a ſtake at C, and another at any place as A, in the line AB. And let two obſervers hang up two lines and plummets at D and B. Then let the obſerver at B take notice when the ſun, or ſome ſtar (agreed on between them) comes in the line BA; and at that inſtant give notice to his correſpondent at D, to ſet his line and plummet ſo, that D and C may be in a line with the ſun or the ſame ſtar. And then CD is parallel to AB. It will be beſt for the obſerver at D, to keep his line and plummet always in a line with C, and the ſun or ſtar.

PROB.

Fig.

PROB. IX.

To meaſure the horizontal baſe of a Hill, as BC.

Let BAD be a horizontal line, CA a perpendicular from the top, BC the ſlope or ſide of the hill. 9.
Set proper marks up, and with a proper inſtrument, take the elevation, that is, the angle ABC or ACB; and one being given the other is known. Then meaſure with a chain the ſide CB. And it will be, as rad : BC : : S.angle BCA : to BA.

If the hill has different declivities in ſeveral places, take the angles in theſe ſeveral places, and the ſide of each part, as before. Then find the baſe of each, by the ſame operation; and the ſum of all, is the whole baſe.

All this may be done by rule and compaſs, by laying down the triangle BAC, or the ſeveral parts thereof.

If there be likewiſe a deſcent on the other ſide, as CD; you may find the part AD in the ſame manner as before. Or proceed thus, take the angles at B and D, and ſubtract their ſum from 180 gives the angle BCD; or the angle BCD itſelf may be meaſured; and meaſuring either ſide of the hill as BC, then, as S.angle D : BC : : S.angle BCD : BD the whole baſe.

Cor. *By the ſame method the height of the hill* CA *may be found. For it will be, as radius* : BC : : S.*angle* ABC : *height* CA.

⁂

PROB.

PROB. X.

To reduce a field to fewer ſides in the field, where an angle is very obtuſe, or when one of the ſides is very ſhort.

10. Let DEFKCBAD be any field, which here conſiſts of ſeven ſides. Then,

1. Suppoſe ABC is a very obtuſe angle or bending, which we want to take way. Produce the longer ſide of the angle CB to cut the ſide AD (adjoining to the other ſide), at G; then take the point I, ſo that it may divide the line GA, in the ſame proportion, as B divides GC; that is, ſo that GI may be to GA, as GB to GC. And as GA is in this caſe very ſhort, this may be done by the eye exact enough, for the moſt part; or as exactly as the two points A and B can be determined. Then draw the line CI, and leaving out the angles B and A, the two lines CB, BA are reduced to one CH, and the field conſiſts of one ſide leſs than before. And the ſame may be done with the angle FED, by producing FE till it cut AD, and finding the point Q as I was found before.

2. Suppoſe ED a very ſhort ſide. Stand at D the end of the ſhort ſide, and obſerve, directly over the angle F (the next but one), ſome very remote object, as a houſe, tree, mountain, ſun, moon, ſtar, cloud, &c. Then immediately remove along AD, till you ſee the ſame object directly over E; at that place Q, put in a ſtake or ſome mark. Then leave out the angles E, D, and take Q inſtead thereof. And the two ſides FE and ED are reduced to one FQ, which takes away another ſide of the field. And the ſame thing may be done with the ſide BA, by obſerving the ſame remote object from A over C, and from I over B.

The reaſon of this proceſs is this; the lines AC and IB are parallel; and the lines DF and QE are parallel; and therefore the triangle CBA is equal to CIA; and the triangle FED is equal to FQD. Fig. 10.

Cor. *This method is very uſeful, when we meet with a field with crooked hedges, by reducing it to fewer ſides.*

For by this means we have the ſame quantity of ground, and fewer hedges. For the lines CI and FQ take in as much as they leave out.

SCHOL.

When any ſide of a field is very irregular, you may draw a ſtrait line inſtead of it, which ſhall take in as much ground in ſome places, as it leaves out in others; and ſo does not alter the quantity of it. And this way (of giving and taking) ſhortens the work; and ſuch a line may be drawn as exactly by the eye, in moſt caſes, as the boundaries themſelves can be aſcertained.

PROB. XI.

To ſurvey a triangular field ABC.

1. *By the Chain.*

Set up ſtation-ſtaves at the angles A, B and C. 11. Then meaſure with the chain from A to C; and when you come at P, where the perpendicular BP falls, ſet up a ſtake there. And the point P is eaſily found, for as you go along from A, ſet your inſtrument where you think the perpendicular will be, then look thro' the ſights back and forward, to ſee that you be in the line AC; that done, look thro' the croſs line upon your index, and if you ſee the vertex B, then you are at the place P, where the perpendicular falls. But if B does not appear, you

Fig. 11. you muſt move back or forward in the line AB, till B can be ſeen along the croſs line, as at P. Then having meaſured AC, and ſet it down; go to P and meaſure the perpendicular PB, and ſet it down likewiſe. And thus you have the baſe and perpendicular, which is ſufficient for finding the content.

Or meaſure all the three ſides AC, CB, BA, ſeverally, with the chain; and note them down. Then the triangle may be laid down, and the content caſt up by Sect. II.

2 *By taking one or more angles.*

Meaſure two ſides AC and CB, and the angle included ACB; by which the triangle may be deſcribed.

Or meaſure one ſide AC, and two angles A and C, then the third angle B will be known.

Or, if you cannot get the angle C; take any point D in the baſe AC, where take the angle ADB, and meaſure AC and DB; and then it may be caſt up by Sect. II.

PROB. XII.

To ſurvey a field in form of a trapezium.

1. *By the chain.*

12. Set up ſtation ſtaves with marks or ſtreamers, at all the corners A, B, C, and D. Then chooſe one of the diagonals AC or DB to meaſure, let it be DB; it ought to be ſuch a one, as that you can ſee the mark at B from D, or the mark at D from B; and likewiſe the marks at A and C from the diagonal DB; and likewiſe the perpendiculars AF and CG ſhould fall within the figure.

Begin at D, and meaſure along the diagonal DB in a right line, till you come at the place of the perpendicular AF, which will be known, by looking backward and forward thro' the ſights to D and B; and then looking along the croſs line to ſee A; removing the inſtrument, till A can be ſeen. Then at F put in a ſtake. Then meaſure forward, and find the place G of the other perpendicular GC as before; and put in another ſtake at G. Then meaſure to the end B, and ſet down the length DB. Then go to F or to A, and meaſure the perpendicular FA, and ſet it down. Laſtly, go to G, and meaſure the perpendicular GC, and ſet it down, with the reſt. And by theſe the content will be found. Fig. 12.

Diagonal DB — 8.37
1 perp. AF — 5.18
2 perp. GC — 4.86

Otherwiſe, by the Chain.

Draw a rude draught of the field to imitate it, upon paper; and having ſet up marks at all the corners, draw a diagonal as DB upon your draught. Then begin at any angle D, and meaſure all the ſides round about to D again; and then the diagonal DB; and ſet them all down as they are meaſured, upon the correſpondent lines of your draught. Then from theſe meaſures, the field may be laid down upon paper; and the content found, by Sect. II.

In meaſuring this way, it is beſt to take the ſhorteſt diagonal.

Or thus, by the Chain.

Marks being ſet up at A, B, C and D. Find the interſection E of the diagonals AC, DB; thus, place the ſtaff and index at E, and if you can ſee D and B forward and backward, and alſo A and C forward and backward, thro' the ſights, then E is the interſection ſought. But if not, remove the ſtaff, 13.

Fig. 13. ſtaff, till you can ſee them all; which is ſoon done by a few trials. Then meaſure the lines in this order, AE, EC, CB, BE, ED, and DA (for proof), with the chain. And write their lengths on their proper lines, in a rude draught of the field. From theſe meaſures, the field may be laid down, and the content found, by Sect. II.

In all theſe methods, there is no occaſion for any inſtrument (beſides the chain), but an index with two ſights and a croſs line, and a ſtaff to ſet it on; whoſe uſe is to make you go ſtrait, by trying foreſight and back ſight; and for finding the perpendiculars, when they are required.

2. *By taking an Angle.*

Having ſet up marks at all the corners; find the interſection E, of the diagonals in the field, as in the laſt method. Then fixing the inſtrument at E, take any one of the angles at E, as AEB, and ſet it down. Then meaſure the diagonals AC and BD, and ſet them down. Then by theſe, the content will be found, by Sect. II.

Angle AEB,	66°	20′
AC,	7.	54
DB,	6.	17

Or thus, by taking ſeveral Angles.

Make a rude draught of the field upon paper, and draw one diagonal as DB, which meaſure and ſet down in the draught. Then with the inſtrument, take the quantity of the angles, DCB, CDB, and ADB, DAB; and ſome other angle or ſide for proof; and ſet all theſe angles down in the draught. Then by means of theſe, the field may be laid down on paper, and meaſured.

Note, it does not ſignify whether your eye draught be true or falſe, provided all the lines lay but in their right poſition; for the conſtruction at laſt Fig. will make all right. 13.

SCHOLIUM.

As the ſurveying a triangle and a trapezium, is the foundation of ſurveying all other ſorts of figures, I have been very particular in ſhewing different methods. For all plane figures muſt be divided into triangles and trapeziums before they can be meaſured; and therefore the learner ought to have a good notion of theſe.

PROB. XIII.

To ſurvey any field by the chain only.

Let ABDF be the field propoſed. Set up ſtation ſtaves, or any long ſticks, with marks or ſtreamers on them, in every corner of the field; pieces of white or red rags will do well enough, if they can but be ſeen, the breadth of the field. And as you walk over the field, conſider how it may moſt commodiouſly be divided into triangles or trapeziums, ſo as to be eaſieſt meaſured. It muſt be ſo divided, that you may ſee from one end to the other, of any diagonal of a trapezium, or baſe of a triangle; and likewiſe ſee the tops of the perpendiculars, which muſt fall within the figure. Here the field is divided into two trapeziums FCDE and FCBG, and a triangle BGA. Here we might have divided the field into the two trapeziums ABCG and GFEC, and the triangle ECD; and a great many other ways. 14.

Having ſet out the parts of the field, as you deſign to meaſure them; begin at any part as FEDC, and meaſure along the diagonal FD, till you come at the perpendiculars IC and HE, which I have ſhewn how to find in the laſt Prob. there put ſtakes

Fig. 14. in. When the diagonal FD is meaſured, and ſet down, go to E and meaſure EH, and then IC, and ſet them down; and then you have the firſt trapezium.

Being at C, meaſure along the diagonal CG of the trapezium CFGB, till you come at the perpendiculars LB, KF, to ſet marks, whoſe places are found as before; then meaſure to the end, and ſet down the diagonal; then go to F, and meaſure the perpendicular FK, and then LB, and ſet them down. So you have the ſecond trapezium.

Being at B, meaſure along the baſe BG to the perpendicular NA, and put in a ſtake at N. Then meaſure to G, and ſet down the baſe BG, and laſtly, meaſure the perpendicular AN, and ſet it down; ſo you have the triangle BGA.

The reſult of the work.

1. *Trapezium.*

The diagonal	11.88
1 perpendicular	3.54
2 perpendicular	2.52

2. *Trapezium.*

The diagonal	12.00
1 perpendicular	4.47
2 perpendicular	3.72

Triangle.

Baſe	10.56
Perpendicular	3.32

In meaſuring any of the lines, take care to go very ſtrait; otherwiſe the perpendiculars will be either too ſhort or too long, by going out of the line. But when the perpendiculars are near together, as HE, IC, the going out of the line cauſes little error; becauſe one gains what the other loſes.

When the perpendiculars are not very long, the places where the perpendiculars fall may be taken true enough by the eye, without an index; for a ſmall error in the place, makes no ſenſible difference in the length. Fig. 14.

Otherwiſe thus.

Let ABCDE be a field; from any angle of the field as D (from which all, or moſt of, the other angles may be ſeen), ſuppoſe lines drawn to the other angles, as A and B; which will divide the whole field into triangles; and marks being placed at theſe angles; take an eye draft of the field, thus divided into triangles. Then meaſure all the ſides of every triangle, and ſet them down in the draft, along their reſpective lines. Thus, meaſure DE, EA, AD, in the triangle ADE; then DB, BA, in the triangle ABD, then BC, CD in the triangle BDC. Having got all theſe meaſures, every triangle may be drawn upon paper one by one, and ſo the whole field will be laid down. And your rough draft will inform you, how to join the triangles together. 15.

If the other angles cannot be ſeen from any angle, then you muſt take a point about the middle of the field as F, from which all the angles A, B, C, D, E, can be ſeen. And ſetting up marks at theſe angles, and ſuppoſing lines drawn to them all from F, take a rough draught of the field as uſual. Then meaſure all the ſides of every triangle one by one, and write down their lengths, as you meaſure them, along their correſpondent ſides in the draft. And then they may be all laid down on paper as before. 16.

Note, the fewer triangles there are, the ſooner they are meaſured; and therefore it is ſooneſt done, when the field is divided into triangles from ſome angle, as in the former method. For the leaſt number of triangles, any field can be divided into, is

leſs

Fig. 16. leſs by 2 than the number of ſides of the figure, when the diviſions run from an angle; and the diagonals leſs by 3. But when the diviſion is made from a point within; the number of triangles, and number of lines, is equal to the number of ſides.

For proof of the work, ſome line or diagonal, ſhould be meaſured over and above.

If you have not a rough draft, meaſure round the field, AB, BC, CD, &c. and ſet them down in a table. Then meaſure the diagonals AD, BD, which ſet down in a table; or meaſure FA, FB, FC, &c. and ſet down. But take notice what way you go about the field.

PROB. XIV.

To ſurvey any field by the plain table.

15. Set up marks at every corner of the field, ABCDE, except the corner, where you take your ſtation: Therefore chooſe ſuch an angle for your ſtation, from which you can ſee all the other angles. Then put a clean ſheet of paper upon your table and fix it at the angle D, ſo that the needle, if it have one, may hang juſt over the flower-de-luce. Then apply the index to D, and direct the ſights to E, and by the edge of the index, draw the obſcure line DE upon the paper; meaſure DE, and ſet it off (from your ſcale) from D to E. Then turn the index about, till thro' the ſights you can ſee A, the ſide of the index ſtill being at D. Then draw the line DA; meaſure the diſtance, and ſet it from D to A. In like manner direct the ſights to B, C, &c. and draw lines upon your paper; and meaſure the diſtances DB, DC, &c. and ſet them from D to B, C, &c. upon the paper. Laſtly, draw the ſides of the field DE, EA, AB, BC and CD; ſo your draught is finiſhed.

Fig. 16.

Otherwiſe thus.

When there is no angle from which all the reſt can be ſeen, take a ſtation in the middle of the field, as at F; from whence all the angles can be ſeen. Then ſet up marks at every corner, and plant the table at F; and turn it about till the needle hangs over the flower-de-luce, and there fix it. Then place the index on the point F, and direct the ſights to A, and draw an obſcure line by the edge of the index upon the paper. Then direct the ſights to B (ſtill keeping the edge of the index cloſe to F), and draw a line. In like manner direct the ſights to C, D and E, and draw lines to C, D, E. Then meaſure all the diſtances, in this order FA, BF, FC, DF, FE; and with your compaſſes ſet the diſtances on their proper lines from F to C, D, E; and laſtly, draw the boundaries of the field AB, BC, CD, DE, EA.

Or thus, by going round the field.

Suppoſe ABCDE a wood or pool of water. Plant the table at A, and chuſing a point on the paper near the edge as A, lay the index upon A, and direct the ſights to E, and draw a black line as AE; then meaſure the diſtance AE, and ſet it from A to E on your paper. Then the table remaining fixt, turn the index about, and direct the ſights from A to B, and draw a line as AB; meaſure the diſtance AB, and ſet from A to B on the paper. 15.

Then remove the table to B, and lay the index upon the line BA; and turn the table about, till thro' the ſights you ſee the mark A. Then fix it, and direct the ſights from B to C, and draw the line BC; then meaſure it, and ſet it from B to C on the paper.

Then remove the table to C, and laying the index on the line CB, turn the table about, till you ſee the mark B; then direct the ſights from C to D,

and

Fig.
18.
19.

Otherwiſe, by going round the field.

Place the inſtrument at any angle A, and direct the index to B, and from the center of the paper draw the line 1, the ſame way from the center that B lies from A. Screw the index faſt upon the line 1, and remove to B, and as you go, meaſure AB, and write its length on the line 1. Then fix the inſtrument at B, and turn it round till the index (by backſight) point to A; there ſcrew the table faſt, and direct the index to C, and draw the line 2 by its edge upon the paper, to lie the ſame way from the center, that C lies from B. Then ſcrew faſt the index to the line 2, and remove to C, and as you go along meaſure the ſide BC, and write its length on the line 2. Fix the inſtrument at C, and turn it round till the index point backward to B; then turn the index to D, and by its edge draw the line 3, and ſcrew it faſt. After this manner proceed to all the angles D, E, F, G; always obſerving, to fix the index upon the line laſt drawn, and when you come at the next angle, to fix the inſtrument, that the index may point backward to the laſt ſtation, and then direct the index to the next ſtation; and in going along, meaſure each ſide, and ſet it upon its reſpective line in the paper. And note, at the laſt ſtation G, the index ought to point to A if the work be right. Here AB is 13.10, BC 6.18, CD 17.75, DE 10.32, EF 10.45, FG 9.00, GA 8.15. And all the lines 1, 2, 3, &c. will be parallel to the reſpective lines AB, BC, CD, &c.

SCHOLIUM.

I might now ſhew how to ſurvey a field by the theodelite, circumferentor, ſemicircle, geodetical ſtaff, &c. but as theſe inſtruments are purely for taking angles, the method whereof has been ſhewn in the firſt Problem; therefore no particular directions need be given further for that purpoſe. Fig. And when theſe inſtruments are fit to be uſed, will 19. be ſhewn in the following Problems, wherein are explained the different methods of ſurveying any field or parcel of ground.

PROB. XVI.

To ſurvey a Field at one ſtation, by meaſuring ſeveral lines.

Chuſe ſome angle in the field for your ſtation, 20. from whence you can ſee all the reſt of the angles, as at S. Then ſet up ſtation ſtaves or marks at every corner A, B, C, &c. Then placing the inſtrument at S, whether it be a Theodolite, Semicircle, Geodetical ſtaff, &c. it is the ſame thing; direct the fixt ſights to A, and there fix the inſtrument; then direct the index to B, ſo that thro' the ſights you may ſee the mark at B; and the degrees cut by the index, will ſhew the angle ASB, which ſet down in the field book. Then direct the index to C, as before; and the degrees cut will ſhew the angle ASC, which ſet down. In like manner take the angle ASD, and the reſt, if there be more, and ſet all down. And this way is more exact, than taking the angles ASB, BSC, CSD, &c.

Then with the chain meaſure the ſides and diſtances in this order SA, BS, SC, DS; ſetting them down in the field book, or writing them on the lines in a rude draft of the field.

Or thus, from a point within.

When all the angles cannot be ſeen from any one 21. of the angles of the field; aſſume ſome ſtation S within the field, where the angles can all be ſeen; there place the inſtrument at S, and directing the fixt ſights to A, there fix the inſtrument, and direct the index to B, and turning it round, till thro' the ſights you ſee the mark at B, obſerve the degrees

Fig. degrees cut by the index, for that is the angle ASB,
21. which ſet down in the field book, or in a rude draft of the field. Then the inſtrument remaining fixt, direct the index to C, and the degrees cut, ſhew the angle ASC. In the ſame manner direct the index to all the other corners D, E of the field; and you will get all the angles about S, meaſured from the firſt line SA, which ſet down. And this is more exact, than taking the angles ASB, BSC, &c. ſingly. But with ſome inſtruments you cannot take the angles quite round from the line SA; and therefore you muſt then take the angles ſeparate, ASB, BSC, CSD, DSE and ESA. And to prove the work, the ſum of all theſe angles ought to be equal to 360 degrees. But if the ſtation S happens to be upon an eminence, the ſum of all theſe angles will be ſomething leſs than 360 degrees.

Having got the angles, meaſure all the diſtances SA, SB, SC, &c. with the chain. And it is quickeſt to meaſure in this order, SA, BS, SC, DS, SE; that is, from the inſtrument to the firſt corner, and back from the ſecond corner to the inſtrument, and ſo on. Still one from the inſtrument, and another to it. Then theſe diſtances muſt all be ſet down in your book or draft.

Otherwiſe, by meaſuring round the field.

22. Take your ſtation, if you can, in an angle or a ſide; let it be at E. Then marks being placed at A, B, C, D; and turning the fixt ſights to A, fix the inſtrument; and direct the index to B, and the degrees cut will ſhew the angle BEA, which note down. Then turn the index to C, and then to D, and note down the degrees cut, which are the angles AEC, AED.

Then meaſure with the chain all the ſides EA, AB, BC, CD, and DE; and ſet them down.

Here if you take all the angles at E ſeparate, the ſum of them all muſt be equal to the angle AED, if the work be right. You muſt note whether the angles at B, C, E, be acute or obtuſe. Fig. 22.

In all theſe methods, care muſt be taken that the inſtrument be not moved out of its place, while you are working; for if it be, your angles will not be true.

PROB. XVII.

To ſurvey a field, by meaſuring but one line, and taking the angles round about.

Let ADF be the field; ſet marks up at all the corners F, A, B, C, D, E; and chuſe ſome ſide, or ſome diagonal or line, to be meaſured, ſuppoſe AF. Then place the inſtrument at F, and take the angles AFB, BFC, CFD, DFE; and note them down. Go to A, and take the angle FAB. In like manner going round the field, take the angles FBC, FCD, FDE; laſtly meaſure FA. Then theſe triangles may be laid down upon paper, one after another, beginning at the triangle FAB. 23.

In like manner any other line, as FB or FC, may be meaſured inſtead of FA; and then the triangle FBC or FCD muſt begin the operation.

Or thus, from a ſtation in the field.

Aſſume the ſtation S, and let FS be meaſured. Place the inſtrument at S, and take all the angles round S, ASB, BSC, CSD, DSE, ESF, and FSA. Then go round the field, and take the angles SAB, SBC, SCD, SDE, SEF, and SFA. From theſe, the field may be laid down, beginning at the triangle SFA. 24.

And any line AS, BS, &c. may be meaſured, inſtead of FS; if AS be meaſured, then the conſtruction muſt begin with the triangle ASB.

These

Fig. Theſe methods are ſubject to errors, eſpecially when the number of ſides is very great.

PROB. XVIII.

To ſurvey a field, by the method of circulation, or going round it.

25. Let ABCDE be the field. In this caſe we muſt reduce the field to as few ſides as we can; and therefore where there are any ſmall bendings, as in crooked hedges, we muſt take offsets by Prob. III. Therefore pitch upon the ſtations, *a*, *b*, *c*, *d*, *e*, as near the hedges as you can conveniently come; and at theſe places ſet up ſtation ſtaves with marks. Begin at ſome angle as *a*, and placing the inſtrument there, take the angle *eab*, which ſet down in the field book. Then meaſure from *a* to *b*; and in going, take the offsets at *a*, and at *f*, *g*, *h*, *i*, and at *b*, and note them down in the field book, and the length *ab*. Then place the inſtrument at *b*, and take the angle *c*, and note it down; and then meaſure from *b* to *c*; and in your way, take the offset at *b*, and where you touch the hedge at *k*, and the offset at *c*; and ſet them down. Then at *c* place your inſtrument, and take the angle *bcd*, which write down; and then meaſure *cd*, taking the offsets at *c* and *d*, which write down. Again place the inſtrument at *d*, and take the angle *cde*, and ſet it down, and the offset at *d*. Then meaſure from *d* to *e*; and as you go, note the point *l* where you touch the hedge, and the offsets at *m* and *e*; and ſet all down. Then ſet the inſtrument at *e*, and take the angle *dea*, and ſet it down, take the offset at *e*, and meaſure from *e* to *a*, being a ſtreight hedge, and take the offset at *a*. Set all down, and your work is finiſhed.

In your book you muſt note which are inward, Fig. 25. and which are outward angles. And to prove your work, add all the angles together, and the ſum will make twice as many right angles, abating 4, as the field has ſides, if the work be right.

When there are many angles or ſides in a field, this method is ſubject to errors; therefore it ſhould not be applied, where there are more than 6 ſides; or at leaſt the field ſhould be reduced to that number; for the fewer ſides, the more exact the work will be. For when the number of ſides is very great, the error is multiplied with the number of ſides, becauſe every angle is taken from a different line; and for that reaſon the draught will never cloſe. For when all is laid down the laſt point ſhould fall upon the firſt, for we end where we begun; and when the plan does not anſwer that, it muſt be wrong.

When you cannot get an internal angle conveniently, produce one of the ſides without the field, and take the external angle. And if all the angles be taken externally, the ſum of them all ought to be equal to 4 right angles. And if a ſide cannot be meaſured directly, it may be done by the method of taking inacceſſible diſtances, in the laſt Section.

If you make uſe of the circumferentor for taking angles, you need only place it at every other angle. And the like for any inſtrument, where a needle is made uſe of.

Otherwiſe, by the needle.

When the number of angles of any piece of ground is very great, the method of circulation, by taking the angles round it, is impracticable, becauſe there is no fixt line to take theſe angles from. In this caſe the needle may be brought in, to our aſſiſtance. And here we have no more to do, Fig. do, but to go from angle to angle, and take the bearings of all the ſides, or the angles they make with the meridian; and to meaſure the lengths of the ſides.

26. Let ABCDEFGH be ſome thick wood or pool of water, which we cannot ſurvey within. Here we muſt take all the boundaries on the outſide, and reduce it to as few ſides as we can, for greater exactneſs; as in this field, where we have but 5 ſtations; but then we ſhall have the more offsets to be meaſured. Therefore having ſet up marks at *a*, *b*, *c*, *d*, *e*, where we take our ſtations; begin at any one as *a*, and placing the compaſs and needle there, ſo that the needle may hang over the flower-de-luce, take the bearing of the line *ab*, and ſet down what angle it makes with the meridian, and which way it lies in reſpect of E. W. N. or S. Then meaſure the length *ab* to the next ſtation, and in going, take the offsets at *a*, *s*, *t*, and where it touches the corner at B; and ſet all down. Then place the compaſs at *b*, and cauſe the needle to hang over the flower-de-luce; there fix it, and take the bearing, of the next ſtation *c*, or of the line *bc*; and then meaſure *bc*, and then take the offsets at *b*, *f*, *h*, *i*, and where it touches the corner at C; and ſet theſe down. Then place the compaſs at *c*, *d*, and *e* ſucceſſively; and take the bearings of *cd*, *de*, *ea*, as before; and the offsets at *k*, E, *d*, and at *m*, *n*, F, *p*, G; alſo at the corners H and A; and ſet them all down in the field book, and the work is done.

Care muſt by taken, that there be no iron near the needle, for that will make it deviate from the meridian. And the inſtrument need only be placed at every other angle, but then you muſt take the bearings both ways, that is, of both ſides of the incloſed angle, the one forward, the other backward.

When the ſurvey is done, if you lay down every distance ſingly, and the bearing, upon paper; and from thence get the eaſting, weſting, northing or ſouthing, and put them into diſtinct columns; then the ſum of the eaſtings and of the weſtings muſt be equal. And the ſum of the northings equal to the ſum of the ſouthings; if the work be right. For it is no more than working a traverſe on the land, by plain ſailing, as it is practiſed at ſea. Fig. 26.

Tho' this method is not to be relied on, or uſed, but in caſes of neceſſity, where there is a great number of angles to be taken, by reaſon one cannot take an angle with the needle, to any great exactneſs; yet there is no greater error in taking one angle, than there is in taking 20, becauſe they are all taken from the meridian, and therefore are independent of one another. But if it was poſſible to take an angle as exactly with the needle as with another inſtrument, it would be the beſt inſtrument to ſurvey with. And as it is, it has the advantage of other inſtruments in ſuch caſes where the number of ſides is very great. The main difficulty is, to fix the inſtrument right by help of the needle, by which it is to be placed truly in the meridian; and to help this, the needle muſt be made very long, and be well touched. In uſing other inſtruments, they are fixt, by directing the index to the laſt ſtation, which admits of greater exactneſs. For all inſtruments with a needle, and alſo the plane table, whether it has a needle or not, require at every ſtation to be placed in the ſame poſition, in reſpect to the meridian, thro' all the operations, which is hard to do; for a needle, tho' never ſo good, will not always ſtand at the very ſame point.

PROB.

PROB. XIX.

To ſurvey a field at two ſtations meaſuring only one line, which is the ſtationary diſtance.

27 Chooſe two ſtations S and T, in the field, from whence all the angles can be ſeen. Then ſet up ſtation ſtaves there, and at every angle, with ſtreamers of different colours on them. Then place the inſtrument at one ſtation S, and directing the fixt ſights to the other T, there fix the inſtrument; then directing the index ſucceſſively to the ſeveral angles in order, take the angles TSA, TSB, TSC, TSD, TSE, TSF; and number them 1, 2, 3, 4, &c. for diſtinction, and write them all down. And the marks at theſe different angles, ought all to be different, to diſtinguiſh them one from another. Then go to the ſecond ſtation T, and directing the fixt ſights to the firſt ſtation S, fix it there; and directing the index to the ſeveral angles, in the ſame order as before, take the angles STA, STB, STC, STD, STE, STF; and number them, and ſet them all down as before. To take the angles in the ſame order, you ought to know them from one another by the marks; for if you do not know which is which, all your work will be confuſion. And as you go from one ſtation to the other, meaſure the ſtationary diſtance ST. Then if all theſe angles be laid down upon paper, and lines drawn, the correſpondent lines will interſect in the angles or corners of the field A, B, C, &c. Then draw the lines AB, BC, CD, &c.

If the plane table is uſed.

Draw the ſtationary line ST on the table, and ſetting the index to the line ST, direct it to T, and fix the table there; and laying the edge of the index to S, direct it ſucceſſively to all the angles A, B, C, &c. and draw lines from S by the edge of the index. Then go to T, and meaſure the ſtationary diſtance ST, which ſet off upon the table. Lay the index upon the line ST, and direct it to S, and fix the table ſo. Then lay the edge of the index to T, and direct it ſucceſſively to the ſame angles A, B, C, &c. and draw lines from T as before; and note where the correſpondent lines (or thoſe drawn to the ſame angle), interſect; for theſe will be the ſeveral angles of the field. Fig. 27.

The two ſtations S and T may be ſometimes taken in one of the ſides, as in FE; and then there will be fewer angles to take. Or they may be taken out of the field, as at *s* and *t*, as when one is not admitted to come into the ground. And it will ſhorten the work, if one or both ſtations are taken in the production of the ſides of the figure, as here, in AF and DE; and likewiſe if two of the angles fall in the ſame right line, drawn from either ſtation. 28.

Note, the corners of the field thus determined by the interſection of lines, will be trueſt, when they interſect the neareſt to right angles. But when they cut very obliquely as at A, ſuch interſection will not give the point exactly; and therefore it would be better to meaſure one of the lines, as SA or TA. And the error of interſection will be reciprocally as the ſine of the angle SAT, *ceteris paribus*.

If it happen that ſome angle cannot be ſeen from one of the ſtations; then ſet up a ſtake for a mark, in the midway between the ſtation and that angle, to ſtand in a right line; which may ſoon be done by an inſtrument, placed at the ſtake, taking backſight and foreſight. Or take a new ſtation, ſo that the angle or angles wanting, may be ſeen from this and one of the others, and proceed as before.

Fig. 28. Thus a map of the chief places in a town may be made; or any part of a river ſurveyed, &c. by taking two ſtations, on two towers, or two high hills; and thus inacceſſible places may be meaſured by means of ſcaffolds erected, and fixing high poles at the angles, with flags of different colours flying in the air.

But in ſuch caſes, it is beſt to have but a few poles up at once, as half a dozen; and when the places of theſe are determined on the draught, fix them up in freſh places; and ſo as many places as you will, may be determined, by taking a few at a time, and repeating the ſame work.

In uſing the plain table, it ſometime happens that the lines run off the table; in this caſe, move the paper ſo far off the table, that the laſt ſtation may be a little way upon it. Then take a freſh ſheet of paper and glue it to the other, at that ſide with mouth glue, and ſo faſten them down. Then lay a ruler upon that ſtation and the ſtationary line, and prolong it upon the freſh paper. Then lay the index upon the ſtationary line, and turn the table, till thro' the ſights you ſee the laſt ſtation; there fix it, and proceed.

PROB. XX.

By means of ſeveral ſtations, to ſurvey a large common, a river, a part of a country, or any hilly ground.

When all the angles of a field or parcel of ground, or all the objects we want to obſerve, cannot be ſeen from one place; then we muſt take ſo many ſtations, as will ſucceſſively diſcover them all to us.

29. Let ADG be a large common, or part of a country; and A, B, C, D, E, &c. the angles or boundaries of it, or any remarkable objects in the country;

country, as trees, towers, houſes, mills, &c. Take Fig. 29. the ſtations S, T, V, &c. upon ſome hills or eminent places, from which all the angles or objects can be ſeen, and at all the places and ſtations, ſet up long poles with marks. Suppoſe the objects K, L, A, B, C can be ſeen from S; place the inſtrument at S the firſt ſtation, and direct the fixt ſights to the ſecond ſtation T, and there fix the inſtrument. Then take all the angles round S, as TSK, TSL, TSA, TSB, TSC; and meaſure all the diſtances SK, SL, SA, SB, SC, and the ſtationary diſtance ST; and ſet all down. But in the plane table, fix the table and direct the index from S to T, and draw the ſtationary diſtance ST. Then direct ſucceſſively from S to K, L, A, B, C; and draw lines on the table. Then meaſure the diſtances SK, SL, SA, SB, SC, and ST, and ſet them all upon their proper lines.

Then remove to your ſecond ſtation T, where you can ſee the objects D, E, F, I. Direct the fixt ſights to the former ſtation S, and fix the inſtrument; then take all the angles from ST, as STD, STE, STF, STI; and to the next ſtation STV, and meaſure the diſtances TD, TE, TF, TI, and TV, and ſet all down. Or in the plane table, lay the index on the ſtationary line ST, and keeping it there, turn the table about, till thro' the ſights you ſee the former ſtation S, there fix it. Then direct the index to all the places D, E, F, I, and V; and draw lines, on which ſet the diſtances as before.

Then go to a third ſtation V, where you can ſee the objects G, H. Then placing the inſtrument at V, direct the fixt ſights to T the former ſtation, and fix the inſtrument, and take all the angles from TV, as TVG, TVH, and to the next ſtation, if there be any more; and meaſure

the

Fig. the diſtances, VG, VH; and ſet all down. But
29. in the plane table, lay the index on the laſt ſtationary line VT, and turn the table about till the former ſtation T appear thro' the ſights, there fix it. Then direct the index ſucceſſively to G and H, and draw lines, on which ſet the meaſured diſtances, VG, VH as before.

And in general when any ſtationary line is drawn on the table, and you come to a new ſtation, lay the index upon that ſtationary line, and turn the table, till thro' the ſights you ſee the ſtation laſt come from, and there ſcrew it faſt, and proceed.

And after this manner, if there be more objects undetermined, you muſt take new ſtations, and ſo proceed, till all is finiſhed.

Then from all theſe angles and diſtances given, the whole work is eaſily laid down by rule and compaſs, and the plan or map deſcribed. But when it is taken by the plain table, it is deſcribed already, by the operations on the table. And if it be a field, lines muſt be drawn from A to B, from B to C, &c.

At every ſtation when the inſtrument is fixt, take care it be not moved out of its place, till all the work at that ſtation be done; which you may know by obſerving ſome remarkable object thro' the fixt ſights; for that object ought always to appear there. Or on the table by drawing a line; or making a mark, by the edge of the index. For then the index being laid to that mark, that object ought always to appear thro' the ſights.

If there be ſome of the objects which cannot be ſeen from any of the former ſtations, then new ſtations muſt be taken from which they can be ſeen. Remembring to take their diſtances, and angles (or poſitions), from ſome of the other ſtations.

If ſome of the objects can be ſeen from two adjoining ſtations, as I from both T and V, then the angles may be taken from both ſtations (ITV and IVT); which will ſave the labour of meaſuring to ſuch objects, the diſtances TI, VI. Fig. 29.

In the plain table, when any ſtation line runs off the table, take off the paper and glue another paper to it with mouth glue; on this produce that ſtation line to its proper length. Then faſten the paper on the table, with the former ſtation near the edge of the table; and laying the index on the new ſtationary line, turn the table about, till you ſee the laſt ſtation come from, thro' the ſights; there fix it and proceed.

It will be proper always to take an eye draught of the field or country, upon paper, before you begin your work. And all your meaſures muſt be written down in this rough draught. Or elſe they muſt be written regularly in a field book on purpoſe.

PROB. XXI.

To ſurvey a lordſhip, or large eſtate of land.

If the eſtate be very large, and contains a great number of fields; it cannot be done by ſurveying all the fields ſingly, and then putting them together; nor can it be done by taking all the angles and boundaries that incloſe it. For in theſe caſes, any ſmall errors will be ſo multiplied as to render it very much diſtorted.

1. Walk over the lordſhip two or three times, in order to get a perfect idea of it, and till you can carry the map of it in your head. And to help your memory, draw an eye draught of it on paper, to guide you, or at leaſt, of the principal parts of it.

2. Chooſe

Fig. 29. 2. Chooſe two or more eminent places in the eſtate, for your ſtations, from whence you can ſee all the principal parts of it; and the fewer ſtations you have to command the whole, the more exact your work will be; and let theſe ſtations be as far diſtant from one another as poſſible; and they will be fitter for your purpoſe, if theſe ſtationary lines be in or near the boundaries of the ground, and eſpecially if two lines or more proceed from one ſtation.

3. Take what angles, between the ſtations, you think neceſſary, and meaſure the diſtances from ſtation to ſtation, always in a right line; theſe things muſt be done, till you get as many angles and lines as are ſufficient for determining all your points of ſtation. And in meaſuring any of theſe ſtationary diſtances, mark accurately where theſe lines meet with any hedges, ditches, roads, lanes, paths, rivulets, &c. and where any remarkable object is placed, by meaſuring its diſtance from the ſtationary line; and where a perpendicular from it cuts that line. And always mind, in any of theſe obſervations, that you be in a right line, which you will know by taking backſight and foreſight, along your ſtationary line; which you muſt never omit. And thus as you go along any main ſtationary line, take offsets to the ends of all hedges, and to any pond, houſe, mill, bridge, &c omitting nothing that is remarkable, and all theſe things muſt be noted down, for theſe are your *data*, by which the places of ſuch objects are to be determined upon your plan. And be ſure to ſet marks up at the interſections of all hedges with the ſtationary line, that you may know where to meaſure from, when you come to ſurvey theſe particular fields, which muſt immediately be done, as ſoon as you have meaſured that ſtationary line, whilſt they are freſh in memory. By this means all your ſtationary lines are to be

meaſured, and the ſituation of all places adjoining to them determined, which is the firſt grand point to be obtained. I would have you lay down your work upon paper every night, when you go home, that you may ſee how you go on.

4. As to the inner parts of the eſtate, they muſt be determined in like manner, by new ſtationary lines. For after the main ſtations are determined, and every thing adjoining to them; then the eſtate muſt be ſubdivided into two or three parts by new ſtationary lines; taking inner ſtations at proper places, where you can have the beſt view; and meaſure theſe ſtationary lines as you did the firſt, and all their interſections with hedges, and all off-ſets to ſuch objects as appear; then you may proceed to ſurvey the adjoining fields, by taking the angles that the ſides make with the ſtationary line, at the interſections, and meaſuring the diſtances to each corner, from the interſections. For every ſtationary line will be a *baſis* to all the future operations; the ſituation of all parts being entirely dependent thereon; and therefore they ſhould be taken as long as poſſible; and are beſt to run along ſome of the hedges or boundaries of one or more fields, or to paſs through ſome of their angles. All things being determined for theſe ſtations, you muſt take more inner ſtations, and continue to divide and ſubdivide; till at laſt you come to ſingle fields; repeating the ſame work for the inner ſtations, as for the outer ones, till all be done. And cloſe the work as oft as you can, and in as few lines as poſſible. And as it may require ſome judgment to chuſe ſtations the moſt conveniently, ſo as to cauſe the leaſt labour; let the ſtationary lines run as far as you can along ſome hedges, thro' as many corners of the fields, and other remarkable points, as you can. And take notice how one field lies by ano-

Fig. ther; that you may not miſplace them in the draught.

5. An eſtate may be ſo ſituated, that the whole cannot be ſurveyed together; becauſe one part of the eſtate cannot be ſeen from another. In this caſe, you may divide it into three or four parts, and ſurvey the parts ſeparately, as if they were lands belonging to different perſons; and at laſt join them together.

6. As it is neceſſary to protact or lay down your work as you proceed in it, you muſt have a ſcale of a due length to do it by. To get ſuch a ſcale, you muſt meaſure the whole length of the eſtate in chains; then you muſt conſider how many inches long the map is to be; and from theſe you will know how many chains you muſt have in an inch, and make your ſcale, or chuſe one already made, accordingly.

7. The trees in every hedge row muſt be placed in their proper ſituation, which is ſoon done by the plane table; but may be done by the eye without an inſtrument; and being thus taken by gueſs, in a rough draught, they will be exact enough, being only to look at; except it be ſuch as are at any remarkable places, as at the ends of hedges, at ſtiles, gates, &c. and theſe muſt be meaſured. But all this need not be done till the draught be finiſhed. And obſerve in all the hedges, what ſide the gutter is on, and to whom the fences belong.

8. When you have long ſtations, you ought to have a good inſtrument to take angles with, which ſhould be exact to a quarter of a degree at leaſt; and hardly any common ſurveying inſtrument will come nearer. And tho' the plane table is not at all a proper inſtrument to ſurvey a whole lordſhip with; yet it may very properly be made uſe of, to take the ſeveral ſmall internal parts; and ſuch as cannot

be taken from the main ſtations; and is a very quick and ready inſtrument. Fig.

Example.

Walking over the lordſhip, I pitch upon the four ſtations A, B, C, D, from which I can command the greateſt part of it, there I ſet up marks. Then I meaſure along AB, which is a right line, and the boundary on one ſide of the land. In meaſuring along, I ſet down the diſtances meaſured, when I come at the corners of the fields *a*, *a*, *a*, *a*, where the hedges come in, and likewiſe where I croſs the brook *bb*. Then having got to B, I ſet down the whole length of AB. 30.

Next I meaſure from B to C, and in my way, I ſet down how far I have meaſured when I croſs the hedges at *c*, *c*, *c*, *c*; and likewiſe where I croſs the brook *bb* again. Thus I meaſure forward till I come at C, and then I ſet down the length of the ſtationary line BC.

After the ſame manner I meaſure along the ſtationary line CA, obſerving to ſet down the interſections with the hedges, as before; till I come at A, where I ſet down the length of CA. Then the three points A, B and C, are determined; and may be laid down in the plan; and all the foreſaid points.

Being come to A again, I go from A towards B, and in my way I ſurvey every ſingle field adjoining to the ſtationary line AB. To do which the ſhorteſt way, I take the angles at every interſection *a*, that the ſides of each field makes with the ſtationary line AB; and then I meaſure their lengths; by which every field is eaſily laid down. In the ſame manner I proceed from B to C, and meaſure every field adjoining to BC. And then I go to A, and meaſure every field in my way thither.

Fig. 30. Next I go from A towards D, and ſet down as before, all my croſſing of the hedges; and the length AD, when I come at D. And in like manner, I meaſure along DC, ſetting down all the croſſings of the hedges as before, with whatever elſe is remarkable, as where a highway croſſes at *d*.

Having finiſhed all the main ſtations, we muſt begin to make inner ſtations. Therefore I take F and G for two ſtations, being in the lines AB and BC, the hedges from F to G running almoſt ſtreight; then I meaſure from F towards G, and at *f*, I find a hedge going to the left, and going on to *g*, I find another hedge going to the right; and at *h* I croſs the burn. At *i* there is an angle, to which I make an offset. Going on further I come at a croſs hedge *l* going to the right; and then meaſure on to G the end of the ſtation. Now in going from F to G, we can take all the angles that the ſides of the fields make with the ſtationary line FG, and then meaſure their lengths; by which theſe fields may be laid down on paper.

Then I take another inner ſtation at I, and meaſuring from A to *o*, I come to the oppoſite corner of the field; then meaſuring on to *p*, I croſs a hedge; then I proceed to my ſtation I. Then I meaſure from I to F, and take an offset to *n* where the hedge croſſes the brook. Then I come to the corner of the laſt field at *m*; and then meaſure to the oppoſite corner at F, the other ſtation. In your going from A to I, you may take the angles that the hedges make with your ſtationary line AI, and meaſure theſe hedges, and then they may be laid down. And the like, in going from I to F.

All this being done, take a new ſtation H, and meaſuring from B towards H, all the hedges lie almoſt in a right line. So going along we come at a croſs hedge, and going further we come at a tree, in the hedge we meaſure along; going further we come at two other croſs hedges; and a piece further we croſs the brook, going on we come at a croſs hedge; going on ſtill we come at another croſs hedge; all theſe hedges are to the left. Then going on ſtill further, we have a windmill to the right; and afterwards a croſs hedge to the left, and then we meaſure on to the ſtation H. Then meaſuring from H towards C, we have a houſe on the left; and then go on to C. And the fields may be all ſurveyed as you go along BH and HC, and then laid down. And after this manner you muſt proceed thro' the whole, taking new ſtations, till all be done. Fig. 30.

PROB. XXII.

To ſurvey a whole county, or large tract of land.

1. Chuſe two, three or four eminent places for ſtations, ſuch as the tops of high hills or mountains, towers, or church ſteeples, which may be ſeen from one another; and from which moſt of the towns, and other places of note, may alſo be ſeen. And let them be as far diſtant from one another as poſſible. Upon theſe places raiſe beacons, or long poles, with flags of different colours flying at them; ſo as to be viſible from all the other ſtations.

2. At all the places, which you would put down in the map, ſet up long poles with flags at them of ſeveral colours, to diſtinguiſh the places from one another; fixing them upon the tops of church ſteeples, or the tops of houſes, or in the centers of leſſer towns. But you need not have theſe marks at many places at once, as ſuppoſe half a ſcore at a time. For when the angles have been taken, at the two ſtations, to all theſe places, the marks may be moved to new places; and ſo ſucceſſively to all the places you want. Theſe marks then being ſet up at a convenient number of places, and ſuch as may

Fig. be ſeen from both ſtations; go to one of theſe ſtations; and with an inſtrument to take angles, ſtanding at that ſtation, take all the angles between the other ſtation, and each of theſe marks, obſerving which is blue, which red, &c. and which hand they lie on; and ſet all down with their colours. Then go to the other ſtation, and take all the angles between the firſt ſtation, and each of the former marks, and ſet them down with the others; each againſt his fellow with the ſame colour. You may if you can, alſo take the angles at ſome third ſtation, which may ſerve to prove the work, if the three lines interſect in that point, where any mark ſtands. The marks muſt ſtand till the obſervations are finiſhed at both ſtations; and then they muſt be taken down, and ſet up at freſh places. And the ſame operations muſt be performed, at both ſtations, for theſe freſh places; and the like for others. Your inſtrument for taking angles, muſt be an exceeding good one, made on purpoſe with teleſcopic ſights; and of three, four or five feet radius. A circumferentor is reckoned a good inſtrument for this purpoſe.

3. And tho' it is not abſolutely neceſſary to meaſure any diſtance, becauſe any ſtationary line being laid down from any ſcale, all the other lines will be proportional thereto. Yet ſome of the lines had better be meaſured, to aſcertain the diſtances of places in miles; and to know how many geometrical miles there are in any length; and from thence to make a ſcale to meaſure any diſtance in miles. In meaſuring any diſtance, it will not be exact enough to go along the high roads, by reaſon of their turnings and windings; and hardly ever lying in a right line between the ſtations, which muſt cauſe infinite reductions, and create endleſs trouble to make it a right line; for which reaſon it can never be exact. But a better way is to meaſure in a right line with a chain, between ſtation and ſtation,

over hills and dales or level fields, and all obſtacles. Only in caſe of water, woods, towns, crags, rocks, banks, &c. where one cannot paſs; ſuch parts of the line muſt be meaſured by the methods of inacceſſible diſtances; and beſides, allowing for aſcents and deſcents, when we meet with them. And a good compaſs that ſhews the bearing of the two ſtations, will always direct you to go ſtreight, when you do not ſee the two ſtations; and in your progreſs, if you can go ſtreight, you may take offſets to any remarkable places. Likewiſe note the interſection of your ſtationary line with all roads, rivers, &c.

4. And from all your ſtations, and in your whole progreſs, be very particular in obſerving ſea coaſts, rivers mouths, towns, caſtles, houſes, churches, wind-mills, water-mills, trees, rocks, ſands, roads, bridges, fords, ferries, woods, hills, mountains, rills, brooks, parks, beacons, ſluices, floodgates, locks, &c. and in general all remarks or rarities.

5. After you have done with your firſt and main ſtationary lines, which command the whole county; then you muſt take inner ſtations, at ſome places already determined, which will divide the whole into ſeveral partitions: and from theſe ſtations you muſt determine the places of as many of the remaining towns as you can. And if any remain in that part, you muſt take more ſtations, at ſome places already determined; from which you may determine the reſt. And thus we muſt go thro' all the parts of the county; taking ſtation after ſtation, till we have determined all we want. And in general, the ſtationary diſtances muſt always paſs thro' ſuch remarkable points as have been determined before, by the former ſtations.

6. Laſtly, the poſition of the ſtationary line you meaſure, or the point of the compaſs it lies on, muſt be determined by aſtronomical obſervation.

Hang

Fig. Hang up a thread and plummet in the ſun, and obſerve when the ſhadow runs along that ſtationary line; and at that moment take the ſun's altitude; then having his declination, and the latitude; the azimuth will be found, by caſe 11th of oblique ſpherical triangles. And the azimuth is the angle the ſtationary line makes with the meridian; and therefore a meridian may eaſily be drawn thro' the map.

Or a meridian may be drawn thro' it by hanging up two threads in a line with the pole ſtar, when he is juſt north, which may be known from the aſtronomical tables. Or thus, obſerve the ſtar *Alliotb*, or that in the rump of the great bear (being that next the ſquare); or elſe *Caſſiopeia*'s *hip*; I ſay, obſerve by a line and plummet when either of theſe ſtars and the pole ſtar come into a perpendicular; and at that time they are full north. Therefore two perpendicular lines being fixt at that moment, towards theſe two ſtars, will give the poſition of the meridian.

County ſurveying is by far the moſt difficult part of ſurveying; for the ſtations are ſo long and ſo many, that if the inſtrument to take the angles, be not very accurate, great errors will be committed; eſpecially after ſo many repetitions of taking angles, at ſo many different ſtations. And to meaſure all the roads and diſtances of places, would be an endleſs labour, and beſides they would not be exact; for no road leading from one place to another, goes in a right line; ſo that any one of them, muſt meaſure to more conſiderably than what it is. Therefore I cannot find that any better method can be practiſed, than what is here laid down. For if a ſtationary diſtance, ſufficiently long be ſet out and meaſured for a baſe; and the angles taken from theſe two ſtations, to all the places that can be ſeen; the ſituation of theſe places will be had more exactly than by any other method. But when we go to theſe ſecond ſtations to take angles, more care is required; and more ſtill, if we go to third, or fourth ſtations, or farther. So that we had need to have good inſtruments to perform this. All this is a work of time and expence. Fig.

PROB. XXIII.

To ſurvey a city or great town.

Suppoſe ABCDEFG be ſeveral ſtreets in any town or city, which you want to ſurvey. Begin at the meeting of two or more of ſome of the principal ſtreets, along which you can have the longeſt proſpect, for getting the longeſt, ſtationary diſtances. Let B be the firſt ſtation; then along theſe ſtreets BC and BF, as far as you can ſee, cauſe two men to ſtand for marks; or rather ſet up ſtation ſtaves in wood pedeſtals. Then with your inſtrument placed at B, take the angle CBF, which note down. Then let the mark at F be carried to B, and the mark at C to D. Then you muſt obſerve at B a ſtreet running off to the right; and going along BC, meaſuring with a chain of 50 feet long, you come to a ſtreet at *b* on the left hand, ſet that down, and how far you have meaſured; and proceeding on to C, ſet down the length of BC in your book of obſervations. 31.

Then planting the inſtrument at C, take the angle BCD, and ſet it down. Then let the mark at B be brought to C, and that at D to E, and obſerve at C a ſtreet running off to the right; then meaſure along from C to D; and at *m* you come to two ſtreets going off, one to the left, the other to the right; then meaſure along to D, and ſet all down.

Fig. 31. Plant the inſtrument at D, and take the angle CDE, and meaſure along from D to E, and at E you have a ſtreet going to the right, and ſet all down. Bring the marks from C to D, and from E to F.

Place the inſtrument at E, and take the angle DEF, which ſet down. Then carry the marks from D to E, and from F to B; and meaſure along EF, and you come to two ſtreets at *n*, one on the right, the other on the left, which ſet down; then meaſure to F, where you have a ſtreet going to the right; ſet all down.

Laſtly, place the inſtrument at F, and take the angle EFB, which ſet down; and at F you have a ſtreet going to the right. Meaſure from F to B, and in your way you have two ſtreets at *p*, one on the right, and the other on the left; ſet them down and proceed to B; and ſet down the length FB, and then you arrive at B where you begun; ſo that part of the ſurvey is finiſhed.

After the ſame manner the part ABFG muſt be ſurveyed, taking the angles at A and B, at F and G; and meaſuring the diſtances AB, BF, FG and GA. And ſo you muſt proceed piece by piece, by this method of circulation, till the whole be finiſhed. And all the parts muſt at laſt be joined together, in their true ſituations.

Obſerve in every part to take in as much of the town as you can at a time; and to cloſe (that is, to come round to the ſame point again) with the feweſt ſides or ſtations you can, not exceeding 5 or 6, if you can help it; for the more ſides the leſs exact. Your chain muſt be 50 feet long, a link to a foot.

At every ſtation, and at other places where it is neceſſary, meaſure the breadth of the ſtreets, taking them as offsets; and as you go along, take offſets to all remarkable places, on the right and left, and to all ends of ſtreets, and to all turnings and corners; and this muſt be done with a rod 10 feet long divided into feet; for all theſe meaſures muſt be taken, and laid down in feet; and ſet all down regularly in your ſurvey book. And to help your memory, it will be convenient to have an eye draught of each part of the town, as you ſurvey it; and always take notice what way you go about. Fig. 37.

Having thus taken your high ſtreets, you may in the ſame manner proceed with the other ſmall ſtreets and lanes, noting down in your book, or eye draught, the offſets, as you find them to the right or left.

Your high ſtreets and croſs ſtreets being taken, you muſt take the meaſures of the houſes to the front, and the breadth of them, with your rod; and the dimenſions of courts and alleys, ſetting them down in your draught. You muſt be very exact and curious in taking the dimenſions of churches, the ſteeple, the buttreſſes, &c. And likewiſe halls, inns of court, colleges, eminent houſes, &c.

SECT. II.

Of Planning or Plotting of Ground; and of Caſting up, Dividing, Reducing, &c.

PROB. I.

To make a ſcale to lay down any plan with.

32. FIRST conſider the bigneſs of your ſcale, or how many equal parts you will have in an inch. Then draw the indefinite line AB, and two more parallel thereto, the one very near. Then take what length you deſign for 10 in your compaſſes; run over the line AB, with the compaſſes, as many of theſe lengths as you pleaſe. Divide the firſt part at A into 10 equal parts; and at all the other points of diviſion, write 0, 10, 20, 30, 40, &c. continued as far as you have occaſion for. And at the ends of the ſcale, and at all the points of diviſion, draw perpendiculars to AB, thro' all the three parallels, and the ſcale is finiſhed.

The uſe is this; take the length of any line you want, to lay down, off the ſcale in your compaſſes, and lay it upon any line in your plan, which you want it ſet off upon, by ſetting the compaſſes upon it; and it will reach to the point required.

And if you want to meaſure any line upon your plan; take the length of that line in your compaſſes, and apply it to the ſcale, and it will ſhew you the length.

There are other ſorts of ſcales, as diagonal ſcales, Fig. 32. where an inch or half an inch, is divided into 100 parts; but theſe are ſo well known, that I need ſay no more about them.

If you have taken a ſurvey, and want to put it into a map; you cannot lay it down, in the bounds intended, without a proper ſcale; and it will be difficult to light of one of a proper length already made; and therefore you muſt be obliged to make one yourſelf.

PROB. II.

To project or draw the plot of a triangular field.

If the field has been meaſured with the chain; 11. then you have all the ſides of the triangle ABC in chains, in your field book. Therefore draw any line AC for the baſe; and take the length of it with your compaſſes from the ſcale, and ſet it from A to C. Then take the length of AB from the ſcale, and ſetting one foot in A, deſcribe an arch towards B. Then take the third ſide BC from the ſcale, in your compaſſes, and ſetting one foot in C, croſs the former arch in B. Then from the point of interſection, draw the lines BA, BC; and BAC is the triangle required.

If the perpendicular and baſe be known, and either ſegment AP. You muſt ſet off AP equal to that ſegment, and raiſe the perpendicular PB of the length given, and draw BA and BC.

Otherwiſe, from an angle or two.

If you have two ſides AC, CB; and the angle ACB. Draw AC, and with the compaſſes ſet off its length in chains from the ſcale. Then by the protractor, or line of chords, make the angle ACB of the given degrees; and draw CB, on which ſet off its length, (from the ſcale) to B; and draw AB.

Fig. 11. Or if the ſide AC, and the angles A and C be given. Draw AC of its due length taken off the ſcale. Then make two angles at A and C, equal to theſe given; and draw AB, CB to interſect in B.

Laſtly, if you have given AC, AD, DB, and angle ADB. Make the angle ADB equal to that given. And DA, DB of the lengths given; and AC equal the given baſe. And draw BC, BA.

Cor. *This is the foundation of deſcribing any poligon; for they may always be divided into triangles. And all the triangles may be deſcribed one after another, as above.*

PROB. III.

To lay down the plan of any field upon paper; from the meaſures taken in the field; and plotting offsets.

15. If the field has been divided into triangles, and all the ſides have been meaſured with the chain; as ABCDE; where the lengths of the ſides are ſet down in the field book, or elſe written upon the ſides, in a foul draught. Draw any line DE, and from the ſcale ſet off its length in chains, from D to E. Then with the length of EA and one foot in E, deſcribe an arch; and with the length of DA and one foot in D, croſs that arch in A; then A is another angle of the field. In like manner having the three ſides AD, DB, AB, lay down the triangle ADB, gives the angle B. And from the three ſides DC, DB, BC, lay down the triangle DBC, and we have all the ſides and angles of the figure.

16. But if all the lines are drawn from a point within as F. Then with the three ſides DE, EF, DF, lay down the triangle DEF; and in like manner the triangle EAF; then ABF; then BCF; and laſtly CDF. For in each triangle, there is given all the three ſides.

If the ſides and included angles are given, as in the figures ABD. Draw the line AS of the length given; and make the angle ASB equal to the given degrees; drawing SB, making it of its due length, and draw AB; then you have the triangle SAB. Then from the ſides SB, SC and the included angle BSC, or elſe ASC, you will project the triangle BSC, in the ſame manner; as alſo CSD; and ſo on till the whole figure is laid down. Fig. 20. 21.

When all the ſides round the figure are given, and an angle oppoſite to each of them, as in the figure ACE. Firſt draw the ſide EA from the given angle, and ſet off its length to A. Then make the angle AEB equal to that given, drawing EB. Then take AB in your compaſſes, and with one foot in A, croſs the line EB in B, and draw AB. Then make the angle AEC or BEC equal to the angle given; and with BC in your compaſſes and one foot in B, croſs the line EC in C; and draw BC. Then make the angle AED or CED equal to that given; and take the length of CD in your compaſſes, and with one foot in C, croſs the line ED in D, and draw CD; and you have laid down the whole figure. But *note*, you ought to know whether the angles ABE, EBC, ECD be acute or obtuſe. 22.

When all the angles at one point or ſtation, oppoſite to the ſides of the field are given; and all the alternate angles round the field; and only one line FA meaſured. Draw the line FA of its due length; and make the angles AFB and FAB as given in your book, or foul draught; and the interſection of FB, AB, gives the point B. Again, make the angles BFC, FBC equal to their given angles, and BC, FC will interſect in C. Proceed thus round the figure, to find the points of interſection D and E. 23. But if the ſtation be taken within the field at S, and FS meaſured. Then SF, and 24.

Fig. 24. the two angles at S and F, will determine the point A. Then SA, and the angles at S and A, will determine the point B. And the like for the points C, D, E. And at laſt you will have the whole figure.

25. When all the ſides and angles, in going round it, are meaſured, as in the figure ABCDE; where *a*, *b*, *c*, *d*, *e*, are the ſeveral ſtations. Make the angle *eab* equal to that given, and make *ab* equal to its given length. Then make the angle *abc* equal to its degrees, and *bc* equal to its number of chains. In like manner make the angles *bcd*, *cde*, *dea*, of their reſpective number of degrees; and the ſides *cd*, *de*, *ea*, of their reſpective lengths in chains; and the figure is deſcribed, all but the offsets.

And to deſcribe and plan all the offsets. At *a* ſet off the diſtances to the hedges at A. Alſo ſet off, in the line *ab*, the diſtances *af*, *ag*, *ah*, *ai*; and at *f*, *g*, *h*, *i*, raiſe perpendiculars, to *ab*, of their proper lengths, all taken from the field book; and at *b* make the offset to the hedge at B. Then thro' A and B, and the ends of all theſe perpendiculars, draw the hedge AB. Then at *b* and *c*, make offsets to B and C, and ſet off *bk* to *k* where it touches the hedge; and draw the hedge B*k*C. Then make the proper offsets at *c* and *d*, to C and D; and draw the hedge CD. Again, make an offset at *d* to D, and ſet off *dl*, where it touches the hedge; and *dm*, and at *m*, the offset; and at *e* an offset to E. Then draw D*l* and *l*E thro' the end of the perpendicular at *m*, for another hedge. Laſtly, ſet off two offsets from *e* to E, and from *a* to A, and draw the hedge EA. Then you have the true figure of the field laid down.

3. Again, let AB be any ſtationary line. Set off from A, the diſtances A*c*, A*d*, A*f*, AB, from the book. And on the points *c*, *d*, *f*, B, raiſe the

perpendiculars *co*, *de*, *fg*, B*h*, of their due lengths, taken from the book; and thro' the ends *o*, *e*, *g*, *h*, draw the hedge or boundary A*oegh*. And the like for any others. Fig.

General Obſervations.

All the lines belonging to the ſame figure muſt be taken from the ſame ſcale, with a pair of compaſſes; and it is beſt to uſe a diagonal ſcale, becauſe the links can be expreſſed by hundredth parts; and the ſcale ought to be as large as the paper will permit, that it may contain the whole draught, and the bigger it is, the more exactly you will work. All offsets belonging to each line, muſt be ſet in their proper places, and of their due lengths, perpendicular to the ſtation line; and to be taken off the ſame ſcale. And every line that is inclined to the horizon (as in a hill ſide), muſt be firſt reduced to a horizontal line, before it be laid down.

And all angles muſt be laid down with a protractor or line of chords of a good radius. And theſe things muſt be laid down in their proper poſitions, places, and magnitudes; whether they be lines, angles, or offsets, or any remarkable objects that are to be determined; and all according to the directions of your field book, or foul draught, you plan it from. And all this according to the known rules of practical Geometry. And at laſt, lines drawn thro' all the points thus determined, will give the true figure of the field.

PROB. IV.

To plan a field, as ſurvey'd by the infallible.

Let PQ be the table with the paper of obſervations; where there was but one ſtation, taken in the middle of the field. Unfold the paper of ob- 17. 18.

Fig. ſervation, and upon a vacant place of the ſame pa-
17. per, take a point S for your ſtation. Then lay one
18. ſide of a parallel ruler upon the line 1, ſet the other ſide to the point S; and by its edge, draw the line SA, always the ſame way from S; that the line 1 lies from the center of the table; and ſet from S to A its length 16.20 chains. Then ſet one ſide of the rule to the line 2, and the other ſide to S, and draw the line SB equal to 10.12 chains. Then ſet to the line 3, and to S, and draw SC 14.68. Then ſet to the line 4 and to S, and draw SD 12.90. Then ſet to the line 5 and to S, and draw SE 7.25. Then ſet to the line 6 and to S, and draw SF 8.84. Then ſet to the line 7, and to S, and draw SG 14.65. Then draw the lines ABCDEFGA for the field.

18. But if the ſurvey was taken by going round the
19. field. Let RT be the table. Unfold the paper, and chuſe any point A for the beginning. Lay one ſide of a parallel ruler upon the line 1, and ſet the other to the point A, and draw the line AB by its edge, to lye the ſame way from A, that the line 1 lies from the center. This muſt be obſerved in drawing all the lines; and make AB 13.10 chains. Set one ſide of the rule to the line 2, and the other to B, and draw the line BC, 6.18. Set one ſide to the line 3, and the other to C, and draw CD 17.75. Set one ſide to the line 4, and the other to D, and draw DE 10.32. Set to the line 5, and to E, and draw EF 10.45. Set to the line 6 and to F, and draw FG 9.00. Set to the line 7, and to G, and draw GA, which ought to be equal to the meaſured length 8.15, if the work is right. So your field is planned.

PROB.

PROB. V.

To lay down the plot of a field, as ſurveyed by the needle.

26. Let ACF be a field ſurveyed by the needle, going round about, taking the ſtations at *a*, *b*, *c*, *d*, *e*. Thro' all the ſtations draw the meridians *am*, *bm*, *cm*, *dm*, *em*, as you come at them. Then beginning at *a*, make the angle *mab* equal to the bearing of the ſide *ab*, north-eaſterly, and thereon ſet its length in chains to *b*. The meridian *bm* being ſuppoſed to be drawn, make the angle *mbc* the bearing of *bc* ſouth-eaſterly, and make *bc* its length. Make the angle *mcd* the bearing of *d* ſouth-weſterly, and make *cd* its diſtance. Make the angle *mde* the bearing of *e*, ſouth-weſterly, and *de* its diſtance. Make the angle *mea* the bearing of *a* north-weſterly, and and *ea* its diſtance. Then the plan is finiſhed, excepting the offsets, which are laid down as directed in Prob. III. Thus, in going from *a* to *b*, you have two offsets at *s* and *t*, at B you touch the hedge: From *b* to *c*, you have four offsets at *b*, *f*, *h*, *i*, and you touch the hedge at C. From *c* to *d*, one offſet at *k*, at E touch the hedge. From *d* to *e*, five offsets at *m*, *n*, F, G, H; at *p* touch the hedge. From *e* to *a*, two offsets at H and A. The places of all theſe, muſt be marked upon the ſtationary lines; and their lengths ſet off perpendicular thereto.

Fig.

PROB. VI.

To lay down the plan of a field, ſurveyed from two ſtations, by taking the angles round about, and meaſuring only the line between the ſtations.

27. Chuſe a convenient point upon your paper, for your firſt ſtation, as S. Thro' S draw a line, and ſet thereon the ſtationary diſtance ST; then T is your ſecond ſtation. Then with a protractor or line of chords, make the angles TSA, TSB, TSC, TSD, TSE and TSF, each of the number of degrees, as you find them in your field book; and from S draw the lines 1, 2, 3, 4, 5, 6, out at length, numbering them as in the book. In like manner, make the angles at T your ſecond ſtation, STA, STB, STC, STD, STE, STF; and from T draw the lines 1, 2, 3, 4, 5, 6, as in the book. Then obſerve where the correſpondent lines interſect; as 1 and 1 interſect at A; 2 and 2 interſect at B; 3 and 3 at C; 4 and 4 at D; 5 and 5 at E; and 6 and 6 at F. Then A, B, C, D, E, F, are the corners of the field. Therefore drawing the lines, AB, BC, CD, &c. from angle to angle, ABCDEF will be the plot of the field. Here you muſt be ſure to take the interſections of the correſpondent lines, or thoſe numbered with the ſame figures; otherwiſe you'll get a falſe plan.

28. If the ſtationary diſtance be taken in a ſide of the field, as EF, or even out of the field as *st*; the work would have been the very ſame. For the correſpondent lines 11, 22, 33, &c. that form the angles which you muſt make about F and E, or *s* and *t*, will always interſect in the points A, B, C, &c. which determine the angles of the field.

And if there had been more ſtations, proceed Fig. in the ſame manner, to lay down the angles taken at them; and to draw lines, to interſect in the points required.

PROB. VII.

To lay down the plan of any part of a country, with ſuch objects as appear; by taking obſervations at ſeveral ſtations.

This is done much the ſame way as the laſt Prob. 29. only there are more ſtations concerned, which require to be laid down, one after another; and the diſtances to the objects meaſured.

Upon a ſheet of paper, take a convenient point S for your firſt ſtation, from which draw the line ST, and ſet thereon the ſtationary diſtance ST as in your book; then T is your ſecond ſtation. With a protractor or line of chords, ſet off the angles TSK, TSL, TSA, TSB, and TSC, as in your book. And draw the lines SK, SL, SA, SB, SC; and thereon ſet the meaſured diſtances of each, as you find them in the book. Then the places of the objects K, L, A, B, C are determined.

Then in like manner at T, make the angles STD, STE, STF, STI, equal to the angles which the ſtationary line makes with TD, TE, &c. as you find them in the book; and drawing TD, TE, TF, TI, ſet thereon, the meaſured diſtances of D, E, F, I; and then their places are determined.

Next make the angle STV equal to the obſerved angle between the ſtationary lines ST and TV; and drawing TV, ſet thereon the meaſured diſtance of the ſtations T and V, as ſet down; then V is a third ſtation. Therefore at V, make the angles with TV as you obſerved them, as TVG, TVH, and perhaps TVI; and on VG, VH, ſet their meaſured

Fig. ſured diſtances, all which you have in the field book; 29. and then the places of G and H are determined. And if you have more ſtations, and more objects, you muſt proceed in the ſame manner, to lay down the angles obſerved, and to ſet off the diſtances. And thus may the plan of a ſquadron of ſhips be taken, or the map of a river, or any remarkable objects, tho' inacceſſible.

PROB. VIII.

To draw the plan of a manor or large eſtate; part of a river, or a great town.

1. Having provided ſome paper, vellum or parchment to draw it upon; conſider how large you deſign to have your map. Then collect from your field book, the dimenſions of the eſtate in chains; and from hence you may make an eſtimate, how many chains will be contained in an inch; and conſequently what ſize your ſcale muſt be of, to plot it with; and it will be beſt to uſe a diagonal ſcale.

30. 2. Then take a convenient point A upon your paper for the firſt ſtation, and from thence draw your firſt ſtationary line AB, of its due length, from the field book; then will B be your ſecond ſtation. Then having the diſtances AC and BC, you will find the place C of a third ſtation, upon the plan. And in like manner, having CD and AD, the point D will be found for a fourth ſtation. For the longeſt ſtationary lines, and the moſt general ſtations muſt be determined firſt; and then the next depending on them, and ſo on to the leaſt or moſt minute. And ſuch lines as ſerve only to find the boundaries, muſt be drawn with black lead, to be rubbed out again.

3. Then for plotting your obſervations taken upon any of theſe lines; you muſt take the ſeveral

diſtances from your diagonal ſcale, and ſet them upon Fig. 30. theſe lines; or with a thin ſcale graduated the ſame way, and applied to theſe lines, prick off theſe diſtances, by the edge. Likewiſe every angle muſt be laid down truly by a protractor or line of chords. And all theſe things muſt be done in that order you took them, as directed by your field book; firſt reducing inclined lines to horizontal ones, by Prob. 9, Sect. 1. Thus in going from A to B, when we come at *a* there is a hedge; therefore ſet the diſtance A*a* from A to *a*; and at *a* lay down the angle that the hedge *ao* makes with A*a*, and ſet the length of it from *a* to *o*, for the corner of the firſt field. Then I ſet off the diſtance from A to the next hedge, and draw that hedge in like manner as the other. Then I ſet off the diſtance of the brook from A to *b*, and draw the brook, making ſuch an angle with AB as I find in my book. In this manner I proceed till all be laid down from A to B. After the ſame manner I lay down every thing from B to C, and from C to A; and likewiſe from A to D, and from D to C. Theſe general ſtations being finiſhed; the inner and leſſer ones muſt be plotted, as from F to G, from A to I, from I to F. Then from B towards H, where I project a tree, and the croſſing of the brook, and a windmill on the right; then from H to C, where I project a houſe on the left hand. Thus you muſt go thro' all the leſſer ſtations, planning every thing you find in your book, in their due places; as all objects, and all offſets to hedges, &c. till all houſes, fields, trees, and other remarkable objects, be put into your plan; proceeding from one part to another till all the parts be laid down; and if all the parts cloſe or fill up the plan, it is a ſign the work is right. And the plan is beſt done upon paper, or at laſt pricked off upon your vellum, by laying the paper upon it. And make always the north

Fig. ſide of the map the uppermoſt. To do all this
30. there is required no more than a little practical Geometry, attending duly to what you have given in your field book. Sometimes it may be convenient to plot different parts in different papers; and put them altogether at laſt.

4. All hills muſt be ſhadowed, to diſtinguiſh them in the draught; likewiſe ſhadow the buttreſſes of ſteeples very deep; and alſo towers, caſtles, houſes, and all ſuch things.

5. Put the trees in each hedge, into their proper places in the map; as they have been taken on a foul draught; and make prick lines on the outſides of the hedges, for the gutters; and theſe will ſhew what hedges belong to each field. And where there are gates, the hedges muſt break off.

6. All the fields and cloſes being laid down in their true order and ſituation; write the name of each field ſomewhere in the middle of it; and if you think proper, the number of acres it contains: tho' ſome people chooſe to have their contents in a book on purpoſe.

7. Colour the hedges with different colours, and put into your map, the manor houſe, and all other out-houſes, barns, ſtables; alſo rivers, roads, bye-lanes; and any windmill, water-mill, bridge brook, pond, wood, coppice, foot-way, or horſeway. Draw trees in woody grounds, ſhadow mountainous grounds with hills and valleys. Expreſs bogs, fens, groves, highways, rivers, gates, ſtiles, &c. Draw the meridian line, and put a mariners compaſs in ſome bye place; and a ſcale of equal parts or chains; and laſtly the title of the map, thus.

A true and perfect ſurvey of the manor
(Townſhip) of ——, in the county of ——,
Being (part of) the eſtate of ——
Anno Domini——.

8. In a very great eſtate, there ought to be drawn vertical and horizontal lines thro' the map, denoted by letters at the top and bottom and ſides; for the ready finding any field therein, mentioned in a table. Fig. 30.

9. The ſame rules, by which any field is laid down, in the method of circulation, will equally ſerve for laying down the ſtreets of a town. For take the point B for the firſt ſtation, and draw BF, and make the angle FBC equal to the degrees obſerved, as in the ſurvey book; and drawing BC, ſet its length upon it to C, taken from the book. And in that line from B to C, make offsets on each ſide, and ſet off ſuch diſtances as you have noted down. In the ſame manner, ſet off the angles BCD, CDE, DEF, and EFB. And ſet off the diſtances CD, DE, EF, FB; and make ſuch offſets on each, as you found in going along them; as the ſurvey book will direct. Then draw the inner ſtreets, till all the part BCDEF be planned. In like manner lay down the part BFGA, and ſo one part after another, till the whole is laid down. But it is beſt to lay down every part as ſoon as it is ſurveyed, whilſt you have it in your mind. 31.

PROB. IX.

To draw a map or plan of a whole county.

The laying down the plan of a county is upon the ſame principle, as laying down a ſurvey, taken from two ſtations, as delivered in the 6th Problem. Only here we are obliged to have more ſtations, and to take more angles. And as a county is of a large extent, every thing muſt be laid down with the utmoſt exactneſs. Therefore to do this, chuſe a convenient point upon your paper for your firſt ſtation. From that point draw your ſtationary line, and

Fig. 31. and ſet upon it, from that point, the diſtance meaſured between your two main ſtations, taken from a proper ſcale; and this gives your ſecond ſtation upon the paper. And the diſtance between them is the only meaſured diſtance you have. Then take a good inſtrument for laying down angles, and place its center at the firſt ſtation; and make an angle there with the ſtationary line, equal to the angle you took (to any certain place) at the firſt obſervation, when at the firſt ſtation; which your book will ſhew. Then make an angle with the ſtationary line, at the ſecond ſtation, equal to that you took (to the ſame place) at your ſecond ſtation; where this line (terminating the angle) cuts the other, is the place at which that particular mark ſtands: thus one place is determined. Again, at the firſt ſtation, make an angle with the ſtationary line; equal to that you obſerved, to a ſecond place. And likewiſe an angle, at the ſecond ſtation, which was obſerved there to the ſame place; and where this line interſects the firſt, is the place of the ſecond mark; ſo a ſecond place is determined. And thus go on, to lay down two correſpondent angles, taken out of the book, one at one ſtation, and the other at the other; and the interſection of the lines of that angle, gives the correſpondent place. And this work is always to be done both at the main ſtations, and at all inner ſtations.

1. All the lines ought to be taken from a diagonal ſcale, of a mile or half a mile to an inch, according to the largeneſs of the county; which may be eaſily collected from the dimenſions of the deſigned map, compared with the dimenſions of the county, in miles.

2. All your work muſt be laid down on a paper ſpread upon a flat table, and paſted down at the ends, or otherways fixt, ſo as not to be altered, till the whole be laid down. Or one may take a fine

copper plate, the bigneſs of the map, to project upon; and one ſet of obſervations may be laid down after another, as they are taken at the two ſtations; till the whole be compleated; and the lines may be drawn with a fine ſteel point. But here ſuch obſervations as are taken to one hand, muſt be laid to the contrary hand, as in all engraving.

3. And the inſtruments made uſe of in laying down the plan, muſt be exceeding good and large. Your protractor or line of chords, or ſector, or whatever inſtrument you uſe to lay down angles, muſt be of as large a radius, as the inſtrument you took the obſervations with, and as accurately divided. For moſt of your operations will be to lay down angles; and if they be not as exactly laid down in your map, as they are taken by obſervation, the map will be erroneous. And you might as well not have taken them exactly, as not lay them down exactly. Here the diſtances muſt all be reduced to horizontal lines, before they be laid down. And all the lines and angles are beſt laid down in the order they were taken; as your book of obſervations will ſhew. And the principal and moſt eminent ſtations muſt be planned firſt; and then others that depend immediately upon them; and the next, thoſe that depend upon theſe; and ſo on. And thus every thing muſt be laid down, according to what you find in your book.

4. When all the towns and remarkable places are thus determined and mapped. The hills and mountains muſt be ſhadowed, the ſea coaſts drawn; all towns, caſtles, houſes, churches exactly drawn, the largeſt of them in perſpective. All roads and rivers delineated, and all bridges drawn; and every thing properly ſhaded, to diſtinguiſh them and ſhew what they are. All the diviſions of the county muſt be drawn in prickt lines. And the name of each part written therein. And the boundaries

Fig. will be more viſible and diſtinct, if they be colou ed with water colours. All this is a work of gre nicety.

5. Write the names of each diviſion in great le ters within it; and the names of all towns and ther places, beſide them. The cities muſt be di tinguiſhed from market towns, and theſe from leſſe towns, &c. by certain marks. Meridians and pa rallels muſt be drawn through the map; alſo a ma riners compaſs in ſome bye place; and a ſcale o miles; the longitude and latitude of places at th ſides and ends, and the title of the map in large characters. And in all maps, take notice, that the north ſide muſt be uppermoſt.

PROB. X.

To reduce one triangle into another of equal area, to have a given baſe, or a given perpendicular.

33. Let ABC be the triangle to be reduced, and AD the new baſe. From the vertex C draw the line CD, and from B the end of the baſe, draw BE parallel to it, interſecting the oppoſite ſide in E; then draw ED, and the triangle AED is equal to the triangle ACB. For the part EBD is equal to the part EBC, being contained between the ſame parallels EB, CD; and the part AEB is common to both triangles.

And if the perpendicular be given; let it cut the ſide at the point E. Draw EB to the oppoſite angle B; and CD parallel to it, from the vertex C, to cut the baſe AB in D; draw ED, then AED is the triangle required; which is proved as before.

Fig.

PROB. XI.

To reduce one triangle to another equal triangle, from a given point in the ſide.

Let ABC be the triangle, and E the point given. 33. From the given point E, to the oppoſite angle B, draw the line EB. And from the vertex C, the line CD parallel to it, to cut the baſe AB in D. Draw the line ED, and the triangle AED is equal to the triangle ACB.

For the triangle EDB is equal to the triangle ECB, being contained between the ſame parallels; therefore to each of them add the triangle AEB, and then AED is equal to ACB.

SCHOL.

This Problem will be uſeful, when the area is not to run beyond a given point E.

PROB. XII.

To reduce a poligon to an equal triangle, from any angle, or point given in a ſide.

This is done by taking away one angle after another, till the whole be reduced. Let the poligon be ABCDEF, and let the point be at the angle D. 34. Produce any ſide, that does not join upon D, as AF. Draw the firſt diagonal AC, and BH parallel to it, to cut the baſe FA in H. Draw HD to the next angle, and CI parallel to it, cutting AF in I. And draw DI for one ſide. Here AC, BH, HD, &c. muſt be obſcure lines.

Again, draw DF, and EG parallel to it, to cut the baſe AF in G; then draw DG for the other ſide.

Fig. *In ſhort thus.*

34. From AC open your parallel ruler to B, to cut AF at H; From HD open to C, to cut at I; and draw ID.

From DF open to E to cut at G, and draw DG. Then IDG is equal to ABCDEF.

For by reaſon of the parallels CA, BH, the triangle CBA is equal to the triangle CHA. Therefore inſtead of the two ſides CB, BA; the ſingle ſide CH may be taken. Again, by reaſon of the parallels DH, CI; the triangle DHC is equal to the triangle DHI; and therefore inſtead of the two ſides DC, CH; the ſingle ſide DI may be taken. In like manner the triangle DEF is equal to DGF; whence the triangle IDG is equal to the figure ABCDEF.

Cor. 1. *Hence any poligon may be reduced to fewer ſides, by taking away one or more ſides of it.*

Thus ABCDEF is reduced to HCDEF, and that to IDEF, and that to IDG.

Cor. 2. *Hence a triangle may be reduced back to a polygon, or a polygon to another of more ſides. Which is uſeful in reduction, when the area is to terminate at a certain point, as* F.

Thus to reduce GD to the point F. Draw from F, to the next angle D, the line FD; and from G, the parallel GE, to cut the other ſide DE in E, if there be ſuch a ſide; otherwiſe take any point E in the parallel GE, and draw the two lines DE, EF.

Likewiſe to reduce DI to the point H; draw HD to the next angle D, and IC parallel to it, in which take the point C, and draw CD, CH.

Again, to reduce CH to the point A: draw AC to the firſt angle C, and HB parallel to it, and take B therein, and draw CB, BA.

SCHOL.

Fig. 34.

SCHOL.

It will be beſt to take ſuch a ſide for the baſe, and ſuch a point for the vertex, of the triangle (which may be in a ſide, as well as an angle), that when the operation is finiſhed, the height and baſe of the triangle may not be very unequal; obſerving that the higher the vertex is, the ſhorter the baſe will be. If the point for the vertex be taken in a ſide, it is the ſame thing, as if the poligon had one ſide more.

PROB. XIII.

To caſt up the content of a triangular field.

Let ABC be the triangle propoſed. Having got the baſe AC, and perpendicular BP, either by meaſuring them in the field; or elſe meaſuring them by the ſcale, upon the draught, laid down upon paper. Then 11.

Multiply the baſe by the perpendicular, in chains and decimals. And take half the product for the ſquare chains, which divide by 10, gives the acres, and decimal parts: or, which is the ſame thing, from half the product, cut off five figures for a decimal. If you have a mind to find the roods and perches, multiply the decimal, firſt by 4, and then by 40.

Note, you may multiply half the baſe by the perpendicular, or the baſe by half the perpendicular, for the content in chains, &c.

Fig. 11.

Examp.

Baſe AC	8.27
Perpen. BP	4.25
	4135
	1654
	3308
Prod. —	35.1475
10)	17.5737
Content	1.757 acres.

Or thus.

When two ſides AC, CB, and the included angle C, are given; or when AC, DB, and angle ADB are given.

Add together the logarithms of AC, CB, and ſine of the angle C; or elſe of AC, DB, and S,ADB; and from the ſum abate 10. Find the number anſwering to that logarithm. Then half that number divided by 10, is the content in acres.

Examp.

AC	8.27	—	0.91750
CB	6.88	—	0.83758
S.∠C	38 20	—	9.79255
	35.29	—	1.54763
Content	1.764 acres.		

Or thus.

When a ſide AC, and the adjoining angles A, C, are given. To twice the log. baſe AC, add the log. ſines of the angles A and C; from the ſum, take the log. ſine of the ſum of theſe angles (or of the third angle B), and throw away 10. Find the number of this logarithm; and divide half of it by 10, for the content. Fig. 11.

Examp.

AC — 8.27	0.91750
	0.91750
S.<C 38 20	9.79255
S.<A 55 30	9.91599
	21.54354
S.<B 86 10	9.99902
35.04	1.54452

Content 1.752 acres.

Or thus, by the laſt data.

Divide the ſquare of the baſe AC, by the ſum of the natural cotangents of the angles A, C. Half the quotient divided by 10, gives the area.

8.27	1.2647			
8.27	.6873			
5789	1.9520	)68.393	(35.04	
1654		58 560	1.752	content
6616				in acres.
		9 8330		
68.3929		9 7600		
		73000		

Otherwiſe thus.

When the three ſides are given. Find half the ſum of the three ſides, and ſubtract each ſide from it. Then multiply that half ſum and the three re-

Fig. mainders continually; and the ſquare root of the
11. laſt product, divided by 10, gives the area.

Rather add their four logarithms together, and take half of the ſum; and the number being found, the tenth part of it is the area in acres.

Examp.

AC	8.27,	
CB	6.88	
AB	5.14	
	20.29	Log.
½ ſum	10.14	1.00604
1 diff.	1.87	0.27184
2 diff.	3.27	0.51454
3 diff.	5.00	0.69897
		2.49139
	17.61	1.24569
Content	1.761 acres.	

PROB. XIV.

To caſt up the content of a ſquare, a rectangle, or of any parallelogram.

Multiply the length, by the breadth (which is the neareſt diſtance from one ſide to the other); and the product, divided by 10, is the area.

Examp.

Fig. 11.

Examp.

Length 16.27
Breadth 10.60

97620
16270

172.4620

Content 17.246 acres, which may be reduced to roods and perches, by multiplying by 4 and 40.

PROB. XV.

To caſt up the content of a trapezium, ABCD.

Having found the diagonal BD, and perpendiculars AF, CG, either by meaſuring in the field, or upon paper, after it is laid down. Then 12.

Multiply the diagonal, by the ſum of the perpendiculars; and half the product, divided by 10, is the content.

Examp.

Suppoſe the diagonal and perpendiculars, the ſame as were meaſured in Prob. XII. Sect. I.

1 perp. 5.18 Diag. 8.37
2 perp. 4.86 10.04

10.04 33 48
83700

84.0348

Content 4.202 acres.

Or thus.

13. Where the lengths of the diagonals, and the angle they make, are given. Multiply the diagonals and the nat. ſine of the angle, continually; and the product divided by 10, gives the content.

Or rather, add the logarithms of the diagonals, and the log. ſine of the angle together, rejecting 10; find the number belonging; and divide half of it by 10 (or the whole by 20), gives the content.

Examp.

Suppoſe the angle and diagonals, as meaſured in Prob. XII. Sect. I.

AC,	7.54,	0.87737
DB,	6.17,	0.79028
S.AEB,	66 20,	9.96184
	42.61,	1.62949

Content 2.130 acres.

PROB. XVI.

To caſt up the content of any field, in form of any polygon.

14. Since we do not always want to make a draught of a field, but only to find its content; in ſuch caſes, we only meaſure ſuch lines as are neceſſary for finding the content; and then fewer meaſures will do, than for plotting.

The ſhorteſt way then to find the content, is to divide the field into trapeziums, as few as you can, or into trapeziums and triangles; and to meaſure the diagonals and perpendiculars in the field, and ſet them down; and likewiſe the baſes and perpendiculars of the triangles, if there be any. Then find the contents of each of the trapeziums, by the laſt Prob. and of the triangles, by Prob. XIII. and the ſum of all is the content of the field. Fig. 14.

Examp.

Let the meaſures be taken as in Prob. XIII. Sect. I.

1. *Trapezium* CDEF.

1 perp.	3.54	Diag.	11.88
2 perp.	2.52		6.06
	6.06		7128
			71280
			71.9928

2. *Trapezium* BCFG.

1 perp.	4.47		8.19
2 perp.	3.72	Diag.	12.00
	8.19		1638
			819
			98.28

The triangle ABG.

baſe	10.56		71.9928
perp.	3.32		98.28
			35.0592
	2112		205.3320
	3168	Content	10.2666 acres.
	3168		
	35.0592		

Otherwiſe

Fig.

Otherwiſe thus.

14. When we have not got the diagonals and perpendiculars, requiſite for caſting up. The field muſt be laid down on paper, by Prob. III. And then, by dividing it into trapezia and triangles, we muſt meaſure theſe as before. Therefore what we have to do, is, to meaſure the lengths of theſe diagonals, and baſes (with the ſame ſcale the field was drawn by) in chains and decimal parts; and their correſpondent perpendiculars, by taking the neareſt diſtance from the vertex or angle, to the baſe or diagonal.

Then multiply the diagonal of each trapezium by the ſum of its perpendiculars; and the baſe of every triangle by its perpendicular; and take half the ſum of all theſe products, which divide by 10, gives the acres and decimals. And the decimal may be reduced to roods and perches, if you pleaſe, by multiplying by 4 and then by 40.

And *note*, if any part of the area caſt up is without the field, ſuch part muſt be ſubtracted from the whole.

As for the offſets, and parts of irregular curves, if there be any; they muſt be computed ſeparately, and added to the former contents, if they lie without the ſtationary lines; or ſubtracted, if within. To find their contents will be ſhewn in the following Problem.

Otherwiſe thus.

34. Reduce the field into a triangle by Prob. XII. And if any ſide in it be very irregular, running in and out, a right line may be drawn by the eye, that takes in, as much as it leaves out. Then the baſe of this triangle multiplied by its height, and half the product divided by 10, will give the area in acres, as uſual.

This reducing a field to a triangle, is as exact a way as caſting up the contents of all the trapeziums and triangles ſeverally; and far more eaſily and ſpeedily done. Fig. 34.

Examp.

Let ABCDEF *be any field; which is reduced to the triangle* IDG. *And let the base* IG *be* 9.00, *and the perp. or the neareſt diſtance from* D *to* AF 6.46.

6.46
9.00

58.14

Content 2.907 acres in the triangle.

Therefore the content of the polygon is 2.907 acres.

SCHOL.

In uneven hilly ground it has been diſputed, whether the horizontal baſe, or the true ſuperficies, ought to be computed; and the greateſt part of ſurveyors ſeem to be agreed, that the ſuperficies ſhould be meaſured. And indeed, ſince ſurveying is but a mechanical practice, we may lay down this general rule, that ſince we meaſure by the chain, we muſt compute by ſuch dimenſions as we find by the chain. But if we compute from a map or draught, we muſt needs take our meaſures from the map, to compute by. And in theſe caſes there will always be a difference between the area meaſured and the area planned; more or leſs, as the ground is more or leſs hilly and irregular; and ſubject to aſcents and deſcents, and to hills and dales. But tho' the ſuperficies ought to be meaſured, only the horizontal baſe muſt be planned; for otherwiſe a map cannot be made too cloſe, *i. e.* for the adjoining parts to come together.

It

Fig. 34. It may be obſerved, that ſince all ground is more or leſs irregular, as well as the fences that incloſe it; therefore if ſeveral perſons ſurvey the ſame piece of ground; no two of them will give exactly the ſame content. Nay, if even the ſame perſon ſurveys it twice, he will differ from himſelf more or leſs; eſpecially if he ſurveys it by different methods.

If any geometrical curve is given to be ſurveyed; you muſt meaſure it by the rules of menſuration, for that purpoſe; for I cannot here lay down rules for all ſorts of figures, but ſuch as are common, and ſuch as come in practice every day. But for geometrical curves of all ſorts, I muſt refer to that book. But theſe things rarely come in queſtion.

PROB. XVII.

To caſt up the contents of offsets and parts of irregular curves.

1. The parts of a field contained by offsets are trapezoids and triangles; and therefore they muſt be meaſured as ſuch. And to find their contents, multiply each part of the ſtationary line by the ſum of the offsets on each end of it, and this being repeated as often as there are offsets; add them all together, and take half the ſum, and divide it by 10, gives the whole area contained by theſe offsets; which muſt be added or ſubtracted, according as they lay within or without the field.

If the offsets are at equal diſtances, find a mean offset, by dividing the ſum of them all, by their number. Then multiply this mean offset by the whole baſe, for the content in chains, which divide by 10.

Examp.

Fig. 3.

Examp.

Suppoſe AB *be the ſtationary line, and* Aoegb *the hedge; and let the offsets be as in* Prob. III. Sect. I. *To find the area* ABbgeoA.

Offset at A	0.	Offset at *c*,	0.87
at *c*,	0.87	at *d*,	0.23
	0.87		1.10
A*c*	1.94	*cd*,	1.48
	3 48		880
	783		440
	87		110
	1.6878		1.6280

Offset at *d*,	0,23	Offset at *f*,	0.81
at *f*,	0,81	at B,	0.47
	1.04		1.28
df,	.75	*f*B,	.28
	520		10 24
	728		25 6
	.7800		.35 84
			1.68 78
			1.62 80
			.78 00
			4.4542

Content .2227 in acres, for the area of AB*bgeo*.

And after the ſame manner muſt all the offset areas be found in the field ADF, Fig. 26, and ſubtracted from the whole content, as they lay without the field.

2. If

Fig. 3. 2. If any part of the area be bounded by a regular curve, and the height be ſmall, multiply the baſe by the height, and take $\frac{2}{3}$ the product for the content in chains.

35. 3. When ſome part of the ground is bounded by an irregular curve, as *abc*, meaſure the baſe AC. Then for finding the content of AC*ca*; erect the perpendicular ordinates or offsets A*a*, B*b*, C*c*, at equal diſtances, and meaſure them. Then

$$\frac{Aa + Cc + 4Bb}{6} \times AC,$$ will be the area of AC*ca*, in chains.

Or more exactly, if *ad* be a curve; divide the baſe AD into 3 equal parts at B, C, and meaſure it, and the perpendicular ordinates A*a*, B*b*, C*c*, D*d*; then $\frac{Aa + Dd + 3Bb + 3Cc}{8} \times AD$, will be the area of AD*da*, in chains.

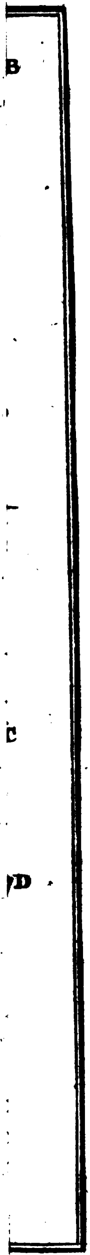
Or more exactly ſtill. Divide the baſe AE into four equal parts, at B, C, D; and meaſure the offsets or ordinates, A*a*, B*b*, C*c*, D*d*, E*e*, (as in Cor. Prob. III. Sect. I.) then will

$$\frac{7 \times \overline{Aa + Ee} + 32 \times \overline{Bb + Dd} + 12Cc}{90} \times AE$$ be the area of AE*ea*, in ſquare chains.

SCHOL.

If a lordſhip is to be ſurveyed; after all the fields have been meaſured and laid down in the plan, the contents of them all are to be caſt up, one by one, by ſome or other of the rules and directions beforegoing; and their contents written in the field, or elſe put into a table, to have recourſe to upon occaſion. And ſuch a table or book is beſt made, to have the names of the cloſes go on alphabetically; eſpecially if the lordſhip be large, and contains a great number of cloſes.

PROB.



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M.

Fig.

PROB. XVIII.

To lay out any number of acres in a triangle, whoſe baſe and containing angle is given.

Multiply the baſe AC by the nat. ſine of the angle C, for a diviſor, by which divide 20 times the acres given; and the quotient is the ſide BC. 36.

Or thus by logarithms. To the log. of the number of acres, add the conſtant log. 11.30103, and reſerve the ſum. To the log. baſe AC, add the log. ſine of the angle C; and ſubtract this ſum from the former ſum. The number of the remaining log. is the ſide CB.

For if A be the number of acres, then $\frac{AC \times CB \times S.C}{2 \times 10} = \text{area} = A$, therefore CB = $\frac{20A}{AC \times S.C}$.

Examp.

Suppoſe 17.64 *acres to be laid out in a triangle, whoſe baſe* AC *is* 24.81 *chains; and angle* C, 38 20. *To find the other ſide* CB.

17.64, 1.24649
conſt. log. 11.30103

12.54752

AC 24.81, 1.39463
S.C 38 20, 9.79255

11.18718

~~CB 29.92,~~ 1.36034 chains.

CB 22.92, 1.36034 chains.

Cor. *If you would lay out any number of acres in a triangle, of a given height.*

Divide

Fig. 36. Divide twenty times the number of acres, by that height or perpendicular; the quotient is the length of the baſe, for any triangle.

PROB. XIX.

To lay out any number of acres in a rectangle, whoſe baſe is given.

Divide ten times the given number of acres, by the baſe given; and the quotient is the height of the rectangled parallelogram.

Examp.

Suppoſe 64.25 *acres laid down in a rectangle, one ſide of which is* 18.27 *chains; to find the other ſide.*

```
18.27)642.5(35.16 the height.
      5481
      ----
       9440
       9135
       ----
        3050
        1827
        ----
        1223
        ----
```

Cor. *If the acres are to lie in a ſquare; extract the ſquare root of ten times the number of acres given, for the ſide of the ſquare.*

Fig. 37.

PROB. XX.

To divide a triangle in any ratio, from an angle; or to cut off any part of it, by a line drawn from an angle B.

Divide the oppoſite ſide AC in the point G, in the ſame ratio that the triangle is to be divided; and draw the line BG.

Or if the area be known; ſay as the whole area ABC, to the part to be cut off ABG; ſo is the whole baſe AC, to the part AG to be cut off, then draw the line BG.

As if you want $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of the triangle cut off, make AG to be $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of AC, and draw BG.

For triangles on the ſame baſe, and of the ſame height, are as the baſes; therefore ABG : ABC : : AG : AC.

Cor. 1. *If you want to cut off a given quantity from the triangle, without knowing its content; proceed by* Prob. XVIII.

Cor. 2. *If you would divide the triangle into any number of equal parts; it is only dividing the ſide* AC *into the ſame number of equal parts; and drawing lines to theſe points, from the oppoſite angle.*

PROB. XXI.

To divide a triangle ABC *in any proportion, by a line* DE, *drawn parallel to a given ſide* BC: *or to cut off any part of it.*

Divide either ſide AC, in the given proportion, at F. And make AE a mean proportional between AF and AC. Thro' E draw ED parallel to BC, which divides the triangle as required. 38.

Fig. 38. *Or thus.*

Say as the whole triangle, to the part next the angle A to be cut off; ſo is the ſquare of either ſide as AB, to a fourth number; extract the ſquare root of it, which ſet from A to D, and draw DE parallel to AC, for the dividing line.

For the triangles ABC and ADE are ſimilar; therefore $ABC : ADE :: AB^2 : AD^2 :: AC^2 : AE^2 :: AC : AF$.

PROB. XXII.

To divide a triangle ABC *in a given ratio, by a line drawn from a given point* D *in a ſide; or to cut off any part of it.*

39. From the given point D, draw a line DB, to the oppoſite angle B. Then divide AC in F in the given ratio; and from F draw FG parallel to DB, to cut CB in G; then draw DG, which divides the triangle as required; the part DGC being at FC; and the part ADGB, as AF.

For drawing BF, the triangles GFB, GFD, contained between the ſame parallels, are equal; add to both CFG, then CFB is equal to CDG; and therefore either of them is to the whole CBA, as CF to CA.

Cor. 1. *If you want to divide the triangle* ABC *into any number of equal parts; divide the baſe* AC *into the ſame number of equal parts; from all which points draw parallels to* BD; *and where they interſect the ſides* AB, CB, *draw lines from thence to* D.

Cor. 2. *If you want to cut off a given quantity from the triangle, without knowing its content; proceed according to* Prob. XVIII.

Fig. 40.

PROB. XXIII.

To divide a triangle ABC *in a given ratio, or cut off any part of it, by a line* GI *perpendicular to any ſide* AC.

Draw BD perpendicular to the ſame ſide AC. And let *r* repreſent the part to be cut off, and *s* the whole area. Then take $AG = \sqrt{\frac{r}{s}\times AD\times AC}$, and at G erect the perpendicular GI, then AGI is the part required. If G falls between D and C, take the other part ($s - r$), and meaſure from C.

For, triangle AGI : triangle ACB : : $AG\times GI : AC\times BD$: : (by ſimilar triangles) $AG^2 : AC\times AD$: : $\frac{r}{s}\times AD\times AC : AC\times AD$: : $r : s$.

PROB. XXIV.

To divide a triangle ABC *into three equal parts, by three lines drawn from the three angles,* A, B, C. 41.

Make AD a third part of AC; thro' D, draw DE parallel to the next ſide AB. Biſect DE in F; and from F, draw to the three angles A, B, C, the three lines FA, FB and FC, which will divide the triangle as propoſed.

For ſuppoſing a line drawn from B to D, the triangle ABD would be $\frac{1}{3}$ of ABC, but ABD and ABF are equal, being contained between the ſame parallels; therefore ABF is $\frac{1}{3}$ the triangle ABC. Therefore the two equal triangles AFC and BFC are equal to $\frac{2}{3}$ of ABC, and each is $\frac{1}{3}$ of it.

Cor. *Hence a triangle* ABC *may be divided into* 3 *parts, by* 3 *lines drawn from the three angles, which ſhall be as any three numbers a, b and c.*

Fig. 41. For make AD to AC as a to $a+b+c$; and draw DE parallel to AB. Then make DF to DE as b to $b+c$. And draw AF, BF and CF. Then it is plain, AFB is as the number a, and AFC as b, and BFC as c. For AFD : BFE :: DF (b) : FE (c) :: DFC : EFC. Or AFC : BFC :: b : c.

PROB. XXV.

To divide a triangle ABC *into* 3 *parts from a given point* D *within it; to be to one another, as the numbers a, b, and c.*

42. In any ſide AC, take AH to AC, as a to $a+b+c$, and draw DH, and from the oppoſite angle B, draw BF parallel to DH; and draw DB and DF, then ABDF is the part repreſented by a.

In like manner take CG to AC, as another of the numbers b, to $a+b+c$; and draw DG, and BI parallel to it; and draw DI. But as I falls without the figure, to reduce it (by Cor. 2. Prob. 12.) draw DC and IN parallel to it, to cut BC in N, and draw DN; then BDN is the part repreſented by b. Therefore FDNC is the third part repreſented by c.

For ſuppoſing BH drawn, then by conſtruction ABH will be equal to $a = \mathrm{ABF} + \mathrm{FBH} = \mathrm{ABF} + \mathrm{FBD} = \mathrm{ABDF}$.

Again, if BG be ſuppoſed drawn; then by conſtruction $\mathrm{GBC} = b = \mathrm{GBI} - \mathrm{CBI} = \mathrm{BDI} - \mathrm{BCI}$ = (DOI being a right line) $\mathrm{BDO} - \mathrm{OCI} = \mathrm{BDN}$, becauſe $\mathrm{DON} = \mathrm{COI}$, by reaſon of the parallels DC and NI.

Cor. *By the ſame way, one may divide a triangle into as many parts as he pleaſes, from a given point* D *within it; and in any ratio aſſigned.*

But ſuch Problems as theſe, are more for the exerciſe of Geometry, than for any great uſe they are of in ſurveying. I ſhall only add another, about triangles, of the ſame ſort. Fig. 42.

PROB XXVI.

To divide a triangle ABC *in a given ratio, by a line* IH, *drawn thro' a given point* D.

Make AF to AC as the part AHI to the whole ABC, and draw BF, and alſo DG parallel to AC. 43. Then put $P = \frac{AB \times AF}{DG}$. And take $AH = \frac{1}{2}P + \sqrt{\frac{1}{4}P - AG \times P}$, and draw IDH. And when AH comes out greater than AB, work for the angle C, inſtead of A.

For, the triangles AHI and AFB will be equal, whence $AH \times AI = AF \times AB$, and $AI = \frac{AF \times AB}{AH}$. And by the ſimilar triangles HGD, HAI, HG : GD : : HA : AI or $\frac{AF \times AB}{AH}$: : AH^2 : $AF \times AB$. Whence $HA^2 \times GD = HG \times AF \times AB = \overline{HA - AG} \times AF \times AB = HA \times AF \times AB - AG \times AF \times AB$, and $HA^2 = \frac{AB \times AF}{GD} \times HA - \frac{AB \times AF}{GD} \times AG = P \times HA - P \times AG$. Which reduced, and the root extracted, gives $HA = \frac{1}{2}P + \sqrt{\frac{1}{4}PP - AG \times P}$.

Cor. *If the point given, is without the field, at* d; *then will* $AH = \frac{1}{2}P + \sqrt{\frac{1}{4}P + Ag \times P}$.

For then g being on the other ſide of A, Ag will be affirmative.

Fig. 44.

PROB. XXVII.

To divide a parallelogram in any ratio, or to cut off any part of it; by a line parallel to one of the ſides.

Divide the other ſide AB at E, in the given ratio, and make CF equal to BE, and draw the line EF, which will divide the parallelogram as was required.

If you want a certain part of it cut off; take BE the ſame part of BA, and CF of CD; and draw EF.

If you deſire to cut off ſo many acres, without knowing the content of the whole; divide your number of acres by the breadth of the parallelogram, EG; and the quotient will ſhew how far you are to meaſure along the ſide BA or CD; where you muſt draw a line, parallel to the ſide BC or AD, as EF.

Cor. *Hence, you may divide a parallelogram into any number of equal parts; by dividing the oppoſite ſides,* AB *and* DC *into the ſame number of equal parts, and drawing lines thro' the correſpondent points.*

PROB. XXVIII.

To divide a trapezoid in a given ratio, or to cut any part of it; by lines croſſing the parallel ſides.

45. Divide the parallel ſides AB, DC in the ſame ratio, and to the ſame hand, as at E and F; and draw the line EF, which will divide the trapezoid in the ſame given ratio.

Otherwiſe, to cut any part off it; make AE the ſame part of AB, and DF the ſame part of DC; and draw EF.

a. 98.

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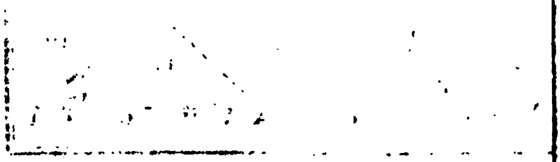


Fig. 45. Cor. *Hence a trapezoid may be divided into as many equal parts as we pleaſe; by dividing the two parallel ſides, into the ſame number of equal parts; and drawing the correſpondent lines.*

PROB. XXIX.

To divide a trapezium in any given ratio of the parts, from an angle A.

46. Draw the diagonal BD oppoſite to the given angle A; and divide it at I, in the ratio given; and draw the lines AI and IC, which will divide the trapezium as required.

But, if you would have one right line drawn from A to divide it; draw the other diagonal AC, and IF parallel to it, to cut DC in F, then draw AF, which will divide it in the given ratio.

For the triangle AID is to AIB, in the ratio of DI to IB, that is in the given ratio; and CID is to CIB in the ſame ratio; therefore the part ADFI is to the part ABCI in the given ratio. But by reaſon of the parallels AC, IF, the triangle AIC is equal to AFC; and therefore ABCI is equal to ABCF; and ADCI equal to ADF.

PROB. XXX.

To divide the trapezium ABCD, *in any given ratio, by a line drawn from* E, *a point in the ſide* AB.

47. Produce the oppoſite ſide DF; and reduce the trapezium ABCD into the triangle GEH, from the point E, by Prob XII. Then divide the baſe GH at F, in the given ratio, and draw EF, which will divide the trapezium as required.

If the point F falls not between D and C, but between C and H; then BC muſt be produced for the

Fig. 47. the baſe, inſtead of DC. Or elſe reduce it back, by Cor. 2. Prob. XII.

For the part DAE is equal to DGE, and ECH equal to ECB. Therefore EADF is equal to EGF, and EBCF equal to EHF, whence theſe parts are as GF and FH, that is, in the given ratio.

PROB. XXXI.

To cut off any quantity of land from a trapezium, by a line parallel to one ſide, as CB.

48. Divide the number of acres, given to be cut off, by the ſide CB, to which you are to go parallel. And the quotient ſet out from B to E, and from C to F; and draw EF. But if EF is greater than BC you muſt ſcarce go ſo far, from B or C; or if it be leſs you may go ſomething further. But when this is ſet out, as the field is irregular, you muſt meaſure the quantity cut off CBEF, and ſee what it is over or ſhort. Then meaſure FE and reduce that difference of the areas, to ſquare, chains, which divide by the length of FE, and the quotient muſt be ſet from E towards A, if you had cut off too little, as to *e*; and from F to *f*. But the contrary way if you had cut off too much. Then draw *ef* for the true line of diviſion.

Note, you may ſet it all to one end if you like; and then you muſt double the quotient, and ſet it from E to *n*, on the greater ſide, and draw *n*F. And indeed the courſe of the ridges in a field often confine us; ſo that when we expect they run parallel to the hedge, we find them otherwiſe; and therefore we may lay out more at one end than the other, in theſe caſes.

Cor. *Hence we may apply this method to any polygon; provided the ſides* AB, CD, *that we work upon, be not far from parallel. Otherwiſe a proper allowance*

ance muſt be made for the increaſe or decreaſe of the Fig.
line EF. 48.

PROB. XXXII.

Any poligon, regular or irregular, being given; to divide it in a given ratio, from a point in an angle or a ſide; or to cut off any quantity from it.

Let ABCDE be the poligon; reduce it into a 49.
triangle CFG, by Prob. XII. whoſe vertex is C, and baſe FG. Then divide FG in the point I, in the ratio given; and draw the line CI, which divides the triangle FCG, and alſo the poligon ABCDE, as was required.

Note, if CI do not cut AE, but ſome other ſide AB or ED, then that ſide it cuts muſt be produced for the baſe inſtead of AE, the point C being ſtill the vertex; or elſe it may be reduced by Cor. 2. Prop. XII.

If a given quantity of acres was to be cut off; reduce that quantity to chains, and take the neareſt diſtance from C to AE for the perpendicular; divide twice that number of chains by this perpendicular, and the quotient will be the length of the baſe, which ſet from F to I, then the line CI drawn, will cut off the quantity propoſed ABCI.

Otherwiſe thus.

If the ratio of the two parts be given, and the whole quantity; then either part is eaſily found.

Therefore to cut off any quantity towards CB, meaſure ſo many triangles next that ſide, as CBA, CAE, &c. till you have ſomething too much (or wants a ſmall matter); then cut off the overplus from the laſt triangle (or the defect from the next); and draw the line of diviſion.

The method to cut off the due part of the laſt triangle is this, let CAE be the triangle, then divide

Fig. vide twice the exceſs (reduced to chains) by the
49. perpendicular from C to AE; and the quotient ſet from the laſt angle E to I, gives the point of diviſion.

Examp.

The field ABCDE contains 42 acres, the part CBAI to be cut off is 27 acres. The triangle CBA is 10.80; the triangle CAE is 24.46. The perpendicular 22.7.

10.80
24.46
———
35.26
27.00
———
over 8.26 = 82.6
——— chains
82.6
2
———
22.7)165.2(7.28 = EI.
1589
———
630
454
———
1760
———

Therefore if EI be made 7. 28. chains, and CI drawn; the part CBAI will contain 27 acres.

Cor. *By the ſame method, a poligon may be divided into any number of equal parts; and even into any number of parts, whoſe proportions are expreſſed by any numbers.*

This is done either by dividing the baſe of the triangle FG in the ratio of the ſeveral parts. Or elſe cutting off the ſeveral parts of the figure one after another. And when there are many parts, it Fig. comes to no more, than dividing a triangle into ſe- 49. veral parts, by lines from the vertex, as explained in Prob. XX.

This problem is of very great uſe in practice; and by theſe methods any pieces of land may be divided more expeditiouſly, and as truly as by any method whatſoever. And for meaſuring any poligon, when it is reduced to a triangle, it is done at once; which otherwiſe would require as many calculations, as there are triangles.

PROB. XXXIII.

To divide a poligon into any number of parts, equal or unequal; by lines drawn from a point within it.

To do this, you muſt have the content of every triangle in the field, and of the whole field. And then beginning at any angle, draw a line to that angle, and take ſo many of the firſt triangles as will make the firſt part required; and what they fall ſhort of that part, cut off the next triangle. Then take the remainder of that triangle, and ſo many of the next, as will make up the ſecond part. And ſo proceed round the figure, till all the parts be laid out.

Examp.

Let ABCDEF *be a field containing* 69 *acres, which* 50. *is to be divided among three men, from a pond in the middle at* G; *ſo that every man may have the benefit of the water. And the firſt man to have* 25 *acres, the ſecond* 20, *and the third* 24. *To ſet out every man's ſhare from the angle* C.

Draw the line GC for the beginning of the diviſion; and alſo lines to all the reſt of the angles, as GB, GA, GF, GE, and GD. Then meaſure all the

Fig. 50. the triangles; and their contents will be as follows.

CGB	11.81
BGA	15.27
AGF	6.07
FGE	6.65
EGD	16.83
DGC	12.37
	69.00

Then ſince the firſt man's ſhare is 25 acres, the firſt triangle CGB 11.81 is too little, and the firſt and ſecond triangles together, CGB and BGA, comes to 27.08, which is more than 25; therefore ſubtract 25 therefrom, and there remains 2.08. And therefore we muſt cut off 2.08 acres from the triangle BGA towards A. To do which, meaſure the perpendicular from G to BA, 13.28 chains, and reduce 2.08 acres to chains, which will be 20.8; divide 41.6 or double theſe chains, by 13.28; and you have for the quotient 3.13 chains; which ſet off by meaſure, from A towards B, as to H, for the dividing point. And draw GH, ſo CGHB is the firſt man's ſhare.

Then for the next man's ſhare. Add this 2.08 acres cut off, to the next triangle AGF 6.07, the ſum is 8.15 which is too little, add the next triangle FGE 6.65, makes 14.80 acres, which is ſtill too little; and adding the next triangle EGD, it will be far too much. Therefore we ſhall cut off an additional part of the triangle EGD at E. Subtract 14.80 from 20 the ſecond man's ſhare, and there remains 5.2 acres, or 52 chains. The perpendicular from G upon ED is 13.9 chains. Therefore divide 104 (the double of 52) by 13.9, and the quotient is 7.48 chains; and this muſt be meaſured

ſured off from E towards D, as to I. Then draw GI, and HGIEFA will be the ſecond man's ſhare. Fig. 50.

Laſtly, the remainder of the triangle GDE, is 11.63 equal to the part IGD; to this add the triangle DGC 12.37; and the ſum is 24 acres for the area GIDC, which is the third man's ſhare. So the field is divided as was required.

If the field was to be divided into any number of equal parts; divide the whole content of the field by the number of parts, the quotient ſhews the quantity of one part or ſhare. As if there were 12 equal ſhares in it, then one ſhare will be 7.5 acres. Therefore cut from the triangle CGB, 7.5 acres; and from that and the next 7.5 acres more, and ſo on; which may be done as before directed; or by Cor. Prob. XVIII.

If you deſire to cut off any number of acres given, without knowing the content of the whole field. You muſt meaſure triangle after triangle, till you get as much as you want, or very near it. And for the defect cut off a part of the next.

PROB. XXXIV.

To transfer a plan or map truly, from one paper to another.

Lay your plan upon the blank paper, and fix them both together upon ſome flat board or table, that they may not move from one another or part aſunder, till the operation is done. Then with a fine needle or pricker, make ſmall pricks thro' all the principal angles, or places on your plan, which will make ſo many marks in your paper. Then take off the plan, and draw lines from one point or mark to another, as you find them in the plan; till all the lines be laid down. If you be in any doubt about ſome point or prick; takes its diſtance from one

Fig. one or two points, you are certain of; and apply it to the plan, and that will ſet all to rights.

Or thus.

Take the map or plan, and rub the backſide all over with black lead; then lay your paper upon a flat table, and upon that lay the leaded ſide of the plan. Faſten them at the four corners, that they may keep together. Then with a ſharp pointed bodkin run over all the lines in your plan, laying pretty hard on; and ſuch other objects as you would expreſs. Then take off the map, and all the lines, that you traced with the bodkin, will appear upon the paper; which you may draw with ink, or may rub out at pleaſure, with crums of bread.

Otherwiſe thus.

Take a paper well oiled with linſeed oil; and laying the map upon a table, lay the oiled paper upon it, and faſten them together; then you may ſee all the lines and furniture of the map, thro' the oiled paper, as thro' a clear glaſs. Then with pen and ink, or with a bodkin, draw or trace all the lines in the map. So will you have a map in the oiled paper, which you may place upon another paper, if you pleaſe. Thus by an oiled paper, may the figure of any flower, fruit, or other curious thing, be taken from any draught of it.

Or thus.

The drawings in a map may be transferred to another paper, by taking every line from the one, and ſetting it upon the other, with a pair of compaſſes: but this is very tedious, and requires a deal of time; but to facilitate the work, there are three legged compaſſes made, to take off three points at once, and ſet them down on the new paper.

PROB.

Fig. 51.

PROB. XXXV.

To reduce any field to a greater or leſſer ſize, in any proportion of the ſides.

Take a point in a ſide or an angle, as at A. From that point, draw lines to all the other angles, as AE, AD, AC. Then on any ſide or diagonal, ſet off the length of that ſide or diagonal, according as you would have it reduced; ſuppoſe along AF, from A to *f*. From *f* draw *fe* parallel to FE, to interſect the next diagonal AE in *e*. From *e* draw *ed* parallel to ED, to interſect AD in *d*. From *d* draw *dc* parallel to DC to interſect AC in *c*. From *c* draw *cb* parallel to CB. Then *afedcba*, wil be the reduced figure.

Or you may take a point within the figure, as at 52.
P; from which draw lines to all the angles, PA, PB, PC, PD, PE. Then make P*a* the reduced length of any line as PA; and from *a* draw *ab* parallel to AB, to interſect the next line PB in *b*. In like manner draw *bc*, *cd*, *de*, *ea*, parallel to BC, CD, DE, EA; each from the interſection of the correſpondent line drawn from P. And *abcde* will be the figure reduced, as was propoſed. And by the ſame rules, they may be reduced to a bigger ſize, as well as a leſſer.

Cor. 1. *Hence the reduced figure is ſimilar to the* 52.
original figure.

For the angles are all equal; and the ſides paralled, and proportional, to thoſe in the given figure.

Cor. 2. *Hence, a figure* ACE *may be divided in a given ratio, by lines parallel to the ſides of the figure; by the ſame method.*

Making Af^2 to AF^2, or Pa^2 to PA^2, in the given ratio.

Fig. 52.

PROB. XXXVI.

To reduce a map or plan, to a leſs or greater ſize, in any proportion of the ſides.

If the draught or plan is not large, lay it down again by a ſmaller ſcale; to do which you muſt take every line from the plan, and apply it to the ſcale it was laid down by; and then take the ſame numbers, off the leſſer ſcale, to be laid down in the new plan. Thus you muſt perform the whole work geometrically, by ſcale and compaſs; and it is beſt to uſe a diagonal ſcale, by reaſon of their minute diviſions. But when the plan is large, this method is tedious.

Otherwiſe.

Make a reducing ſcale with a ſmall hole, in a line with the edge of it, and nearer one end than the other, in proportion of the ſize it is to be reduced to. Theſe are both graduated with equal parts, in the ſame proportion. Then glue or fix your map to a table, and cloſe by it edge by edge a paper, on which it is to be reduced into a leſſer or greater form. Put a ſtrong needle thro' the hole in the center of the reducing ſcale, and fix it in the table about the middle of the line where their edges join. Then, if you would reduce it to a leſſer ſize; turn the edge of the longer end of the ſcale, to any object on the plan; and ſee what number of equal parts it cuts; and find the ſame number on the other end of the ſcale, and by its edge make a mark on the paper, for the place of that object. And thus you muſt proceed till all things on the map be laid down.

But if you would make a larger draught, you muſt turn the ſhorter end of the ſcale to the objects on the map, and the longer end to the paper.

The rule muſt be a thin plate, that the numbers may be pricked off by the edge of it. Fig.

But plots or maps are beſt reduced by an inſtrument contrived on purpoſe. This is made with two drawing pens; ſo that tracing one of them over all the lines in the plan, the other pen will deſcribe it anew on a clean paper, either in a leſſer or a greater ſize, as one pleaſes.

Or thus.

Incloſe the map in a large ſquare, and divide that 53. into ſeveral ſmall ſquares. Alſo make a ſquare upon 54. your new plot, ſo much leſs than the other, as you would have it diminiſhed; and divide it alſo into the ſame number of ſmall ſquares. Then obſerve in what ſquare, and what part of the ſquare, any object, on the map, is placed; then place it in the like ſquare, and in the ſame part of it, on your new plan. And thus transfer every remarkable point in the map, into your new plot; putting it in its proper ſquare, and into the ſame ſituation. Thus in fig. 53. you have ſeveral cloſes to the weſt, and a hill near them; likewiſe a river running thro' the ground, two trees near it, and a houſe a little below; with a road leading to it. And in the reduced plan, fig. 54. you have alſo the cloſes to the weſt, and hill near them; and the river running thro' the ground, and two trees beſide it, and a houſe below, and the road going to it; and all placed in the correſponding ſquares, and the ſame parts thereof. And thus all other things therein, ought to be placed. And the ſame way, a plan is reduced from a leſſer to a greater ſize; but this ſeldom comes in practice.

This method is very proper for a large plan, becauſe it contains a great variety of things.

PROB. XXXVII.

To reduce one ſort of meaſure into another.

In ſome places, thro' cuſtom, they reckon more feet to the perch than the ſtatute allows; and therefore an acre with them will be greater than a ſtatute acre. But to know how many ſtatute acres there are in any field, ſurveyed by a different perch; and the (contrary); by having that perch, and the correſpondent number of acres given. Multiply the number of acres by the ſquare of the length of the perch it was ſurveyed and planned by; and divide the product by the ſquare of the other perches length; and the quotient gives the number of acres, according to that perch.

Examp.

Suppoſe a field of 40 *acres was planned by a perch of* 18 *feet; how many ſtatute acres does it contain?*

18	$16\frac{1}{2}$	272.2|5)12960)47.60
18	$16\frac{1}{2}$	10890
144	96.	20700
18	16	19054
	$16\frac{1}{4}$	
324		16460
40	$272\frac{1}{4}$	16332
12960		1280

Anſ. 47.6 acres.

Cor. *The length of the perch, a field is meaſured by; is reciprocally as the ſquare root of the number of acres, it meaſures to, by that perch.*

PROB.

PROB. XXXVIII.

Having given the content of a field; to find what ſcale it was laid down by; or how many parts to an inch.

Take any number of parts to an inch for your ſcale, by which meaſure the content of the field; then multiply the content given by the ſquare of your ſcale (or number of parts) and divide the product by the content found by that ſcale; the ſquare root of the quotient, is the true ſcale, or number of parts in an inch.

Examp.

There is a field contains twelve acres, but meaſured by a ſcale of 10 *parts in an inch, its content is* 16$\frac{1}{2}$ *acres. To find the ſcale.*

```
   10     24
   10    100
  ----   ----
  100
  ----
  16.5)2400(145.5,        145.5(12.07
        165.               1
        ----              ----
         750              22)45
         660                 44
        ----                 --
          90                 1.50
          82
        ----
           8
        ----
```

So the ſcale contains 12.07 parts in an inch.

Cor. *The ſcale or number of parts in an inch, is as the ſquare root of the content found by it, in a given field.*

SECT. III.

Of Meaſuring inacceſſible Heights, and Diſtances.

PROB. I.

To meaſure the height of any perpendicular object AB, *where you can come to the baſe* A.

55. LET ABD be the object, as ſuppoſe a tower, whoſe height AB is required; AC a horizontal line; for here C is ſuppoſed to be level with the foot of the object at A.

Take your ſtation at C, about as far from the object as the height of it; and where you can ſee both the top and foot of the object; and there place your inſtrument, with which take the angle BCA; that done, meaſure the diſtance CA, from the inſtrument to the foot of the tower. Then by plain trigonometry ſay,

As radius:
to the diſtance CA::
So the tangent of the angle BCA:
to the height of the tower AB.

To the height thus found, add the height of the inſtrument above the ground.

Examp.

Fig. 55.

Examp.

Suppoſe the angle BCA 51 52, and diſtance CA, 64 feet.

Rad	—	10
CA 64	—	1.80618
Tan. 51 52	—	10.10510
AB $81\frac{1}{2}$	—	1.91128

To which add 4 feet the height of the eye, the ſum is $85\frac{1}{2}$ feet.

This may alſo be reſolved by projection; draw CA, on which ſet 64 feet from any ſcale, to A. Erect the perpendicular AB. Then make the angle ACB 51 52; and draw CB interſecting the perpendicular in B, the top of the tower. Then AB meaſured on the ſame ſcale, gives $81\frac{1}{2}$.

Otherwiſe, by the ſhadow.

Upon plain ground, ſet up the ſtaff *ab* perpendicular to the horizon; ſo that the ſun ſhining, it may caſt the ſhadow *ac*, and let AC be the ſhadow of the tower AB. Then meaſure the ſhadows *ac* and AC, and the length of the ſtaff *ab*. Then ſay,

As the length of the ſtaff's ſhadow ac :
is to the height of the ſtaff ab : :
So is the length of the object's ſhadow AC :
to the height of the object AB.

Or thus by the ſhadow without the ſtaff. Take, with an inſtrument, the altitude of the ſun, when the ſhadow of the top of the tower B, falls at C. Then meaſure CA, and you have the angle C, and the baſe CA; to find the perpendicular BA; which is done by the firſt method.

Fig. 55.

SCHOLIUM.

In taking the angle ACB, if you move backwards or forwards, till that angle be juſt 45 degrees; then the diſtance CA, being meaſured, will be equal to the height AB.

If you be on the top of the tower, you may take the angle ABC, to any mark at C; then ſubtracting the angle ABC from 90, gives the angle ACB; then meaſuring AC; from theſe, AB will be found as before. And this comes to the ſame thing as meaſuring a depth.

Hence alſo if the height AB be given, the diſtance of any point C, as AC, may be found; having the angle ACB. It is only varying the proportion, given in the firſt method.

We may obſerve, that the altitude AB, may be found otherwiſe than by calculation, if we can get to the top of it. And that is, by help of a line and plummet, let down from the top B, to the bottom at A. For the length of the line, reaching from B to A, being meaſured, gives the height BA.

PROB. II.

To meaſure an inacceſſible height DB.

56. Suppoſe DB to be a tower and a ſpire, where we cannot come at the bottom of it; or in caſe of a mote or other hindrance, between us and the object; that we can get no nearer than A; we muſt proceed thus.

Suppoſe the line of ſtation BAC, drawn from the foot of the perpendicular DB, to be horizontal. In this line we muſt take two ſtations A and C, at a good diſtance from one another. Then place the inſtrument at A, and take the angle DAB, ſuppoſe 58°; then go to the other ſtation C, and take the angle DCB, 38 deg. Then meaſure the diſtance between the ſtations CA, 26 yards. Fig. 56.

Then to project it, draw a line CB, and at one end A, make the angle DCB 38 deg. and draw CD. Then ſet off the ſtationary diſtance 26 from C to A; and at A make the angle BAD 58 deg. and draw AD, to interſect CD in D. Then the diſtance of D from the line CB, applied to the ſcale, gives the height 40 yards.

Or thus, by Trigonometry.

Subtract BCD 38°, from BAD 58°; there remains 20°, for the angle ADC. Then,

As	Rad —	10.
	× ſ. ADC, 20	9.53405
To	CA, 26	1.41497
So is	S.C, 38	9.78934
	× S.A, 58	9.92842
		21.13273
To the height BD, 39.7,		1.59868

To this height, add the height of the inſtrument above the ground for the whole height.

For by trigonometry, $S.<D : CA :: S.<C$. $AD = \frac{S.C \times CA}{S.D}$ and rad : AD or $\frac{S.C \times CA}{S.D}$: : S.A : BD; that is,

Rad × S.ADC : CA×S.ACD : : S.BAD : height BD.
or Rad × S.ADC : S.ACD × S.BAD : CA : BD.

Or thus, by nat. tangents.

Find the difference of the cotangents of the angles BAC, and BAD. Then,

As the dif. cotangents :
Radius : :
So the ſtationary diſtance CA :
to the height DB.

Cot.

Fig. 56.

Cot. 38	1.27994
Cot. 58	.62487
diff.	.65507

then .655 : 1 : : 26 : DB.
1

.655)26.00(39.7 = DB.
19 65
6350
5895
455

For if DB be radius, CA will be the difference of the tangents of ADB and CDB, which are the complements of A and C. And therefore rad : that difference : : DB : CA.

Otherwiſe thus.

If you take two ſtations at A and C, ſo that the angle DAB ſhall be double to DCB. Then CA will be equal to AD.

Then *Radius* :
the ſtationary diſtance AC : :
S.angle DAB :
to the height DB.

Or thus.

Take the ſtation at A, ſo that the angle DAB may be juſt 45 degrees; and the angle DCB 26° 34′. Then the height DB will be equal to the ſtationary diſtance CA.

For when DAB is 45, DB = BA = CA, and CB = 2DB. But it is, as CB (or 2DB) : DB, or as 2 : 1 : : rad : tan. 26° 34′.

Cor.

Fig. 56. Cor. 1. *Hence one may find the height of one object,* DEF, *ſtanding upon another* GEF.

Firſt find the whole altitude DB, and then the height of the tower EG, by this Prob. And their difference, is the height of the ſpire EDF alone.

Cor. 2. *If the height of the tower* EG *be known; any diſtance* CA, *in the line* GC, *may be meaſured from* E.

For that is only the reverſe of this Prob. and therefore CA may be found by working the proportion backwards, by the natural tangents; or by the ſines; as laid down before.

PROB. III.

To meaſure an inacceſſible height BD, *by means of any two ſtations; ſo that one of them* A, *may be level with the baſe* B *of the object.*

Draw the horizontal line BA, and taking one 57. ſtation at A in that line, take another any where as at C, no matter whether it be higher or lower. Place the inſtrument at A, and take the angles BAD and DAC, and go to C, meaſuring the ſtationary line AC; and at C plant the inſtrument, and take the angle ACD. Then ſubtract the ſum of the angles DAC and DCA from 180, to get the angle ADC.

Then ſay, as $\left\{\begin{array}{l}\textit{Radius}\\ \times\ \text{S.ADC}\end{array}\right.$

to $\left\{\begin{array}{l}\text{S.ACD}\\ \times\ \text{S.DAB}\end{array}\right.$

So the ſtationary diſtance AC:

to the height BD.

Examp.

There is a tree DE *ſtanding upon a high hill* BE; *to find the height of its top* D, *above the level of* A.

First

Fig. 57. Firſt I place the inſtrument at A, and take the angle BAD 58 deg. and the angle DAC 72° 10′. And then I meaſure the ſtationary diſtance AC 26 yards, and at C I fix my inſtrument, and take the angle ACD 64° 30′. Therefore, ADC is 43 20. Whence,

Rad.	10.
S.ADC, 43 20,	9.83647
S.ACD, 64 30,	9.95548
S.DAB, 58 0,	9.92842
AC, 26	1.41497
	21.29887
height BD, 29	1.46240

Then to this height, add the height of the inſtrument. The demonſtration is the ſame here, as in the laſt Prob.

Or thus.

Take the ſtation A, ſo that the angle BAD may be 30 deg. Then will DA be equal to 2DB; and it will be,

As S.ADC,
to $\frac{1}{2}$CA;
So S.ACD,
to the height DB.

Otherwiſe thus.

This ſuppoſes both ſtations A and C to be level with the baſe B of the object.

Here are given the angles of altitude BAD, BCD, and the horizontal angle at one of the ſtations, as ACB, and the ſtationary diſtance CA.

Make as tan. BCD : tan. BAD : : S.ACB : S.BAC, obſerve whether acute or obtuſe.

Then ſubtract the ſum of the angles BCA and BAC from 180, gives the angle ABC. Then ſay,

As

UNIV.
OF

As S.ABC × radius : S.ACB × tan. BCD : : ſo Fig. 57.
the ſtationary diſtance AC : to the height DB.

This method is deduced from Prob. 125 of my Algebra.

In the former method, the point C may be either in or out of the plane DAB, as is evident. For DAB is perp. to the horizon; but ADC may be any way inclined to it. But in the latter method both ADB, and CDB are perp. to the horizon; and therefore the plane DCA muſt always be inclined to it.

Cor. 1. *A depth may be found the ſame way as an altitude; by aſſuming two ſtations, from both which, any ſpot in it may be ſeen, where you can ſet up a mark.*

This is eaſily underſtood, by only conceiving the point D to be below B.

Cor. 2. *If* CA *coincided with* AB, *this would come to the ſame caſe as the laſt Prob. And if* CA *was perpendicular to the horizon, this Prob. would be the ſame as finding one altitude* BD, *from having another altitude* AC *given; which therefore is eaſily reſolved.*

After this manner the height of a hill or a mountain, or any object above the level of the place, is eaſily found.

Cor. 3. *Hence may be found the height* DE *of one object upon another; by firſt finding the height* DB, *and then* EB; *and the difference will be the height* DE.

PROB. IV.

To find the height of a very high mountain.

This might be reſolved by what went before, if 58.
it was not for the refraction of the rays of light, which do not come ſtreight to the eye from ſuch a height in the atmoſphere, where the air muſt be greatly

Fig. 58. greatly rarified. For this reaſon, a ray coming from the top of a mountain to the earth, deſcribes a curve line, concave to the earth. Therefore an obſerver at the earth, taking the angle of altitude of an object upon the top of it, directs his eye along the tangent to that curve; for in that direction the rays come to the eye. But as that tangent runs above the mountain, he makes the angle of altitude higher than it ſhould be; and of conſequence the mountain itſelf higher than it really is. Now to rectify this, let marks be ſet up at the ſtations A and C; and one at the top of the mountain at D. And let the obſervers go, one to A, and the other to D; and let them both at the ſame time, make their obſervations thus; let the obſerver at A take the altitude of D above the horizon; where the rays, inſtead of coming in the right line DA, will come in the direction FA, the tangent of the curve at A: and therefore he really takes the altitude of the point F. Alſo let the obſerver at D take the depreſſion of A below the horizon; where the rays, inſtead of coming to him in the line AD, will come in the direction FD, the tangent of the curve at D. And therefore he has taken the depreſſion of the point F, inſtead of D. Now by comparing notes, the angle at A will be greater than that at D, whereas they ought to be equal. Therefore take half the difference of the obſerved angles, and ſubtract it from the altitude taken at A, and that gives the true altitude. For the altitude at A was taken too great by the quantity DAF, and that at D too little by ADF equal to the former. The ſame thing is to be done at the ſecond ſtation C. Theſe angles being rectified, you may then proceed by either method laid down in the laſt Prob. (eſpecially the latter) to find the altitude; and for this purpoſe make your ſtationary diſtance AC as long as you well can.

It may happen that two ſtations may be taken Fig.
upon a mountain, more conveniently than below; 58.
and then you will only want one ſtation below. But this creates no material difference in the work; for it is only finding a depth inſtead of a height.

PROB. V.

To find the height of a cloud by its ſhadow.

Obſerve ſome ſmall cloud which is coming direct- 59.
ly towards you, or going directly from you; as C. Then obſerve when the middle of its ſhadow is at ſome remarkable point upon the ground, as at A; at that inſtant take the altitude ABC of the middle of the cloud. That done, take the ſun's altitude at the place of your ſtation B, and that will be equal to the angle BAC; alſo meaſure the diſtance BA between your ſtation and the ſhadow. Then you have a triangle ABC, where all the angles and the ſide BA are given; which is eaſily laid down by rule and compaſs, and the height of C, above BA, found. Or by trigonometry. Subtract the ſum of the angles at B and A from 180, to get the angle C. Then ſay, as rad × S.ACB : S.BAC × S.ABC : : ſo the ſtationary diſtance AB : to the height of the cloud.

For $\text{S.C} : \text{AB} :: \text{S.B} : \text{AC} = \frac{\text{S.B}}{\text{S.C}} \times \text{AB}$. And rad : AC or $\frac{\text{S.B}}{\text{S.C}} \times \text{AB} :: \text{S.A} :$ perpendicular; or rad × S.C : AB : : S.A × S.B : height.

If the cloud be directly over your head at the time of obſervation, then CBA will be a right angle. Then rad : AB : : tan. ſun's altitude CAB : height CB.

You muſt have a ſmall cloud to obſerve by, becauſe the obſervation muſt be made to a point. But

Fig. 59. But if you would obſerve a large cloud, take the altitude of the very edge, and obſerve the very edge of the ſhadow. And you muſt be upon ſome large plain or open ground, where you can ſee about you.

PROB. VI.

To find the altitude of an object whoſe top appears juſt in the horizon; having the diſtance in miles given.

This is a matter which frequently happens at ſea, in ſailing to or from the land.

Square the diſtance in miles, which multiply by the decimal .2223, and the product is the height of the object in yards.

60. For let AD be an arch of the earth's ſurface, DB the height of an object, AB a tangent at B. Then in the right angled triangle CAB, CB^2 or $CD^2 + 2CD \times DB + DB^2 = CA^2 + AB^2$. But becauſe $CA = CD$, $2CD \times DB + DB^2 = AB^2$, or (becauſe BD is very ſmall) $2CD \times DB = AB^2$ $= AD^2$, and $DB = \frac{AD^2}{2CD}$. But 2CD, or the earth's diameter, is 7916 miles; therefore $\frac{AD^2}{7916} = DB$ in miles, and $\frac{AD^2 \times 1760}{7916} = DB$ in yards; that is, $.2223 \times AD^2 =$ the height DB in yards. And the point B juſt appears or diſappears, to a ſpectator in the tangent at A.

PROB. VII.

To meaſure the length of a horizontal line AB *acceſſible at both ends,* A *and* B.

61. A line may be acceſſible at both ends, and yet cannot be meaſured, by reaſon of ſome impediment which happens to be in the way, as wood,

water, houſes, &c. In ſuch a caſe, ſet up a ſtake at ſome diſtance from the line, as at F; and thro' F produce the lines AF, BF, beyond F, in which take FD equal to one of the lines as FA, FC, equal to the other FB; and leave two marks at D and C. Then meaſure the length of the line CD, which will be equal to that required AB. Fig. 61.

If there is not room to meaſure out the lengths AF, BF; take half of them and meaſure from F to D and C; then meaſure CD and double it. Or take $\frac{1}{3}$ of each, and then CD is $\frac{1}{3}$ of AB.

Or thus.

Take a ſtation at ſome place where you can ſee A and B, as at F; and with an inſtrument take the angle AFB, then meaſure the two lines AF, BF. And then the triangle AFB is eaſily laid down with ſcale and compaſs.

Or take the angles at A and B, or any two angles; and meaſure one ſide; and then the triangle may be laid down, or computed by plain trigonometry.

PROB. VIII.

To meaſure a diſtance AB, *acceſſible only at one end* B.

Let the end B be one ſtation, and chuſe another at a convenient place as C; and having an object at A, produce the lines AB and AC to D and E, of any length. Then meaſure the ſides of the triangle BCD, and alſo the ſides of the triangle BCE. And by theſe the diſtance BA may be determined; all this with the chain only. 62.

Upon a paper draw the line BC, of its due length, and on that conſtruct the triangle BCD with the ſides you meaſured. Alſo upon the ſame line BC, make the triangle BCE, with the ſides, as you meaſured

Fig. 62. ſured them. Then produce DB and EC till they interſect in A; then BA meaſured, on the ſame ſcale, will ſhew the diſtance of A from B, or the length of the line BA.

It may be alſo computed by plain trigonometry. In the triangle CBD there are given all the ſides; to find the angle DBC, and its ſupplement ABC. And in the triangle BCE, all the ſides are given, to find the angle BCE, and the angle BCA its ſupplement. And laſtly, in the triangle ABC all the angles will be known, and the ſide BC, whence AB will be found.

Or thus.

Take one ſtation at B, and another at a convenient diſtance as C; and ſet up marks. Then placing the inſtrument at B, take the angle ABC, between ſome mark or object at A, and the mark at C. Then meaſure the ſtationary line BC; and place the inſtrument at C, and take the angle ACB, between the ſame object at A and the mark at B. Then ſubtract the ſum of theſe angles B and C from 180, to get the angle A. Then ſay,

As ſine of the angle A :
to the ſtationary diſtance BC : :
So ſine of the angle C :
to the diſtance AB.

The diſtance BA may alſo be found here by rule and compaſs, by laying down the triangle ABC.

Or thus.

Make the angle ABC of 60 degrees, or if you cannot at B, take ſome other point in the line BD. Then chooſe ſome other point C, in the line BC, that the angle BCA may alſo be 60. Then meaſure BC, and that is the length of BA. For if ABC be 60°, and BCA 60, then CAB will be 60; and the triangle ABC will be equilateral.

Or elſe make the angle ABC a right angle, and ſomewhere in the line BC, take the point C, that the

Fig. 62. the angle BCA may be 45 degrees. Then meaſure with the chain the length of BC, and that is the length of BA. For the angle at A is alſo 45 degrees; and therefore the ſides oppoſite to the equal angles, AB and BC, will be equal.

Or you may make the angle ABC a right angle; and the angle ACB 60 degrees. Then meaſure the length of BC; then BA will be equal to the ſquare root of $3BC^2$, or equal to $BC\sqrt{3}$. For in that caſe $AC = 2BC$, and $AB^2 = AC^2 - BC^2 = 4BC^2 - BC^2 = 3BC^2$.

Or make the angle ABC what you will, and the angle BCA half the ſupplement of it; then meaſuring the length of BC, you will have the length of BA. For then the angle BAC will alſo be half the ſupplement of ABC, ſince they are both together the whole ſupplement. Therefore their oppoſite ſides are equal.

PROB. IX.

To meaſure an inacceſſible diſtance AB; *or one that cannot be come to, at any end.*

63. Set up a ſtake ſomewhere in the line BA produced, as at C; then chuſe another ſtation as at D. Then with an inſtrument at C, take the angle ACD. Then meaſure the diſtance CD; and placing the inſtrument at D, take the angles CDA, ADB. Subtract the ſum of the angles ACD and ADC from 180, and there remains the angle CAD, or rather add the two angles ACD and ADC together, and you have the angle DAB. Likewiſe ſubtract the ſum of the angles DAB and ADB from 180, and you have the angle ABD. Then make, as S.BAD × S.ABD : S.ACD × S.ADB : : ſo is the ſtationary diſtance CD : to the diſtance ſought AB.

Fig. 63. For S.A : CD : : S.C : AD $= \frac{S.C}{S.A} \times$ CD.

And S.B : S.ADB : : AD or $\frac{S.C}{S.A} \times$ CD : AB.

Or S.A $\times$ S.B : S.C $\times$ S.ADB : : CD : AB.

It may alſo be laid down by projection. Draw the line CB, and make the angle BCD equal to that obſerved; ſet off CD, the diſtance meaſured; and at D, make the obſerved angles CDA, ADB; and the part AB intercepted between DA and DB, is the diſtance required.

Or thus.

64. When you cannot get a ſtation in the right line AB; take a proper ſtation at F, where you can ſee A and B; and produce the lines AF and BF, to D and C, and make FD and FC, of any convenient lengths, by meaſuring them. Then placing the inſtrument at F, take the angle AFB. Then go to C and take the angle ACB; and likewiſe at D, the angle ADB. Then the work may be laid down thus.

Draw any line AD, and at any point as F make the angle AFB equal to that obſerved, and draw the line BFC; and ſet off the diſtances FD and FC. Then at the point C, make the angle BCA, equal to that you meaſured, drawing CA to interſect DA in A, one end of the line. Again, make the angle ADB equal to its meaſured quantity, and draw DB, interſecting CB in B, the other end of the line; then AB meaſured on the ſcale is the diſtance required.

To reſolve it by trigonometry. Subtract the angle ACB from the angle AFB, and you have the angle CAF; then in the triangle CAF, all the angles and the ſide CF are given, therefore to find AF, it will be,

As ſine of the angles CAF :
to the ſide CF : :
So the ſine of the angle ACF :
to the ſide AF.

Fig. 64.

Again, ſubtract the angle ADB from AFB, and there remains the angle FBD. Therefore to find FB in the triangle DFB; it is,

As ſine of the angle DBF :
to the ſide FD : :
So the ſine of the angle FDB :
to the ſide FB.

Then in the triangle AFB, there are given the two ſides AF, FB, and the included angle AFB; by which the ſide AB will be found.

If the points C, D, be taken in the lines BF, AF produced, ſo that the angles ACB and ADB may be each of them half the angle AFB; then the line AB will be equal to CD. For then the angle ACF is equal to CAF, and FDB equal to FBD; therefore CF is equal to AF, and DF equal to BF; whence AB is equal to CD.

If there is not room to go back to C and D, you may take a ſtation ſomewhere about G; and find the length of AF (by Prob. VIII.) by the method of meaſuring diſtances, acceſſible only at one end. And the ſame way, find the length of BF. And then the line AB will be found as before.

Otherwiſe thus.

Having choſen two points of ſtation C and D at a proper diſtance, in the ſame plane with AB, whoſe diſtance may be meaſured; and from which, A and B may be ſeen. There ſet up two marks; and placing the inſtrument at C, take the angles ACB, BCD. Then meaſure the ſtationary diſtance CD; and at D, placing the inſtrument, take the angles CDA, ADB. Then you have the angles ACD, and CDB. Take the ſum of the angles ACD and CDA from 180, and there remains

Fig. the angle CAD. And in the ſame manner the an-
64. gle CBD is found.

Now we have in the triangle ACD all the angles, and the ſide CD, from which the ſide AC will be found. Then in the triangle CBD, we have all the angles and the ſide CD, whence we can find the ſide CB. Laſtly in the triangle ACB, we have the two ſides AC, BC, and the included angle ACB; from whence the required ſide AB is found.

It may alſo be laid down by rule and compaſs, to ſave the trouble of ſo many calculations. Draw any line as CD, on which ſet off the diſtance of the ſtations at C and D. Then at C, make the angles BCD, BCA, drawing the lines CB, CA. Again make at D, the angles CDA and ADB, all equal to theſe meaſured; and draw DA, which interſects CA in A. And draw DB, which interſects CB in B, and AB drawn and meaſured is the diſtance ſought. And thus you may meaſure any of the lines CA, CB, or DA, DB, by applying them to your ſcale.

It is no matter whether the line AB be horizontal, perpendicular, or inclined, provided CD be in the ſame plane with AB. For then the length of AB may always be determined by the two ſtations C and D. And therefore if AB be an inacceſſible altitude, and C, D, two ſtations either in a perpendicular CD, or upon any hill ſide; the height AB may be found as above; as will be evident by ſuppoſing the plane CADB perpendicular to the horizon.

Otherwiſe thus.

If we cannot get two ſtations C, D, in the ſame plane with AB, the line to be meaſured; then we muſt take at C, the three angles ACB, BCD, and ACD; and at D the three angles CDA, ADB, and CDB; and meaſure CD; and then proceed in the very ſame manner, to find AB, as before. And this may be done two ways, 1. finding AC and CB, and then AB; or 2. finding AD and DB, and then AB. And if it be done both ways it will be a proof of the work. And this method is univerſal, however the ſtations C, D, be taken; and whatever poſition the line AB is in, whether parallel to the horizon, or perpendicular, or any way inclined; provided that A and B can but be ſeen from both the ſtations C and D. Fig. 64.

PROB. X.

To take an inacceſſible diſtance AB, *when both ends of it,* A *and* B, *cannot be ſeen from any two ſtations.*

Aſſume ſome ſtation D, from which both A and B are viſible. Take two other ſide ſtations, C and E, ſo that A may be ſeen from C, and B from E; and both C and E from D. Then by Prob. VIII. find the inacceſſible diſtance DB from the two ſtations D, E; and the inacceſſible diſtance DA, from the ſtations D and C; which is to be done thus. Meaſure the diſtance DE, and the angles BDE, BED, from which find the ſide BD. Alſo meaſure CD, and the angles ADC and ACD; from which the ſide AD will be found. Then meaſure the angle ADB; and in the triangle ADB, there are given the two ſides AD, BD, and the included angle ADB, to find AB as required. 65.

The conſtruction of this, by rule and compaſs, will be eaſy. From any point D, draw DE of a proper length, and at D make the angles EDB, BDA, and ADC; on DC ſet its meaſured diſtance to C. Then at E, make the angle DEB; and at C, the angle DCA. Then the lines EB and CA will cut DB and DA in B and A; therefore AB drawn and meaſured, will be the diſtance ſought.

Fig. 66.

Or thus,

When no ſtation can be found, from which both A and B can be ſeen; take two ſtations C and D, ſo that A may be ſeen from C, and B from D. Then take two ſide ſtations E and F; by help of which, find the inacceſſible diſtances DB and CA (by Prob. VIII.) by meaſuring the diſtance DE, and angles EDB and DEB; and alſo CF and the angles FCA, CFA. Then meaſure the angles ACD, and CDB, and the ſtationary diſtance CD. Then in the triangle ACD, the ſides AC, CD, and included angle ACD being given; the ſide AD and the angle ADC will be found; and ADC ſubtracted from BDC, gives ADB. Then in the triangle ADB, the two ſides AD, DB, and the included angle ADB are given; to find AB the diſtance required.

The conſtruction is eaſy enough. Draw CD, and make the angles FCA, ACD, CDB and BDE; and ſet off the diſtances CF, DE; then make the angles CFA, DEB; and FA, EB, will interſect CA, DB, in A and B; and AB is the diſtance ſought.

Other Problems concerning heights and diſtances might be deviſed, from the various ſituations that objects may be ſuppoſed to have. But the foundation of all is contained in what has been here laid down; and to add more, would only confound the memory, and ſwell the book to little purpoſe.

PROB. XI.

To find the level of two places; or to find the aſcent or deſcent from one place to the other.

This is beſt done by a ſpirit level with teleſcopic ſights; which may be ſet level by ſcrews, which raiſe or lower one end of it. You muſt alſo have Fig. two poles, four yards long, with white papers or 66. marks upon them to ſlide up and down, and croſs the middle of the mark, a black line is drawn; and you muſt have two aſſiſtants to carry theſe two poles.

To take a level from A to I. Let one of the 67. men ſtand at A and the other at C, and at a convenient place as P, between A and C, place the level, and ſet it horizontal by help of the ſcrews. Let the man at A hold the pole upright in his hand at A, whilſt from P, you look through the ſights towards A, and cauſe the man at A to ſlip the mark up or down, till the black ſtroke on the mark at B, be level with the ſights; then meaſure BA. Then cauſe the other man to ſtand at C, and hold the pole upright, then turning the inſtrument round at P, and looking towards C, cauſe him to raiſe or lower his mark, till thro' the ſights, you can ſee the black ſtroke at D; then meaſure the height above the ground CD. And the difference of the heights AB and CD, is the aſcent or deſcent, if there be no more ſtations. But if there are many ſtations make a table with two columns, one for the back ſtations, and the other for the fore ſtations. Now to proceed, ſet down the two heights AB, CD; and let the man at A go to F with his pole, and remove the inſtrument to Q, and level it; then direct the ſights towards C, and let the man at C ſlip his mark up or down till the black line appears thro' the ſights, as at E; and meaſure CE. Then turning the inſtrument about, look towards F, till thro' the ſights you ſee the black mark at G, and meaſure GF; in the mean time the man at C removes to I, and placing the inſtrument at R, direct towards F backward, and to K forwards. And ſo proceed from ſtation to ſtation to the end. And ſet down the meaſures you took of the back ſtations,

Fig. 67. ſtations, AB, CE, F*o*, and the fore ſtations CD, FG, IK, in their proper columns. And at laſt ſum up both the columns, and take the difference of them; and if the fore ſtations exceed, you have a deſcent; but if the back ſtations exceed, it is an aſcent. If they be equal, you have a level.

Sta.	Back ſta.		Fore ſta.	
	f.	in.	f.	in.
1	4	0	2	6
2	5	4	7	3
3	0	0	3	0
	9	4	12	9
			3	5

So as the fore-ſtations are greater, there is a deſcent of 3 feet 5 inches, from A to I. But in the length of a mile, 8 inches deſcent muſt be allowed, upon account of the earth's curvature; that is, if you meaſure along the tangent, at the point A. But if the water is to run, a greater allowance muſt be made, in proportion to the velocity it is to run with. And thus it will be known whether or no, water can be conveyed from one place to another. But it is difficult to take a height, the length of a mile to a few inches. For many propoſitions may be firm and true in geometrical rigor; which reduced to practice amount to nothing at all.

Otherwiſe thus.

To know if water can be conveyed in an open channel; take two boards half a yard long; make two holes in the middle of them, in which ſet up two ſmall ſticks, perpendicular, and exactly of the ſame height; at the tops of which, fix white marks, or pieces of paper. Set the boards in the water to float, at a competent diſtance from one another, the further the better. Then as far from the pond as you can, ſet up a third ſtick, with a mark to ſlip up or down, which muſt be in a line with the other two. Then looking along the tops of the two firſt ſticks, ſlip the mark on the third, up or down, till it be exactly in the ſame horizontal line. Then meaſuring the diſtance of the mark from the ground, and comparing it with the length of the ſticks, you will know, which place is higher, and how much. And thus you may proceed to further places, by ſetting up more ſticks, or removing the boards thither. Fig. 67.

After this manner ſetting your boards in a ditch, and a ſtick at a good diſtance, with a mark on it, in a line with the tops of the other two; you will find if the ground deſcend.

If water is to be conveyed to any aſſigned place, in an open channel, no part of the channel muſt be lower than the place it leads to; for if it is, the water can never riſe again to the height of the laſt place. Therefore, that you may always cauſe it to go by an eaſy deſcent, carry the trench by any turnings and windings, by the ſides of hills or otherwiſe, keeping near a level. Not being ſolicitous to have it go ſtreight, tho' the nearer to a ſtreight line the better, if the ground will permit. But if there happen to be any valleys or low grounds; the water muſt be carried over them in troughs, ſupported by bridges or pillars. And if a hill interferes, it muſt be cut through to the ſame level.

But if water be to be conveyed in cloſe pipes, you may deſcend into valleys, becauſe water in the pipes will riſe to the ſame height. But where there is a hill it muſt be cut thro'. If the deſcent be great, the pipes muſt be laid one up, and another down; or in and out, to check the violent motion of the current; and not in a right line.

Fig. 67. If a canal is to be cut thro' a country, for the benefit of carriage; be ſure to take it from a place high enough to carry the water over the intermediate places; and where there is plenty of water. For you may deſcend when you will, but you cannot riſe; by this means, by turnings and windings, you may conduct it over any riſing grounds. Yet it will often be better to build bridges over hollow places, to carry the canal, than go too far about. And when there is plenty of water, the navigation will be quick; and the water, loſt by opening the locks or flood gates, will ſoon be ſupplied. But if water be ſcarce, theſe defects and loſſes cannot be ſupplied readily enough, which will retard the navigation, and prolong the time; and therefore ſuch a canal can never anſwer the purpoſe.

And tho' we muſt neceſſarily take the water from a height at leaſt equal to the place, where the goods muſt be taken up or laid down; yet it will be a fault to take it higher; becauſe then we ſhall have a greater deſcent to make; the conſequence of which is, that we muſt have more flood gates. And the number of flood gates is known by dividing the whole deſcent by the deſcent of one. And the deſcent at one flood gate may be 4, 5, or 6 feet, according to the bigneſs of the veſſels, and the largeneſs of the canal

68. To illuſtrate the nature of this; let FA be the part of a canal, EC, DB, two flood gates, which may be opened and ſhut at pleaſure; let FED be the ſurface of the water, when the gate DB is ſhut, and EC open. And CBA the ſurface of the water, when EC is ſhut, and DB open. Now a veſſel coming up the part AB, has to make an aſcent into the part EF. In this caſe DB muſt be open, and conſequently the veſſel comes into the part BC, upon the ſame level; and when ſhe is got in there, the gate DB is ſhut, and EC opened, and then the part ECDB inſtantly fills from the part EF; and the veſſel riſes with

the water from the level CB to the level ED, and the Fig.
gate EC being open, ſhe can go along the level ſur- 68.
face of the water DEF, and thus ſhe has aſcended from the height of AB to the height of EF. And then ſhe can go along EF till ſhe meet with another flood gate, to make another riſe.

In like manner a veſſel coming from F towards E, if EC be open, ſhe goes into the part ED. Then ſhutting EC and opening DB, ſhe goes out of EB, into the part BA, and ſo away.

It may be obſerved, that both gates muſt never be open at once; for then the water in FE would flow into BA, and would all be loſt. And when the gate DB is ſhut, and EC opened, the part EB is filled inſtantly, without ſenſibly decreaſing the height of EF. For EF is a long part of the canal, and EB a very ſhort part, and ſoon filled. And as the loſs of water, is the quantity ECDB; therefore the gates E and D ought to be ſet as near together as they can, to take in one or more veſſels.

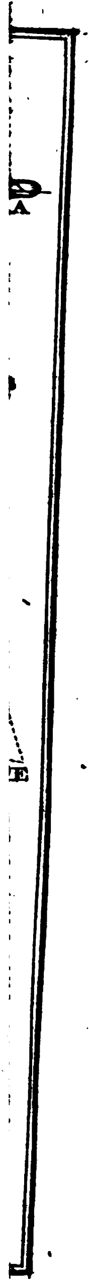
I ſhall only add this; when a canal is to be cut thro' a country, it is often attended with difficulties that make it impracticable. For in digging the canal, they often meet with rocks, thro' which they can hardly make any progreſs; and when that is done, the clefts of the rocks will probably take away all the water. Likewiſe when they meet with looſe ouzy ground; by its porous nature it drinks up, and deſtroys a great part of the water. And even when they meet with ſolid ground, yet the expence of digging, levelling, &c. is ſo great, and requires ſo much time to execute it; eſpecially when they are to go many miles in length; that conſidering all things, hardly any of them can afford a profit anſwerable to ſuch a monſtrous expence. When the ground is very much upon the deſcent, a great number of locks or flood gates are required, and therefore a great number of hands will be neceſſary, which

Fig. which loſes a deal of time; and ſuch accidents 68. make the thing extremely difficult, if not impoſſible.

And now having gone thro' all the principal branches of the Mathematics; I ſhall conclude with a few directions, to ſuch as deſign to make theſe Sciences their chief ſtudy; which are by far the moſt difficult, as well as the moſt uſeful parts, of human learning. And firſt, the young ſtudent muſt take care to be well inſtructed in Arithmetick; for this is the baſis of all manner of computations; not only in theſe ſciences, but in all ſorts of buſineſs. In the next place, he muſt get a complete knowledge of Geometry; for on this depends all computations relating to lines, which are infinite. Without a thorough knowledge of this, he will be obliged continually to turn back, or elſe ſtick faſt in the road. Theſe two branches lay the foundation of every thing that is to follow. Then the practical parts may take place, and Trigonometry and the doctrine of the ſphere may be read, which make way for Aſtronomy. And it will be proper then, to ſtudy the Conic Sections, and the higher Geometry, which conſiders the nature of all ſorts of curves. As Algebra is a univerſal ſcience, and applicable to any thing; he may begin to read that, after he is acquainted with Arithmetic and Geometry; but then he can apply it to nothing but numerical and geometrical problems; till ſuch time as he has learned the principles of any other ſcience, that he would apply it to. And in this book of Algebra, there is a ſufficient fund for exerciſing all the branches of the Mathematics. Fluxions being a ſcience of the higheſt nature, and greateſt extent, cannot be attained without the knowledge of all the reſt; and particularly Algebra, by which all computations therein are performed. And this is the method that the young mathematician muſt proceed in, to make the beſt progreſs, and in the Fig. leaſt time. But he muſt frequently expect to meet with obſtacles and difficulties, thro' which his own natural ſagacity muſt carry him, joined with the delight he finds in the knowledge of theſe ſciences. For I doubt he will meet with no other reward for his induſtry, nor even for any diſcoveries he makes, but the pleaſing ſatisfaction of finding out truth. And it is this pleaſure that muſt be his main ſupport under the great fatigue of ſtudying theſe difficult ſciences. For when truth at laſt emerges out of a *chaos* of intricate calculations, and appears in its native brightneſs and luſtre, it poſſeſſes the whole mind; and the labour and fatigue of ſeeking after it inſtantly vaniſhes, and is no more thought on or remembered.

The little encouragement there is in this nation, for promoting theſe ſciences, is the reaſon, that few people go any great length this way. If they have gained ſo much knowledge, as to be able to teach a common ſchool, to get a living by, they think it ſufficient. Few make any farther progreſs; and if they do, they get the pleaſure and pains of it for their labour. Or when a man is eminent for his diſcoveries, perhaps he is dignified with the title of F. R. S. but then he is to pay a quarterly Ceſs for the honour of it. This is the way that ingenuity is rewarded in England.

As to this courſe of Mathematics, that I have here drawn up, it has been the work of my life; for in almoſt all the books I peruſed, I always found either a defect of matter or a defect of method; and ſometimes a redundance of things unneceſſary, and of little conſequence. This put me upon writing, firſt one branch, and then another, till I had got together a tolerable ſyſtem of mathematical learning; where I believe every branch is treated with as much perſpicuity, and contains as much matter, Fig. as any one can wiſh for, in that compaſs. For the whole is only deſigned as an introduction for young ſtudents, and to enable them to go thro' larger Treatiſes. Theſe things were all originally deſigned only for my own uſe, and have always proved extremely uſeful to me; and would have been much more ſo, if I had had them ſooner in life. And becauſe what has been ſerviceable to myſelf, I ſuppoſe will be of the ſame ſervice to others, under like circumſtances; therefore I determined to publiſh them. But before this could be done, I had them all to reviſe, correct, and methodize, and to add ſuch things as later improvements had found out; which has coſt me a great deal of time and labour. But then the reader is furniſhed with ſubjects, both eaſy and common, as well as difficult and ſublime; which I expect will fully anſwer his greateſt expectations. However, I have uſed my beſt endeavours. But, if not, I have taken a great deal of pains to no purpoſe; being ſenſible that without pains and patience nothing is to be done: *Sed tandem poſt ardua quies.*


A
E

APPENDIX.

Containing Corrections and Additions to all the foregoing Volumes.

ARITHMETIC.

PAGE 7, line laſt; (*read*) of them, taken all together.

Page 31, line 3, &c. The operation being wrong, correct it thus,

```
 C.  ſt.  lb.  C.  ſt.  lb.
6)72  6  11(12  1  1 the quotient.
  72
  --
  0   6)6(1
        6
        -
        0   6)11(1
               6
               -
               5 remains.
```

Page 79, l. 8 (*read*) *Ex.* 5.
ib. l. 17 (*read*) *Ex.* 6.
ib. l. 27 (*read*) In *Ex.* 5.
Page 80, l. 18 (*read*) In the 6th *Ex.*
ib. l. 22 (*read*) *Ex.* 7.

Pag. 104, l. 16 (*read*) *the ſingle rule of three.*

Pag. 114, l. 12 b (*read*) of 4 degrees of hardneſs, and $232\frac{1}{2}$

ib. l. 10 b (*read*) dig a trench, of 7 degrees

ib. l. 6 b (*read*) * 4 deg. —— 7 deg.

Pa. 146, l. 14 b (*read*) to the leaſt error; to be added, if the number was too little; but ſubtracted, if too great. Put out all the 2d article to the bottom of the page, as being needleſs.

Pa. 150, l. 1 (*read*) the correction ... 5.8694077.
ib. after l. 16, *add*,

Note, this rule will anſwer all queſtions that the ſingle rule of falſe will anſwer.

Page 156, l. 10 (*read*) 6 ..... 0.7781512
Pa. 157, l. 8 (*read*) ſquare of 426?
Pa. 170, l. 3 b (*read*) and 38 l. a year
Pa. 171, l. 3 (*read*) 38 ... 684
ib. l. 4, &c. correct it thus,

172)1774(10.32 months
172 .
54
51
3

P. 186, l. 12 (*read*) two the leaſt whole numbers
ib. l. 13, &c. (*read*) 5 and 3 every where for 365 and 219.
ib. l. 16, 17 (*read*) Therefore the time of meeting will be $3 \times 24\frac{1}{3} = 73$ days.

Or thus,

$$\begin{array}{lr} \text{C} = & 10 \\ \text{B} = & 08 \\ \hline \text{diff.} & 2 \\ \hline \end{array}$$

Then $2^{m} : 1^{d} :: 73^{m} : 36\frac{1}{2}$ days, the time in which C will overtake B again; then find the leaſt number (by Prob. III. Ch IV. B. II.) which $24\frac{1}{3}$, $14\frac{3}{5}$ and $36\frac{1}{2}$, meaſure; and that is the anſwer, viz. 73.

Pa. 223, l. 8 b (*read*), circulate or come about again, according

Pa. 229, at the end of Prob. I. *add*,

SCHOL.

Fig.

SCHOL.

If one or both numbers (in any operation) be fractional, reduce them to a common denominator. Then proceed with the numerators as before directed; and at laſt, ſubſcribe the common denominator.

All the examples throughout the book ſhould be in italic.

PROPORTIONS.

There wants Sect. I, II, III, all along in the running title.

GEOMETRY.

Page 26, 27, inſtead of Prop. XXV and XXVI. inſert theſe.

PROP. XXV.

If an angle A *of a triangle be biſected by a right line* AD, *which cuts the baſe; the ſegments of the baſe will be proportional to the adjoining ſides of the triangle;* BD : DC : : AB : AC. 1.

And if the external angle EAC *be biſected by the line* A*d*, *cutting the baſe in d; then the diſtances (of the interſection) from the angles, are as the adjoining ſides;* B*d* : C*d* : : BA : CA.

Produce BA, and make AE = AC, and draw the line CE; becauſe AE = AC, the $\angle$ACE = E (Prop. III.) = $\frac{1}{2}$BAC (Prop. I.) = BAD (hyp.); therefore DA, CE are parallels (cor. 3. Prop. IV); and therefore BA : AE or AC : : BD : DC (Prop. XII.)

Again, make C*e* = CA; then the $\angle$ A*e*C = *e*AC = EA*d* = FAB; therefore *e*C, AB are parallel; whence *d*B : *d*C : : AB : C*e* or CA.

Cor. 1. *If the ſides be as the ſegments of the baſe; the line* AD, *biſects the angle* A.

For ſince BA : AC or AE : : BD : DC; DA and CE are parallels (cor. 1. Prop. XII); and BAD = $\angle$E, and DAC = ACE = E (Prop. III). Whence BAD = DAC, and A is biſected by AD.

Fig.

1. Cor. 2. *If a line biſecting the vertical angle of a triangle, cuts the baſe; it will be*

As the ſum of the ſides, $BA + AC$:
To their difference, $BA - AC$ ::
So the baſe, BC :
To the difference of the ſegments, $BD - DC$.

For $BA : AC :: BD : DC$ (Prop. XXV), and $BA + AC : BA - AC :: BD + DC\ (BC) : BD - DC$ or $2DO$ (Prop. XIII. Proportion); where O is the middle of the baſe BC.

Cor. 3. *Hence* $DB : DC :: dB : dC$.

2, Cor. 4. *If* AB, AC, AD, *be continually proportional, and* AE *equal to* AC; *then* CE *drawn, biſects the angle* BCD.

For the triangles ABC, ACD are ſimilar; whence $CB : CD :: AB : AC$ or $AE :: AC$ or $AE : AD :: AE - AB\ (BE) : AD - AE\ (ED)$. Therefore (Cor. 1.) angle $BCE = ECD$.

PROP. XXVI.

3. *If an angle* A *of a triangle* ABC, *be biſected by a right line* AD, *which cuts the baſe; the ſquare of the biſecting line, together with the rectangle of the ſegments, is equal to the rectangle of the ſides;* $AD^2 + BDC = BAC$.

Produce AD, and make the angle $\angle DBP = DAC$. Then the 3 triangles CDA, PDB, and PBA are ſimilar. For $CAD = PBD = PAB$, and $CDA = PDB$ (2.I), whence $\angle C = P$, and $ADC = ABP$ (cor. 1. Prop. II). Therefore $CD : DA :: PD : BD$ (Prop. XIII), whence $DA \times PD = CD \times BD$ (12 Proportion). Again, $CA : DA :: AP$ or $AD + DP : AB$ (Prop. XIII); therefore $CA \times AB = AD^2 + DA \times DP$ (12 Proportion) $= AD^2 + CD \times BD$ (Ax 3).

Cor. *If the external angle* EAC *be biſected by the line* Ad, *cutting the baſe in* d; *then the rectangle* BdC $-$ *the ſquare of the biſecting line* Ad, *is equal to the rectangle* BAC.

Fig. 3. For produce dA to F, and make $\angle dBF = dAC$; then the triangles CdA, FdB, and FBA are ſimilar. For $CAd = dAE = FAB$; and $CdA = FdB$. Whence $ACd = AFB$, and $AdC = ABF$ (cor. 1. Prop. II.) Therefore, $Cd : dA :: Fd : dB$ (XIII); and $dA \times Fd = Cd \times Bd$ (12 Proportion). Again $CA : dA :: AF$ or $dF - dA : AB$ (13). Therefore $CA \times AB = dA \times dF - dA^2$ (12 Proportion) $= Cd \times Bd - dA^2$.

PROP. XXVII, &c.

Page 73, l. 20, (*read*) *between the ſum and difference of the two radii*;
P. 107, l. 16, (*read*) DFKHCILG =
P. 121, l. 2, (fig. 191)
ib. l. 25, (*read*) DCH, be 3
P. 125, l. 18, fig. 195.
P. 126, l. 2, fig. 195.
ib. l. 8, fig. 196.
ib. l. 19, fig. 197.
ib. l. 4 b, fig. 198.
P. 127, l. 2, fig. 198.
P. 128, l. 2, fig. 198.
ib. l. 2 b, fig. 199, 200.
P. 129, l. 2, fig. 199, 200.
P. 137, l. 5, fig. 209.
P. 156, l. 2, (*read*) *Practical Geometry, or The Conſtruction*, &c.
P. 157, 8, Prob. V and VI, (*read*) *To biſect, or divide a given*
ib. After Prob. VII, *add*,

Or thus,

If the angle be given in degrees; from the given point B, draw the line BD. Then upon a line of Chords extend your compaſſes from the beginning to 60 degrees (called the *Sweep of* 60), then ſetting one foot in B, with the other deſcribe the arch DC. Then extend from the beginning, to the number

Fig. of degrees given, and ſet that extent from D to C, upon the arch DC. Thro' C draw the line BC, and DBC is the angle required.

Page 162, at the end of Prob. XII. *add,*

Cor. *If it be required to ſet any number of equal parts upon a right line, it is done thus.*

Take with your compaſſes, ſo many equal parts from a diagonal ſcale, or any convenient ſcale of equal parts; and ſet that extent upon your line.

TRIGONOMETRY.

Page 100, after Prop. III. *add,*

Cor. 1. *If two right angled triangles, have the ſame or equal hypothenuſes; the perpendiculars, are as the ſines of the angles at the baſe; and the baſes are as the ſines of the angles at the vertex.*

For they are both in the given ratio of the hypothenuſe (AC) to the radius.

Cor 2. *If two right angled triangles have equal baſes, the perpendiculars are as the tangents of the angles at the baſe; and the hypothenuſes are reciprocally as the ſines of the angles at the vertex.*

For the perpendiculars and tan. oppoſite angles, are in the given ratio of the baſe to the radius.

And if B be the baſe; H, b, the hypothenuſes; V, v, the ſines of the angles or the vertex. Then by this Prop. as $V : B :: \text{rad} : H = \frac{B}{V} \times \text{rad}$; and $v : B :: \text{rad} : b = \frac{B}{v} \times \text{rad}$. therefore $H : b :: \frac{B}{V} \times \text{rad} : \frac{B}{v} \times \text{rad} :: \frac{1}{V} : \frac{1}{v} :: v : V$.

Cor. 3. *If two right angled triangles have equal perpendiculars; the baſes are as the tangents of the angles at the vertex; and the hypothenuſes are reciprocally as the ſines of the angles at the baſe.*

This is plain by Cor. 2, making the baſe to become the perpendicular. Fig.

Page104, l. 8 b, (*read*) greater ſide :

Page 154, after Prop. XX. *add*,

Corol. *In a right angled ſpherical triangle, if the hypothenuſe be a quadrant, one of the ſides will be aquadrant, and its oppoſite angle a right angle.* fig. 37.

Let BAC be the triangle, A the right angle, the hyp. BC a quadrant. AB (produced at leaſt) paſſes thro' the pole of AC ; and if the quadrant CB be ſuppoſed to revolve about C, it will cut AB in the pole of AC ; but B is the point where they cut ; therefore B is the pole of AC ; and conſequently BA, BC are both quadrants ; and A being a right angle, C will alſo be a right angle (by Prop. XIV).

Page 158, put out the two Corols. at bottom. 4.

Page 159, line 1, (*read*) Cor. 1.

ib. Inſtead of the two Corols. at bottom, inſert

Cor. 1,

In right angled ſpherical triangles ABC, ADF, *that have equal angles at the baſe ; then,*

1. *The ſines of the hypothenuſes, are as the ſines of the perpendiculars.*
2. *The ſines of the baſes are as the tangents of the perpendiculars.*
3. *The tangents of the hypothenuſes, are as the tangents of the baſes.*
4. *The coſines of the hypothenuſes are as the cotangents of the angles at the vertex.*
5. *The coſines of the baſes, are as the coſines of the vertical angles.*
6. *The coſines of the perpendiculars, are reciprocally as the ſines of the angles at the vertex.*

All theſe are eaſily demonſtrated by the help of the two laſt propoſitions ; by extending the ſides of the triangles to quadrants, after the manner of fig. 41, and finding the proportions, in the complemental triangles EDF, ADC, ECB ; which will come out as follows.

Fig. For rad : S.A : : S.AC : S.CB : : S.AF : S.FD.
4. and rad : tan. A : : S.AB : tan. BC : : S.AD : t. DF.
and rad : coſ. A : : tan. AC : tan. AB : : t. AF : t.AD.
and rad : *t*.A : : coſ. AC : : cot. C : : coſ. AF : cot. F.
and rad : S.A : : coſ. AB : : coſ. C : : coſ. AD : coſ. F.
and rad × coſ. A = coſ. CB × S.C = coſ. FD × S.F.

Cor. 2.

In right angled ſpherical triangles, AFB, AFD, *that have equal hypothenuſes; then,*

1. *The ſines of the baſes are as the ſines of the vertical angles.*

2. *The tangents of the baſes, are as the coſines of the angles at the baſe.*

3. *The tangents of the angles at the baſe, are as the cotangents of the angles at the vertex.*

4. *The coſines of the baſes, are reciprocally, as the coſines of the perpendiculars.*

For rad : S.AF : : S.AFB : S.AB : : S.AFD : S.AD.
and rad : t. AF : : coſ. FAB : t. AB : : coſ. FAD : t. AD.
and rad : coſ. AF : : t.FAB : coſ. AFB : : t. FAD : cot.AFD.
and rad × coſ.AF = coſ. AB × coſ. BF = coſ. AD × coſ. DF.

Cor. 3.

In right angled ſpherical triangles ABC, ABF, *having equal baſes; then,*

1. *The tangents of the perpendiculars, are as the tangents of the angles at the baſe.*

2. *The ſines of the angles at the baſe, are as the coſines of the angles at the vertex.*

3. *The ſines of the perpendiculars, are as the cotangents of the vertical angles.*

4. *The coſines of the perpendiculars, are as the coſines of the hypothenuſes.*

5. *The cotangents of the hypothenuſes, are as the coſines of the angles at the baſe.*

6. *The ſines of the hypothenuſes, are reciprocally as the ſines of the vertical angles.*

For,

$$\text{For, rad : S.AB} :: \begin{cases}\text{tan. CAB : tan. CB.}\\ \text{tan. FAB : tan. FB.}\end{cases}$$

Fig. 4.

$$\text{and rad : coſ. AB} :: \begin{cases}\text{S.CAB : coſ. ACB}\\ \text{S.FAB : Coſ. F:}\end{cases}$$

$$\text{and rad : tan. AB} :: \begin{cases}\text{cotan. ACB : S.CB.}\\ \text{cotan. F : S.FB.}\end{cases}$$

$$\text{and rad : coſ. AB} :: \begin{cases}\text{coſ. CB : coſ. AC.}\\ \text{coſ. FB : coſ. AF.}\end{cases}$$

$$\text{and rad : cotan. AB} :: \begin{cases}\text{coſ. CAB : cotan. AC.}\\ \text{coſ. FAB : cotan. AF.}\end{cases}$$

And $\text{rad}\times\text{S.AB} = \text{S.ACB}\times\text{S.AC}$, or $= \text{S.F}\times\text{S.AF}$.

Note, either ſide may be taken for the baſe.

Page 196, after line 7, *add*,

If the ſolution is neither found in ABF nor EDF; produce the ſides from F, thro' A and B to quadrants as before, to find a 3d triangle; and then the ſolution will be found in this new complemental triangle

P. 208, l. 6 b, (*read*) tan. $\frac{1}{2}$AB.

p. 211, l. 7, (*read*) let fall FI a perp.

p. 213, l. 6 b, (*read*) if AF + FB < 180.

p. 239, l. 4 b, (*read*) proportionals between

Tab. natural ſines.

8° 38′——ſine .1501106

24°, at bottom right hand, (*read*) 114 deg.

41° 5′——tan. 84 35

43° 53′——ſine .6931922

Tab. artificial ſines.

15° 60′——ſine 9.4403381

29° 60′——coſine 5306

Tab. Logarithms.

Num. 354

Num. 987

1927——log. 8817

Num. 2804

ALGE-

Fig.

ALGEBRA.

page	line	read
4	7 b	compound quantity; and the ſum taken all together.
47	1 b	$= \frac{\sqrt[n]{a}}{\sqrt[n]{x}}$.
119	9 b	$agc - bcf$.
148	4	wants a line underneath.
258	8	$a = \frac{d}{b + a} =$
282	6 b	$\frac{.26106}{10.598} =$
300	1	fig. 4.
328	3 b	$a = \frac{3s}{5} =$
336	8	$bfa - bdc$
342	13	$x^9 - x^4 = bx^6$
347	2	fig. 22.
353	9 b	$R^t - 1 = R^t$.
ib.	8 b	$\left(\frac{R^t}{rR^t}\, a = \right)$
358	6	2 miles, the ſecond 3, the third 4,
	9	$\overline{x - 1} + 2 =$ his laſt
364	4	*of the means, or their ſquares* (c);
401	1 b	area, and AC $= y$.
408	1	fig. 54, put A for C, B for A, and C for B.
409	5 b	$+ \overline{a - c}^2 = 2aa + 2cc =$
413	6	$\overline{s + b} \times \overline{s - b} =$
	7	$z \times \overline{z - a} \times \overline{z - b} \times \overline{z - c}$.
430	12	$\sqrt{tt \times \overline{c + y}^2 -}$
441	1 b	GPV, VPM,
442	7	ſpherical triangle ABC,
465	5 b	(mechan. 56 and cor.)
	3 b	*leave out (ib. cor. 5.)*
466	3	$n \times \overline{\sqrt{2} - 1}$.

(mechan.

page	line		Fig.
477	1	(mechan. 27. cor. 2)	
	2	dele fig. 132.	
497	10	laſt term will.	
524	1 b	(mec, 18. cor. 3)	
525	16 b	(mec. 23. cor. 1)	
526	10	(mec. 23. cor. 1)	
	6 b	(mec. 23. cor. 1)	
		Poſtſcript.	
1	13	met with ſuch	
2	7 b	but Dr. Halley and Fr. Robarts, eſq; have	
	5 b	leave them	
		ARITHMETIC of INFINITES, &c.	
3	1	$= \frac{1}{4}n \times \overline{na}^3$, or	
	3 b	$s = \overline{\overline{n+1.a}}\vert^m$	
7	13	all the *x*'es	
12	11	$c = -A + 2b + C =$	
17	21,	&c. Prop. V. ſhould be italic	
29	4	and thus you may	
44	0 b	FINIS.	
		CONIC SECTIONS.	
9	17	$CA^2 : CN^2 :: BPA : PM^2$.	4
18	6 b	(XIII. Cor. 1) =	
20		at the end of Prop. 23. *add,* Cor. 1. *rectangle* $NMV = SMF - CE^2$. Cor. 2. $SC^2 : CE^2 ::$ *rectangle* $SMF - CE^2 : PM^2$. fig. 18. For (Prop. 19.) $CA^2 : CE^2 :: PN^2 : PM^2$, and $CA^2 - CE^2 : CE^2 :: PN^2 - PM^2 : PM^2$; that is, $SC^2 : CE^2 ::$ rect. $NMV : PM^2 ::$ (cor. 1.) $SMF - CE^2 . PM^2$.	
29	6	Cor. 1.	
		after line 7, *add,* Cor. 2. $FM : FT :: \sqrt{FMS} : CD$.	

Fig. page	line	read
		Cor. 3. *Draw* MO *perp. to the tangent* MH. *Then* $CI \times MO = CD^2$. For (cor. 2. Prop. 35.) $MO : CD :: CK : CA ::$ (cor. 1. of this) $CD : CI$.
34	5 b	$OD \times DN :: CF^2 : CK^2$.
56	7	*more lines el,*
60	2	(ib. Prop. III.)
131	4	multiplying by 4, $QZ^2 =$

PLATES.

Plate	iii	*f.* 28. draw MO perp. to MH.
plate	x	pa. 66, (at bottom).
pl xii	fig.	102, draw CD, PM.
plxxi		pa. 148, (at bottom).

CURVE LINES.

89	7	whence y^2
90		after line 5, *add,* Cor. *In a very ſmall part of any curve of finite curvature, as* AG; *the ſubtenſe of the angle of contact, is as the ſquare of the tangent, cord, or arch; for theſe in their evaneſcent ſtate, are all equal; that is,* $BG \propto AB^2$ or AG^2. fig. 86. *It is alſo obſervable, that the cords, ſines, tangents, verſed ſines,* &c. *of a very ſmall arch of a curve, have the ſame properties, as thoſe of the circular arch.*
94	12 b	line MF, are
-	7 b	Since $Mo = ms$,
95		at the bottom, *add,*

SCHOLIUM.

It may be obſerved, that the circle of equal curvature with the curve at M, is that which paſſes thro' three points of the curve, ſuppoſed to approach infinitely near one another.

For

page	line	read	Fig.
		For two points do not determine it, becauſe an infinite number of circles may paſs thro' two points. But there is only one that can paſs thro' three. And if a circle paſſes thro' two points that approach infinitely near, and at laſt coincide, it only denotes that this circle touches the curve in that point; which an infinite number of circles may do.	
97	3 b	$\dot{q} = \dot{p} + \ddot{p}$;	
98	14 b	*axis*, $\ddot{y} = 0$,	
99	2	$= y\dot{x} \times \frac{\dot{y}\ddot{y}}{\dot{z}}$	
114	6	$Mn = \dot{x}$, *mn*	
Pl. 8		fig. 100, wants o.	

OPTICS.

page	line	read	Fig.
85	5	$\frac{Cf \times x}{\frac{rr}{2Cf} - \dot{x}}$	
99		after Cor. 3, *add*, Cor. 4. *In different mediums, it will be*, AB : BC : : AD : DE.	
150	2	*the focal diſtance. And the ſame for a water Lens*, (by Schol. Prop. II.)	
159	14 b	attends old people,	
169	14	$+\left.\overline{\frac{q-s}{q+s}}\right\|^{2} t,$	
170	4 b	in the ſame point P; and a ſtop with a hole in it muſt be placed in the middle between AB and CD, or nearer AB, to cut off ſtraggling rays, that colour the object; and then the teleſcope is fit for uſe. The teleſcope——	
173	15	thro' the lens B,	

Fig. page	line	read
179	12	*power of a refracting teleſcope.*
181	3	*thro' a refracting teleſcope.*
184	2 b	*in different refracting te-*
185	15	*In ſimilar refracting teleſcopes ;*
193 194 195	1	fig. 116.
203	1	p. 203.
204	5	*leave out*, (by Prop. XVII. B. III.)
	9 b	to do; its magnifying
220	8 b	Microſcope, which is ſhorter. Where
224	7 b	focus of AB, it
pl.viii		fig. 106, GLI ſhould be ſtreight.
pl.ix.		fig. 107, put Q near the end of P.
pl. x		fig. 116, the two axes at C, D ſhould croſs one another.
pl.xiii		fig. 131, the ſquare LMIK to be prick'd.

PERSPECTIVE.

Fig. page	line	read
63	1	Diſtances of 13, &c.
pl. ii		fig. 16, the whole plane RGCQ ſhould be ſhaded.
		fig. 19, for *t* put *l*.
pl. ix		fig. 55, 1, 1 ſhould be at the angles above.
pl. x.		fig. 57, the prick'd lines 41, 1S, wanting, and ſome others.
pl.xv		fig. 77, produce R*l*, a little beyond *l*.

MECHANICS.

Fig. page	line	read
32		After line 11, *add*, Cor. 5. *The ſpace deſcribed by a falling body in the time of* 1 *vibration* : *is to half the length of the pendulum* : : *as the ſquare of the circumference* : *to the ſquare of the diameter.*

page	line	read	Fig.
		For the ſpaces being as the ſquares of the times, the ſpace fallen : ½length of the pendulum : : as the ſquare of the time of vibration : to the ſquare of the time of falling thro' ½length of the pendulum : : (by this Prop.) as the ſquare of the circumference; to the ſquare of the diameter.	
35	10	Cor. 1. Hence,	
145	14	Cone ABC	
146		after line 11 b, *add*,	
		I lately contrived another ſort of barometer, which ſhews the aſcent and deſcent of the mercury at the bottom.	
		ABC is a recurve tube, cloſe at the top, where the bucket C is, and open at the end A. The length of CB is 32 or 33 inches, and of AB 6 or 7. The bucket C ſhould contain about as much as the end AB. And the bucket and end CB muſt be quite filled with mercury, as far as B, a little beyond the turn. The wider the bucket C is, the better. The ſcale ſet to the end AB, muſt be graduated downwards, for the mercury falls in this, when it riſes in the other ſort. This being placed againſt a wall, will ſhew the height of the mercury, as in the common ones. And this way is more commodious, as it ſaves the labour of clambering up, upon chairs, to ſee it, as one muſt do, in the common ſort, to ſee exactly.	7.
		A WATER BAROMETER	

PROJECTION.

5	1	fig. 2.	
8	18	fig. 5.	off

Fig. page	line	read
9	3 b	off the ſines.
13	17	fig. 12.
14	2 b	the two laſt lines ſhould be roman and indented.
	21	$CpA + CAp$;
15	3	ſhould be indented.
		3, 4, 5 lines roman.
16	5	ECI = CAD
18	4	if *p*, *q*, be
	12	harmonically
20	7	projection C,
	9	equal arches EF
21	15	$\frac{OG + OD}{2}$:
25	9 b	points A, B, G,
26	8	draw EB to the point B, and BF perpendicular to it, cutting CH in F. To &c.
27		After *l* laſt *add*, But if the circle DLE was to lie on the other ſide of DGE, the arch HI muſt be taken on the other ſide of H.
29	6 b	interſection P,
30	2	*g*C*t* required.
	4	circle DEH, or
	11 b	pole *p*, draw
		After l. 8 b, *add*, *For a right circle* ACB. Thro' C (fig. 34) draw DE perpendicular to AB, then E is the pole of AB. Set the number of degrees from D to H, and draw EGH, then GC contains the degrees required. Or if GC be given, draw EGH, then HD in degrees, is the meaſure of GC. And in like manner AG contains the degrees in AH, or AH is the meaſure of AG.

page	line	read	Fig.
31	7 b	be ſet from G to L, and LPE drawn, then,	
33	17	of this circle,	
36	6 b	TV, and	
	2 b	at S cut	
	9 b	any angle $Qst = Qt$.	
		Prop. V. read little *s* throughout.	
37		at the end of Cor. 2. *add*,	
		Cor. 3. *If* P *be the point where the great circles* PC, PF, *&c. meet,* (*as at the pole*); *and if* CB, *the radius of projection, be perpendicular to* CP, *at* C *the point of contact. Then the dividing center* A *of* PF, *is diſtant from* P, *the length of* PB.	5
		For ICA is perpendicular to PF, and $IA = FC$; whence $PA^2 = PI^2 + IA^2 = PI^2 + FC^2 = PI^2 + FI^2 + IC^2 = PC^2 + FI^2 = PC^2 + CB^2 = PB^2$, and $PA = PB$.	
38	2 b	to *b*S;	
41		after l. 16, *add*,	
		Or thus, ſet CD from I to F, and FC from I to A, and draw AP, &c.	
43	2	CL = radius	
44	20	A to *d*,	
45		after l. 17, *add*,	
		Note, the dividing center of all the lines, PM, PL, PN, &c. is always diſtant from P, the length of PW. Therefore all the dividing centers are in the circumference of a circle, whoſe radius is PW, and center P; ſee below, Cor. 2.	
46	12	or $px = P\lambda$;	
	8 b	Cor. 1.	
		after Prop. 15, Cor. 1, *add*,	

Fig.	page	line	read
6.			Cor. 2. *If* P *be the point where the great circles*, AP, BP, *&c. meet; then to find the dividing centers of all the circles* AP, BP, *&c.* Thro' C the center of projection, draw PCG. Thro' C draw CW perpendicular to PG, and equal to the radius of projection. Draw PW, and WG perpendicular to it, to cut GP in G. At G and the middle of GP, erect the perpendiculars GA and DS. With the radius PW deſcribe the circle EWO. Then to find the dividing center of any circle AP; with the diſtance LP, and one foot in L, cut the circle OWE in R, the dividing center required. For ſince PWG is a right angle, PG is 90 degrees, and ſo is PA. Therefore the dividing center of AP is in a ſemicircle deſcribed round AP, and it is alſo in the circle OWR (by Cor. 3. Prop. V.), and therefore at the interſection R. But DS biſects all theſe circles; and therefore the center of that ſemicircle is in L.
	47	6 b	of meaſures GH, and equal to the radius of projection. To
	49	2, 3	by reaſon ſeveral
	57	11 b	projected from what is The plates ſhould be paged at the bottom; pl. I, 10. II, 18. III, 22. IV, 24. V, 30. VI, 38. VII, 48. VIII, 50. IX, 54. X, 56. XI, 56. Pl. VII, fig. 44, wants F, and FC.

CENTRIPETAL FORCES.

	1 Pref	11	irregularities only
	1	11	drawn or impelled to

page	line	read	Fig.
13	6	moved from A thro' C	
	1 b	$=\pi t\sqrt{\frac{2AC}{p}}$.	
16	14	whence A*a* × *b*A +	
22		after line 5, *add*, Cor. 3. *If* R *be the radius of curvature in* I, *the centripetal force at* I *will be as* $\frac{d}{p^3R}$ or $\frac{SI}{R\times SP^3}$. For $R=\frac{SI\times IN}{Pq}$, and $\frac{Pq}{IN}=\frac{SI}{R}$; whence the force $\frac{Pq}{PS^3\times IN}=\frac{SI}{R\times PS^3}$.	
28	8	Then univerſally the force is always as	
	10	But *ms* or *rs* is	
43	15	Such motions as	
59	19	P♎C and P♈C.	
65	1	4, R recedes from M,	
66	4	Curve RDB.	
78	6	its axis 1B*z*	
		Pl. I. fig. 5. the body *m* ſhould be ſhaded.	
		Pl. II. fig. 23. P*n* muſt be perp to F*p*.	

ASTRONOMY.

page	line	read	Fig.
103	12	fig. 90.	
	3 b	90th degree of	
130	1	fig. 31.	
162		after l. 4, *add*, Cor. 6. *The horizontal parallax of one body ſeen from another, is equal to the apparent ſemidiameter of this other ſeen from the firſt.* For the parallax of P is the angle at P made by the lines AP, CP continued	

Fig.	page	line	read
			to the fixt ſtars, and that is equal to the alternate angle APC, under which the radius AC is ſeen, (by Cor. 2), which will be the horizontal parallax, when PAC is a right angle. (fig. 48.)
	168		after line 5, *add*, 1. *To find it by Calculation.*
	193	8 b	Dec. 30 20 26, 1742
	199	8	(becauſe
		9	= 3.1416)
	205	7	the planet's angular
		13	the planet's angular
	225	5 b	is to its
			after Cor. 3. *add*, Cor. 4. *The velocity at any place* N : *is to the velocity in a circle at the mean diſtance* SA : : $\sqrt{NF}$: $\sqrt{NS}$. For (by Cor. 5. Prop. 16, Centripetal Forces) the velocity in the ellipſis at A, is equal to the velocity in a circle at the diſtance SA.
	233	1	fig. 78.
		1 b	♀'s geocent. place
	243	15	as *p* and P,
		10	⊕ place
	266	6	or PΓ,
	267		after Cor. 2. *add*, *Or thus.* Let a = apparent diameter of the central body ſeen from the revolving body, and p = the periodic time. Then (Prop. 19. Centr. Forces) the denſity is $\frac{1}{a^3p^2}$. For a is the parallax of the revolving body (by Cor. 6. Prop. 16. Sect. IV.) And the quantity p is known for all the planets and their ſatellites ;

and

page	line	read	Fig.
		and a may be eaſily found for them all. Whence the denſities are had, without any regard to the ſun's parallax or diſtance.	
		Cor. 3.	
285	13	moved backward.	
318	17	diſtance $ms = MS - Mo + Sp$, very nearly, when MS is large.	
319		after line 3, *add*, But when MS is ſmall or Mm large, Mo will need ſome correction. Therefore to correct it, take $mo = \sqrt{Mm^{2} - Mo^{2}})$, and find its tangent. Then Tan. MS ; tan. mo :: $\frac{1}{2}mo$: correction, which muſt be ſubtracted from Mo, when the angle M is acute ; but added when obtuſe. When Ss is conſiderable (which ſeldom happens), find Sp ($\sqrt{Ss^{2} - Sp^{2}}$), and add it to mo, before you make the proportion. When DM is greater than a quadrant, the ſigns of Mo and this correction, will be contrary to what they were. The reaſon of this operation in correcting Mo, is, that in the former computation mo is perpendicular to MD, whereas Dmo ſhould be an iſoſceles triangle ; and this correction only ſhews how far we are to go beyond o, or how much Mo is to be ſhortened.	
320	16	point s is	
		After the Scholium, *add*, *Otherwiſe thus.* The foregoing method will, in moſt caſes, give the anſwer exceeding near the	

the truth; but being only an approximation, and gained piece by piece, it will deviate a little in ſome particular caſes, as all other patched up methods will do. And therefore it will be better to uſe the following general and exact method, by help of the natural ſines.

Let $Z =$ coſ. $ZM + ZS$, $Z =$ coſ. $Zm + Zs$.
$C =$ coſ. $ZM - ZS$, $c =$ coſ. $ZM - Zs$.
$B =$ coſ. obſerved baſe MS, $b =$ coſ. true baſe ms, required.
$P = c - C$, $Q = z - Z$, $R = C - Z$, $S = C - B$, then
$b = B + P + \frac{Q - P}{R} S$, obſerving the ſines, and that the coſines of arches greater than 90, are negative.

For by Cor. 1. Prop. 42. B. III. Trigonometry, $C - Z : C - B :: 2r :$ verſ.$<Z$ $= \frac{C - B}{C - Z} \times 2r$, and $c - z : c - b ::$ $2r : \frac{C - B}{C - Z} \times 2r :: C - Z : C - B$. Whence $\overline{c - 2} \times \overline{C - B} = \overline{C - Z} \times \overline{c - b}$. and $c - b = \frac{\overline{c - z} \times \overline{C - B}}{C - Z}$, whence $b = c - \frac{\overline{c - z} \times \overline{C - B}}{C - Z}$, which is one rule. But $b = c + \frac{c - z}{C - Z} \times \overline{B - C} = c + \frac{c - z + C - Z - C + Z}{C - Z} \times \overline{B - C} = c + \frac{C - Z}{C - Z} \times \overline{B - C} +$

$c -$

page	line	read	Fig.
		$\frac{c-z+Z-C}{C-Z} \times \overline{B-C} = c+B-C + \frac{z-Z-c+C}{C-Z} \overline{C-B} = B+P+\frac{Q-P}{R} S.$	

Examp.

Let $\begin{cases} MS = 2^\circ\ 20' \\ ZM = 80\ 21 \\ ZS = 78\ 46 \end{cases}$

moon's parallax	$56'$	$10''$
refraction	4	40
$Mm =$	51	30
Ss ſtar's refr.	4	40

$ZM = 80\ 21\ 0$
$ZS = 78\ 46\ 0$
$Zm = 79\ 29\ 30$
$Zs = 78\ 50\ 0$

		Coſ.
$Z =$	159 7 0	— .9343082
$z =$	158 19 30	— .9292938
$C =$	1 35 0	.9996182
$c =$	0 39 30	.9999340

$P = c - C = .0003158$
$Q = z - Z = 0050144$
$R = C - Z = 1.9339264$
$S = C - B = .0004473$
$Q - P = .0046986$

.004698
000447
188
18
3
1.93)20.9(11
193
16

$B = .9991709$
$P = 3158$
quotient 11

$.9994878 = \text{coſ.}\ ms = 1^\circ\ 50'\ 2''$

Almoſt all the work conſiſts in ſetting down the data.

fig.

Fig. page	line	read
321	1	fig. 110.
331	0	p. 331.
333	7 b	from the node:
355	16	if x = number
	18	$27.54x = 29.53a$
	6 b	$\doteq x = 50.39^d$
356	7	$\doteq x = 239$ *rev.*
357		after l. 9, *add*, Put a, x, e, y for the periods of the change, apſis, node, and year; then any of them may be found as before, from the equations; $29.53a = 27.54x = 27.21e = 365.25y$.
367	8	ſooner at London; or weſt if later. Or it is eaſt, if it happen later in reckoning the hours; but weſt if ſooner. This

In the Plates.

Plate I. at bottom, pa. 8. Pl. II. pa. 24.
Pl. XI. pa. 260.
Pl. I. fig. 1, the bodies of the Sun, Saturn, Jupiter wanting.
Pl. VI. fig. 40, the lines ſhould interſect in B.

GEOGRAPHY.

page	line	read
4	17	length of days
5	9	*Aſcii.* At
	8 b	frigid zone, theſe
19	1,2,3b	longitude (for lat.)
20	11	and in both ways,
23	1	whoſe diſtance
40	9 b	pole, A and B
57	8	oblate ſpheroid
72	8 b	into a horrible

page	line	read	Fig.
79	9	eaſt by *Spain*,	
82	6	as the land of	
123	2 b	long night be	
128	14	dif. lat. AR,	
	4 b	velocity A*k*.	
130	13	$ts + sr$, &c.	
132	16	ſer than A*z*;	
137	10	whence $z = 2.30$	
144	10	$HI = BE \times \overline{1 + \frac{1}{2}qss}$.	
162	22	fig. 18.	
169		after Cor. 2, *add*, Cor. 3. *And from hence will be known how much the watch is too faſt or too ſlow; which now may be ſet to the true time.*	
		DIALLING.	
9	15	*tag, his rays.*	
16	17	*removed weſtward or eaſtward.*	
17	10	*carried back thro'*	
18	5	*ſituation, on that meridian.*	
58	11	fig. 20.	
	5 b	put out fig. 20.	
77	1	fig. 43.	
131	1	put out fig. 12.	
151		at bottom, *add*, See Cor. 3. Prop. V. Sect. III. Projection.	
163	14 b	Chambalu Pl. IX. fig. 34, 9 and 3 are wrong placed; they ſhould ſtand where *qb*, *qd* cut AB.	
		COMBINATIONS.	
25	5 b	but there ſhould	

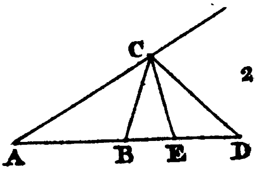
MENSURATION.

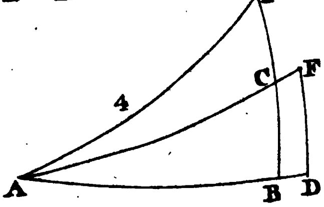
Fig. page	line	read
26	4	$37^{l}\ 12^{s}\ 6^{d}$
66	12	$3 ; \frac{9}{2}$
69	1	fig. 8.
89	o b	dele *Examp.*
118	12	— 1.895091
125	12	9 176.16

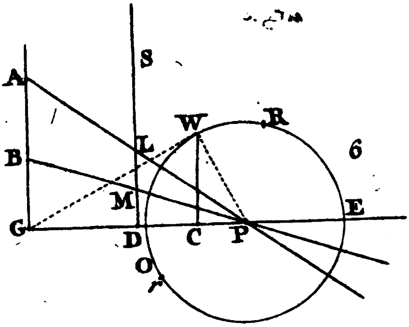
SURVEYING.

38	19	angle *abc*, and [illegible]
89	12 &c.	880 440 110 1.6280

FINIS.







the end